

TEXTBOOK

Yan Liu
Liang Li *Editors*

Principles of Environmental Engineering



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Preface

In the face of increasingly serious global environmental problems and increasing requirements for environmental quality due to improvements in living conditions, the environmental engineering discipline has gradually developed and improved into a new independent discipline since the 1960s in terms of water supply and drainage engineering, chemical engineering, biological engineering, and so on. Therefore, the principles of environmental engineering have not only derived from the main principles involved in water supply and drainage engineering, chemical engineering, and biological engineering, but they also combine the characteristics of environmental engineering itself.

“Principles of Environmental Engineering”, as a professional basic compulsory course for undergraduates majoring in environmental engineering, environmental science, and environmental management at the Department of Environmental Science and Engineering of Fudan University, has been offered for 23 years since 2000. The teaching contents of the “Principles of Environmental Engineering” course derive from hydraulics, hydrodynamics, chemical principles, chemical reaction engineering, and bioengineering, which are closely related to environmental engineering and have been combined with the characteristics of environmental engineering. At the same time, repetition has been avoided with professional courses such as “water pollution control”, “air pollution control”, “solid waste treatment and disposal”, and “ecological restoration engineering”. This course coincides with the teaching concept of fewer teaching hours and a wide foundation at Fudan University, and a handout entitled “Principles of Environmental Engineering” has been edited and used at Fudan University for the teaching of the course since 2004.

China Science Publishing & Media Ltd. (Science Press) published *Principles of Environmental Engineering* in Chinese in October 2018 and printed it three times in October 2018, July 2019, and October 2019 after the handout was revised. The book is suitable for undergraduate students majoring in environmental engineering, environmental science, environmental management, water supply and drainage, and other related majors at universities, especially those with fewer teaching hours. It can also be used as a reference book for postgraduate students of related majors and other professional technicians and scientific researchers engaged in environmental

protection. The contents are suitable for a course of 48 ~ 64 teaching hours. The textbook is divided into five chapters, mainly covering the flow and transport equipment of fluid in pipes and open channels, heat transfer, absorption, reaction kinetics, and basic concepts of reactors, as well as their applications in environmental engineering. At the same time, the readers will learn the basic viewpoints and methods commonly used in engineering technology, such as the balance method, reasonable simplification, dimensional analysis method, boundary layer theory, optimization, and mathematical modeling methods. This information can improve the ability to understand and solve problems in environmental engineering and increase student awareness of industrialization. The textbook involves a large number of symbols, which are unified and listed in the appendix for easy reference, and basic terms and concepts are also listed in the appendix.

Principles of Environmental Engineering was edited by Yan Liu and Liang Li as editor-in-chief, as well as Prof. Guangtuan Huang, Dr. Ruihua Dai, and Prof. Dong An as subordinate editors. The undergraduate and postgraduate students in the Department of Environmental Science and Engineering, Fudan University, also participated comprehensively in related works of the handout and the textbook, including editing, drawing figures, and inputting words and equations. They are Yunlu Chen, Chenxi Li, Shuya Li, Ling Lin, Yun Zhang, Yangwei Yan, Xiaosong Cha, Jin Wu, Jia Li, Anan Song, Yuanlong Huang, Chunlin Li, Haowen Lu, Jingchao Han, Yuhang Jing, Sanbao Zhang, etc. A hearty thanks to all of them here!

The English version of *Principles of Environmental Engineering* was translated by Yan Liu and Liang Li, editor-in-chief of the Chinese version, as well as Chenxi Li, a Ph.D. student at the Department of Environmental Science and Engineering, Fudan University. Other postgraduate students in Fudan University also participated related works for finding errors, giving their suggestions for the revision of errors, and polishing the English for author proofs. They are Xinyue Cao, Shenghan Cheng, Wanlin Lei, Ganlin Xia, Yuhao Zhu, etc. Thank them very much! Thanks to both Springer Nature Singapore Pte Ltd. and China Science Publishing & Media Ltd. (Science Press) as well as their related editors!

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Some teaching materials, monographs, manuals, norms, papers, articles, and network materials have been referred to and used in the compilation of this textbook without attribution. Please kindly understand and accept our heartfelt thanks to these intellectual property owners.

We would like to thank the Department of Environmental Science and Engineering of Fudan University for its support and help in publishing *Principles of Environmental Engineering* in both Chinese and English versions.

Due to the limited knowledge level and writing ability of the editors, mistakes and shortcomings are unavoidable. Readers are welcome to criticize and correct the text, and peer experts are welcome to give their advice.

Shanghai, China
September 2021

Yan Liu
Liang Li

Contents

1	Prolegomenon	1
	Yan Liu, Liang Li, and Chen-Xi Li	
1.1	Overview	1
1.2	Mass Balance and the First Law of Thermodynamics	1
1.3	Equilibrium and Rate	6
1.4	Units of Physical Quantities and Unit Systems	7
	Reference	11
2	Fluid Flow and Transport Equipment	13
	Yan Liu, Liang Li, Rui-Hua Dai, and Chen-Xi Li	
2.1	Overview	13
2.1.1	Methods for Investigating Fluid Flow	14
2.1.2	Basic Concepts of Fluid Flow	16
2.1.3	Internal Friction in Fluid Flow	20
2.1.4	Types of Fluid Flow and Its Discriminant Method	23
2.1.5	Flow Boundary Layer	29
2.2	Friction Loss of Fluid Flow in a System	33
2.2.1	Factors Affecting Friction Loss	34
2.2.2	Straight Pipe Friction Loss and Dimensional Analysis	35
2.2.3	Local Loss	44
2.3	Balance Equation for a Steady Flow System and Its Application	47
2.3.1	Mass Balance Equation	48
2.3.2	Energy Balance and Bernoulli Equation	50
2.3.3	The Calculation of Pipeline System	63
2.4	Open Channel Uniform Flow and Thin Wall Weir	81
2.4.1	Formation Conditions and Characteristics of Uniform Flow in Open Channels	83
2.4.2	Calculation of Uniform Flow in Open Channel	84
2.4.3	Thin Wall Weir	104
2.5	Common Measuring Instruments	108

2.5.1	Venturi Flowmeter	108
2.5.2	Ubbelohde Viscometer	109
2.6	Two-Phase Flow	112
2.6.1	Motion Resistance and Resistance Coefficient of Spherical Particles in Fluid	112
2.6.2	Gravity Settlement	115
2.7	Fluid Transport Equipment	120
2.7.1	Centrifugal Pump	121
2.7.2	Centrifugal Compressor, Root Blower and Jet Aerator	142
2.7.3	Reciprocating Compressor and Reciprocating Pump	146
2.7.4	Other Commonly Used Fluid Conveying Equipment	152
	References	167
3	Heat Transfer	169
	Yan Liu, Liang Li, Guang-Tuan Huang, and Chen-Xi Li	
3.1	Overview	169
3.1.1	The Basic Mode of Heat Transfer	169
3.1.2	Heat Transfer Problems in the Field of Environmental Engineering	170
3.2	Heat Conduction	171
3.2.1	Fourier's Law	171
3.2.2	Thermal Conductivity	173
3.2.3	Stable Heat Conduction of Flat Wall	177
3.2.4	Heat Conduction on Cylinder Wall	182
3.3	Convection Heat Transfer	188
3.3.1	Mechanism of Convection Heat Transfer	188
3.3.2	Convection Heat Transfer Rate	192
3.3.3	Empirical Equation for the Convection Heat Transfer Film Coefficient	194
3.3.4	Critical Diameter of Insulation Layer	204
3.4	Calculation of Heat Transfer Process	209
3.4.1	Heat Balance	209
3.4.2	Total Heat Transfer Rate Equation	210
3.4.3	Overall Heat Transfer Coefficient	213
3.4.4	The Mean Temperature Difference of Heat Transfer	215
3.4.5	Examples of Heat Transfer Calculation	221
3.5	Heat Exchangers	224
3.5.1	Direct Contact Heat Exchanger	224
3.5.2	Regenerative Heat Exchanger	225
3.5.3	Interwall Heat Exchanger	225
3.6	Measures to Strengthen the Heat Transfer Process of Heat Exchangers	236
	References	243

4 Absorption	245
Yan Liu, Liang Li, Guang-Tuan Huang, and Chen-Xi Li	
4.1 Overview	245
4.1.1 Mass Transfer Process in Environmental Engineering	247
4.2 Mass Transfer Mechanism as well as Mass Transfer Rate and Flux	250
4.2.1 Molecular Diffusion	251
4.2.2 Convection diffusion	263
4.3 Gas-Liquid Phase Equilibrium	266
4.3.1 Overview	266
4.3.2 Solubility Curve	267
4.3.3 Henry's Law	269
4.3.4 Phase Equilibrium and Absorption Process	274
4.4 Kinetic Basis of Absorption	277
4.4.1 Overview	277
4.4.2 Two-Film Theory	278
4.4.3 Absorption Mass Transfer Flux Equation	280
4.5 Design and Operation Calculation of the Absorption Process	295
4.5.1 Overview	295
4.5.2 Material Balance and Operating Line Equation of the Absorption Tower	297
4.5.3 Calculation of the Absorbent Dosage or Molar Flow Rate (L) and the Specific Absorbent Dosage (L/V)	300
4.5.4 Calculation of Packing Layer Height of Absorption Tower	305
4.5.5 Calculation of Theoretical Plates and Tower Height	319
4.5.6 Determination of Tower Diameter	325
4.5.7 Calculation of the Absorption Process	326
4.6 Desorption	329
4.6.1 Overview	329
4.6.2 Gas Stripping Desorption	329
4.7 Multicomponent Absorption and Chemical Absorption	336
4.7.1 Multicomponent Absorption	336
4.7.2 Chemical Absorption	338
References	361
5 Chemical and Biological Reaction Kinetics and Reactors	363
Yan Liu, Liang Li, Dong An, and Chen-Xi Li	
5.1 Chemical and Biological Reaction Kinetics	363
5.1.1 Classification of Reactions	363
5.1.2 Reaction rate and Equation	364
5.1.3 Homogeneous Reaction Kinetics	369
5.1.4 Biochemical Reaction Kinetics	388
5.2 Reactor	403
5.2.1 Flow Model of Material in Reactor	404

5.2.2	Reactor and Operational Mode	406
5.2.3	Calculation of Ideal Homogeneous Reactor	408
5.2.4	Optimal Selection of Ideal Homogeneous Reactor	426
5.3	Microbial Reactor	436
5.3.1	Batch Microbial Reactor	437
5.3.2	Continuous Microbial Reactor	440
	References	451
Appendix A: Physical Properties of Water (Cardarelli, 2018)		453
Appendix B: Important Physical Properties of Some Gases (Cardarelli, 2018)		457
Appendix C: Important Physical Properties of Some Liquids (Cardarelli, 2018)		461
Appendix D: Physical Properties of Dry Air (101.33 kPa) (Cardarelli, 2018)		465
Appendix E: Absolute Roughness of Pipe Wall (Cardarelli, 2018)		467
Symbols and Units		469
Index		517

Chapter 1

Prolegomenon



Yan Liu, Liang Li, and Chen-Xi Li

1.1 Overview

Continued developments in industry, agriculture, and services, as well as the rapid growth in population, the economy, and urbanization, have led to the generation of large amounts of pollutants. At present, it is impossible to achieve pollution control and environmental protection by solely relying on the self-purification ability of the earth. Therefore, environmental engineers are tasked with controlling pollution and protecting the environment by establishing environmental engineering facilities for the collection, treatment, and discharge of wastewater or polluted gases and wastes. The design, construction, and operation of these facilities require at least a basic knowledge of other disciplines, such as hydraulic, hydrodynamics, chemical principles, chemical reaction engineering, and bioengineering. The basic knowledge mentioned above forms the foundation and principles of environmental engineering. This creates the basis for further study of environmental engineering, including water and air pollution control, solid waste management, and ecological restoration engineering.

1.2 Mass Balance and the First Law of Thermodynamics

The mass balance based on the law of conservation of mass and the energy conservation based on the first law of thermodynamics form the basic principles, methods, and foundations for solving the problems in principles of environmental engineering.

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(1) Conservation of mass

To account for mass balance, the first step is to determine the scope of the system. Depending on the situation, either the whole system or a part can be selected as a defined mass accounting system. According to the principle of mass conservation (law of mass conservation), the total amount of input materials is equal to the sum of the total amount of output materials, the total amount of accumulated materials, and the total amount of reacted materials in the system for a certain time. The mathematical expression is as follows

$$\Sigma q_{m-in} = \Sigma q_{m-out} + \Sigma q_{m-accumulation} + \Sigma q_{m-reaction} \quad (1.1)$$

where

Σq_{m-in} —The total input materials of the system per unit time, kg/s

Σq_{m-out} —The total output materials of the system per unit time, kg/s

$\Sigma q_{m-accumulation}$ —The total accumulated materials of the system per unit time, kg/s

$\Sigma q_{m-reaction}$ —The total reacted materials of the system per unit time, kg/s.

The material in the equation can be either the quantity of a material (such as mass, molar, chemical oxygen demand, biochemical oxygen demand, etc.) or the quantity of elements (such as sulfur, oxygen, carbon, etc.).

Example 1.1 A large municipal wastewater treatment plant uses coagulation sedimentation coupled with a centrifugal dewatering process to treat suspended solids (SS) in municipal wastewater. Figure 1.1 shows a schematic diagram of the treatment process. The wastewater flow rate is $2.10 \times 10^6 \text{ m}^3/\text{d}$, and the concentrations of SS

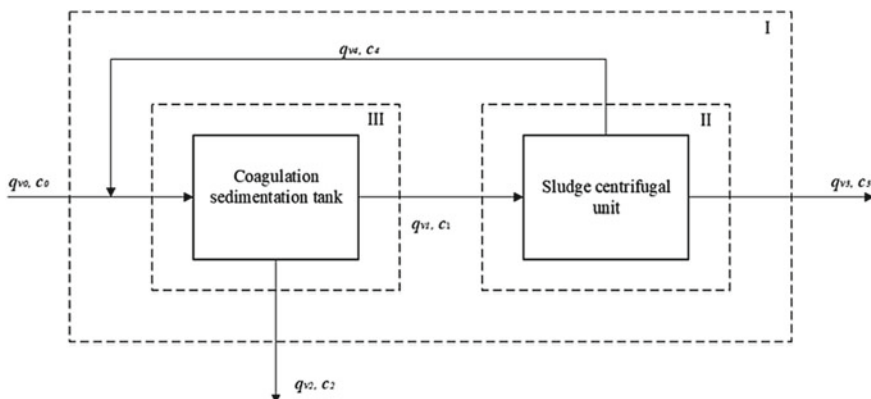


Fig. 1.1 I. Schematic diagram of the coagulation sedimentation-centrifugal dewatering process; II. Sludge centrifugal unit; III. Coagulation sedimentation tank

in the influent and effluent of the coagulation sedimentation tank are 240 mg/L and 15 mg/L, respectively. After coagulation and sedimentation, the sludge (water content 95%) enters the sludge centrifugal unit and is dewatered by a high-speed centrifuge. After dewatering, the sludge moisture content is 65%, and the solid content of the centrifuged liquid is 0.8%. The separated liquid is treated again after returning to the coagulation sedimentation tank. Assuming that the system is in a stable state, there is no biological effect in the process, and the amount of coagulant can be neglected. Please calculate the flow flows rates of the whole treatment system effluent and discharged sludge, as well as the effluent flow rates of the coagulation sedimentation tank and the backflow of the centrifuged liquid. The wastewater and sludge densities are both assumed to be 1000 kg/m³.

Known: $q_{V0} = 2.10 \times 10^6 \text{ m}^3/\text{d}$, $c_0 = 240 \text{ mg/L} = 0.24 \text{ kg/m}^3$, $c_2 = 15 \text{ mg/L} = 0.015 \text{ kg/m}^3$,

The water content of the sludge is the mass ratio of water in the sludge to the total sludge, and thus, the suspended solid concentrate in the sludge is as follows:

$$c_1 = (100 - 95)/(100/1000) = 50 \text{ g/L} = 50 \text{ kg/m}^3$$

$$c_3 = (100 - 65)/(100/1000) = 350 \text{ g/L} = 350 \text{ kg/m}^3$$

The solid content of the wastewater is the mass ratio of sludge solid to wastewater, and thus, the suspended solid concentrate in the wastewater is as follows:

$$c_4 = 0.8/(100/1000) = 8 \text{ g/L} = 8 \text{ kg/m}^3$$

$$\rho = 1000 \text{ kg/m}^3, \Sigma q_{m-\text{accumulation}} = 0, \Sigma q_{m-\text{reaction}} = 0$$

Try to calculate: q_{V3} , q_{V2} , q_{V1} , and q_{V4} .

Solutions:

- (1) System I (including the coagulation sedimentation tank and sludge centrifugal unit) is taken as the target of the mass balance calculation.

$$\Sigma q_{m-\text{in}} = \Sigma q_{m-\text{out}} + \Sigma q_{m-\text{accumulation}} + \Sigma q_{m-\text{reaction}}$$

$$\Sigma q_{m-\text{in}} = \Sigma q_{m-\text{out}}$$

Input flow rate $q_{m-\text{in}} = c_0 q_{V0}$.

Output flow rate $q_{m-\text{out}} = c_2 q_{V2} + c_3 q_{V3}$.

The following mass balance equations can be obtained:

$$\begin{cases} c_0 q_{V0} = c_2 q_{V2} + c_3 q_{V3} \\ q_{V0} = q_{V2} + q_{V3} \end{cases}$$

Substituted with relative data:

$$\begin{cases} 0.24 \times 2.10 \times 10^6 = 0.015 \times q_{V2} + 350 \times q_{V3} \\ 2.10 \times 10^6 = q_{V2} + q_{V3} \end{cases}$$

Result: $q_{V3} = 1.35 \times 10^3 \text{ m}^3/\text{d}$, $q_{V2} = 2.10 \times 10^6 \text{ m}^3/\text{d}$.

- (2) With system II (sludge centrifugal unit) as the target of mass balance calculation, the results can be obtained with the same method:

Input flow $q_{m-\text{in}} = c_1 q_{V1}$.

Output flow $q_{m-\text{out}} = c_3 q_{V3} + c_4 q_{V4}$.

The following mass balance equations can be obtained in the same way:

$$\begin{cases} c_1 q_{V1} = c_3 q_{V3} + c_4 q_{V4} \\ q_{V1} = q_{V3} + q_{V4} \end{cases}$$

Substituted with relative data:

$$\begin{cases} 50 \times q_{V1} = 350 \times 1.35 \times 10^3 + 8 \times q_{V4} \\ q_{V1} = 1.35 \times 10^3 + q_{V4} \end{cases}$$

Result: $q_{V1} = 1.10 \times 10^4 \text{ m}^3/\text{d}$, $q_{V4} = 9.64 \times 10^3 \text{ m}^3/\text{d}$.

Answer: The flow rates of the whole treatment system effluent and discharged sludge are $2.10 \times 10^6 \text{ m}^3/\text{d}$ and $1.35 \times 10^3 \text{ m}^3/\text{d}$, respectively. The effluent flow rate of the coagulation sedimentation tank is $1.10 \times 10^4 \text{ m}^3/\text{d}$, and the recycled flow rate of the centrifuged fluid is $9.64 \times 10^3 \text{ m}^3/\text{d}$.

(2) The law of conservation of energy and the first law of thermodynamics

Energy can exist in many forms. One form of energy can be converted to another form following the law of energy conservation. That is, the sum of all energy in a system is unchangeable. Energy can only be changed from one form to another or transmitted from one object to another within the system. According to the first law of thermodynamics, for any system, if the heat transferred from outside the system into it is Q , then the internal energy of the system changes from an initial state E_{n1} to a final state E_{n2} . At the same time, if the system doing external work is W , then no matter the process, there will always be:

$$Q = E_{n2} - E_{n1} + W \quad (1.2)$$

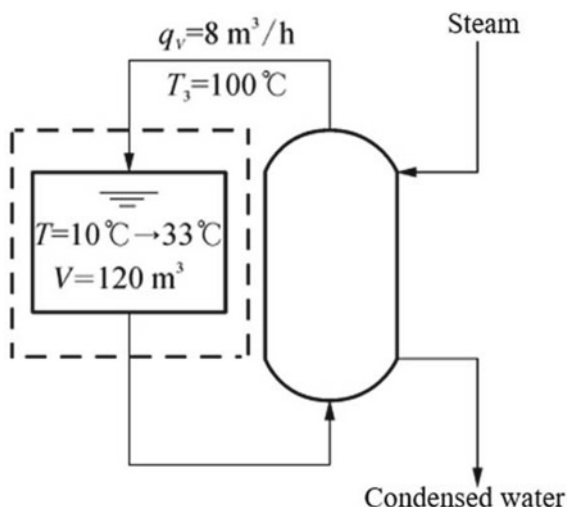
where

Q —The total heat absorbed by the system from the outside is positive, and the opposite is negative, J

E_{n1} —The internal energy of the system at the initial state, J

E_{n2} —The internal energy of the system at the final state, J

Fig. 1.2 Sludge heating system



W —The system receiving external work is negative, and the opposite is positive, J .
For differential state change processes, Eq. (1.2) can be written as follows:

$$dQ = dE_n + dW$$

The scope of the system and the reference benchmark need to be defined before the application of the first law of thermodynamics.

Example 1.2 As shown in Fig. 1.2, there is a continuous reaction tank with 120 m^3 of mixed sludge. This mixed sludge needs to be heated from 10 to $33 \text{ }^\circ\text{C}$ for mesophilic anaerobic digestion. By using external circulation, the sludge is heated by the steam in the heat exchanger with a flow rate of $8 \text{ m}^3/\text{h}$. Its outlet temperature is constant at $100 \text{ }^\circ\text{C}$. Assuming that the sludge in the tank is evenly mixed and the density of the sludge is 1100 kg/m^3 , how long does it take for the sludge to be heated to the required temperature (neglecting the heat dissipation of the tank and the energy loss in the sludge recycling process)?

Known: $V = 120 \text{ m}^3$, $q_v = 8 \text{ m}^3/\text{h}$, $T_1 = 10 \text{ }^\circ\text{C}$, $T_2 = 33 \text{ }^\circ\text{C}$, $T_3 = 100 \text{ }^\circ\text{C}$. Because the sludge is evenly mixed in the tank, the temperature of the sludge discharged from the tank at any time is the same as that in the tank, and it can be set as T .

The time required for the sludge to be heated to the required temperature is calculated.

Solution: The sludge tank is selected as the accounting system, and sludge at $0 \text{ }^\circ\text{C}$ is used as the temperature reference benchmark.

Within the time period of $d\tau$:

Input enthalpy of the system: $dH_F = C_p T_3 q_v \rho d\tau$.

Output enthalpy of the system: $dH_p = C_p T q_v \rho d\tau$.

Accumulated enthalpy of the system: $dE_q = C_p V \rho dT$.

Known: $d\mathcal{H}_p - d\mathcal{H}_F + dE_q = 0$. It can be deduced that $d\mathcal{H}_F - d\mathcal{H}_p = dE_q$. Thus,

$$C_p T_3 q_V \rho d\tau - C_p T q_V \rho d\tau = C_p V \rho dT$$

$$d\tau = \frac{V dT}{q_V (T_3 - T)}$$

Boundary conditions: $\tau_1 = 0, T_1 = 10^\circ\text{C}, \tau_2 = \tau, T_2 = 33^\circ\text{C}$

$$\int_0^\tau d\tau = \frac{120}{8} \int_{10}^{33} \frac{dT}{100 - T}$$

$$\tau = 15 \ln \frac{100 - 10}{100 - 33} = 4.43 \text{ h}$$

Answer: It takes 4.43 h for the sludge to be heated to the required temperature.

1.3 Equilibrium and Rate

Equilibrium and rate are two necessary factors used in the analysis of any unit operation process.

The equilibrium indicates the direction of the process and the limit that can be reached. If the process has reached equilibrium, it will no longer proceed. During an environmentally engineered reactor process, a disequilibrium state far from equilibrium usually transforms into another disequilibrium state near equilibrium. For example, if several liquids with different temperatures are poured into a container, as long as the temperature at any two places in space is different, that is, the temperature does not reach equilibrium state, the heat will be transferred from the high-temperature place to the low-temperature place until the temperature is the same everywhere, then this process reaches equilibrium. For example, in the condensation tube of the chemical oxygen demand (COD) measuring device, the temperature in the gas phase is higher than that in the liquid phase, i.e., the temperature does not reach equilibrium state, and the heat will transfer from the gas phase to the liquid phase. The temperature difference between the gas phase and liquid phase at the outlet must be less than that at the entrance, that is, from a disequilibrium state far from equilibrium (greater temperature difference at the entrance) to another disequilibrium state near equilibrium (smaller temperature difference at the outlet).

The rate of the process refers to the degree—fast or slow—of process conduction, which is the basis for judging how fast it can proceed from disequilibrium to equilibrium. If a process is carried out at a very slow rate, the volume of the equipment required for the process will be very large, which makes it difficult to apply in practical engineering.

The rate of a process is proportional to the driving force of the process and inversely proportional to the resistance of the process, which can be expressed as follows:

$$\text{The rate of process} = \frac{\text{driving force}}{\text{resistance}} = \text{process coefficient} \times \text{driving force} \quad (1.3)$$

The characteristics of the driving force depend on the process. For example, the driving force of the heat transfer process is the temperature difference, while the driving force of the gas absorption process is the concentration difference or pressure difference. Resistance is the overall reflection of the influence of various factors on the process rate. The process coefficient is the reciprocal of the process resistance.

1.4 Units of Physical Quantities and Unit Systems

Units of physical quantities are divided into basic units and derived units. The basic unit is several independent basic units of physical quantity. There are seven basic physical quantities commonly used in the international system of units (SI) and the corresponding basic units (see Table 1.1 for details): length (m), mass (kg), time (s), current (A), thermodynamic temperature (K), luminous intensity (cd), and quantity of material (mol). Physical quantities other than basic physical quantities can be derived from basic physical quantities according to the definition of physical quantities and the law between physical quantities. They are called derived physical quantities, and their units are called derived units. For example, the unit of length is “m”, which is a basic unit. The volume unit “m³” is a derived unit from the unit of length. In addition, there are some derivative units with proper names, such as Watt for power, but they have a relationship with the basic unit: 1 W is 1 kg·m²/s³. The unit for force is Newton, and 1 N is 1 kg·m²/s². Table 1.2 displays derived units for some international units commonly used in environmental engineering.

Apart from the international system of units that is widely used at present, there are two main types of units system: absolute unit system and engineering unit system. These two types of unit systems can be divided into British system and metric system (Cardarelli F., 2005). The main difference between the absolute unit system and the engineering unit system is that the absolute unit system takes mass as the basic physical quantity, its unit (kg) as the basic unit, and the unit of force (kg·m/s²) as

Table 1.1 Basic units of physical quantity in the international unit system (Hu et al., 2015)

Name of physical quantity	Symbol	Unit of physical quantity	Unit symbol
Length	<i>l</i>	Meter	m
Mass	<i>m</i>	Kilogram	kg
Time	<i>τ</i>	Second	s
Quantity of matter	<i>n</i>	Mole	mol
Electric current	<i>I</i>	Ampere	A
Thermodynamic temperature	<i>T</i>	Kelvin	K
Luminous intensity	<i>I_v</i>	Caldera	cd

Source of Table 1.1: Reprinted from Table 2.1.1 in book “*Principles of Environmental Engineering*” with kind permission from Prof. Hongying Hu

Table 1.2 Frequently used derivative SI units in environmental engineering (Hu et al., 2015)

Name of physical quantity	Name of unit	Unit symbol	Other unit symbol
Area	Square meter	m ²	—
Volume	Cubic meter	m ³	—
Velocity	Meter/second	m/s	—
Density	Kilogram/cubic meter	kg/m ³	—
Concentration	Mole/cubic meter	mol/m ³	—
Volume flow rate	Cubic meters per second	m ³ /s	—
Mass flow rate	Kilogram per second	kg/s	—
Flow rate of mole	Mole per second	mol/s	—
Specific volume	Cubic meter/kilogram	m ³ /kg	—
Force	Kilogram meter/second ²	kg·m/s ²	N
Pressure	Kilogram/(meter·second ²)	kg/(m·s ²)	Pa, N/m ²
Energy, work, heat	Kilogram·meter ² /second ²	kg·m ² /s ²	J, N·m
Power	Kilogram·meter ² /second ³	kg·m ² /s ³	W, J/s
Dynamic viscosity	Kilogram/(meter·second)	kg/(m·s)	Pa·s, N·s/m ²
Kinematic viscosity	Meter ² /second	m ² /s	S·t
Specific heat capacity	Meter ² /(second ² ·Kelvin)	m ² /(s ² ·K)	J/(kg·K)
Diffusion coefficient	Meter ² /second	m ² /s	—
Thermal conductivity	Kilogrammeter/(second ³ ·Kelvin)	kgm/(s ³ ·K)	W/(m·K)
Heat transfer coefficient	Kilogram/(second ³ ·Kelvin)	kg/(s ³ ·K)	W/(m ² ·K)

Source of Table 1.2: Reprinted from Table 2.1.3 in book “*Principles of Environmental Engineering*” with kind permission from Prof. Hongying Hu

the derived unit, while the engineering unit system takes force as the basic physical quantity, its unit (kgf) as the basic unit, and the unit of mass (kgf·s²/m) as the derived unit. The relationship between force and mass is

$$F_y = ma_j \tag{1.4}$$

where

F_y —Forces acting on an object

m —The mass of an object

a_j —Acceleration of an object in the direction of force.

Table 1.3 shows the relationship of the absolute metric unit system, the engineering unit system, and the SI. The value of the same physical quantity can be different in different unit systems, but it is “equivalent”. The conversion factor is the numerical ratio of the same physical quantity with different unit system quantities. For example, in the SI, the energy of 1 J (or 1 kg·m²/s²) is 0.1021 kgf·m in an engineering unit system, and its conversion factor is 0.102.

Table 1.3 The relationship of the same physical quantity under different unit systems (Hu et al., 2015)

Unit systems	SI unit		Absolute metric unit		Engineering unit
Physical quantity	Symbol	Name	CGS	MKS	
Length	m	Meter	cm	m	m
Mass	kg	Kilogram	g	kg	kgf·s ² /m
Time	s	Second	s	s	s
Temperature	K	Kelvin	—	—	—
Quantity of matter	mol	Mole	—	—	%
Electric current	A	Ampere	—	—	—
Luminous intensity	Cd	Caldera	—	—	—
Force	kg·m/s ² = N	Newton	g·cm/s ² = dyn	kg·m/s ² = N	kgf
Power	kg·m ² /s ³ = W	Watt	—	—	—
Pressure	kg/(m·s ²) = Pa	Pascal	—	—	atm mmHg mH ₂ O
Energy, power, heat	kg·m ² /s ² = J	Joule	—	—	—

Source of Table 1.3: Reprinted from Appendix 1 in book “*Principles of Environmental Engineering*” with kind permission from Prof. Hongying Hu

Example 1.3 It is known that 1.000 atm is equal to a 760.000 mmHg column and 1.033 kgf/cm², as well as 1 kgf = 9.807 N. Calculate how much is 1.000 atm equal to in Pa·N/m², kg/(m·s²) and mH₂O column.

Solution: 1.000 atm = 1.033 kgf/cm² = 1.033 × 9.807 / (1 / 10,000) = 1.013 × 10⁵ N/m².

Since $p = F / A$, in which F refers to force, its unit is N. A refers to area, and its unit is m². It is known that 1 N/m² = 1 Pa. Thus,

$$1.000 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

From Tables 1.1, 1.2 and 1.3, 1 Pa = 1 kg/(m·s²); thus, 1.000 atm = 1.013 × 10⁵ kg/(m·s²).

Known: $\rho_{\text{Hg}} = 13.600 \text{ g/cm}^3$, $\rho_{\text{H}_2\text{O}} = 1.000 \text{ g/cm}^3$. Thus,

1.000 atm = 760.000 mmHg = 760.000 × (13.600/1.000) mmH₂O = 10,336.000 mmH₂O = 10.336 mH₂O.

Exercises

1. Convert the following physical quantities into specified units:

Density: $3600.000 \text{ kg/m}^3 = \text{_____ g/cm}^3$.

Pressure: $8.900 \text{ atm} = \text{_____ Pa}$, $900.000 \text{ mmHg} = \text{_____ Pa}$, $5.800 \text{ Pa} = \text{_____ kgf/cm}^2$.

Specific heat capacity: $11.300 \text{ kcal/(kg} \cdot ^\circ\text{C)} = \text{_____ J/(kg} \cdot \text{K)}$.

Surface tension: $10.000 \text{ N/m} = \text{_____ dyn/cm}$, $8.000 \text{ kgf/m} = \text{_____ N/m}$.

Mass: $4.400 \text{ kgf} \cdot \text{s}^2/\text{m} = \text{_____ kg}$.

Power: $20.000 \text{ horsepower} = \text{_____ kW}$.

Flow rate: $300.000 \text{ m}^3/\text{h} = \text{_____ L/s}$.

2. As shown in Fig. 1.3, a PAC (polyaluminum chloride) aqueous solution with a 20.0% concentration (mass percentage, the same below) is fed into the evaporator at a flow rate of 1000.0 kg/h , and a part of the water is evaporated at a certain temperature to obtain a PAC aqueous solution with a 50.0% concentration. Then, the PAC solid containing 4.0% water was cooled and removed continuously. The saturated solution of PAC at a 37.5% concentration was returned to the evaporator for recycling treatment. The process is a continuous and stable process. Try to calculate:

- (1) the flow rates of solid PAC product P and water evaporation W ;
- (2) the flow rates of the saturated solution R and concentrated solution S .

Answer: ($P = 208.33 \text{ kg/h}$, $W = 791.67 \text{ kg/h}$, $S = 975.00 \text{ kg/h}$, $R = 766.67 \text{ kg/h}$).

3. As shown in Fig. 1.4, a tank contains 30 tons of heavy oil at a temperature of 30°C . An external circulation heating method is used to heat the heavy oil, and the circulation flow rate of the heavy oil is $8 \text{ m}^3/\text{h}$. The circulating heavy oil is heated by steam in a heat exchanger. The outlet temperature of the heat exchanger is constant at 100°C . The oil in the tank is evenly mixed. Assuming

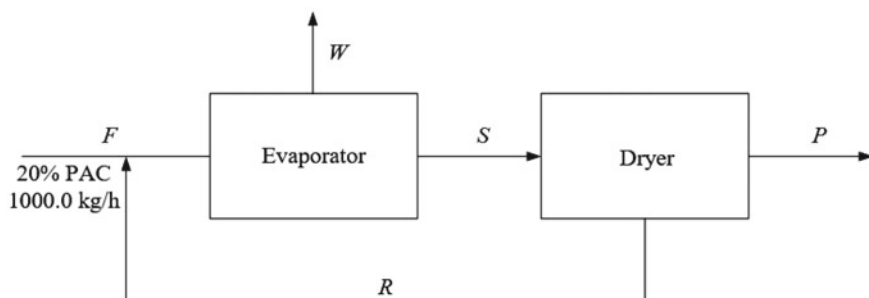
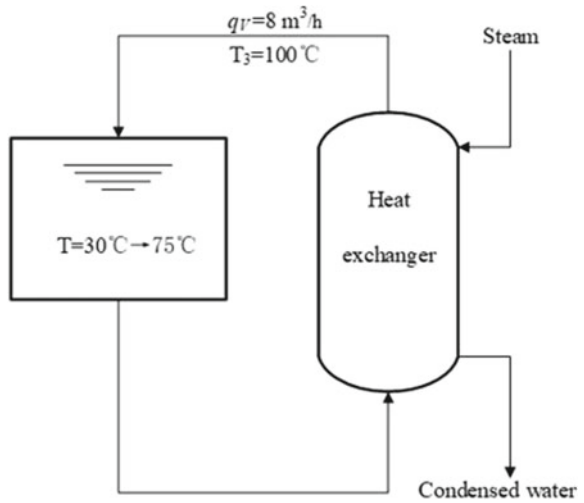


Fig. 1.3 The PAC drying process

Fig. 1.4 Heavy oil heating system



that the tank is insulated from the outside, please calculate how long it takes for the oil in the tank to be heated from 30°C to 75°C . The density of the oil is set to 990 kg/m^3 .

Answer: (3.90 h).

Reference

Hu, H. Y., Zhang, X., Huang, X., Wang, W., & Xi, J. Y. (Eds.). (2015). *Principles of Environmental Engineering* (3rd ed.). Beijing: Higher Education Press. In Chinese.

Chapter 2

Fluid Flow and Transport Equipment



Yan Liu, Liang Li, Rui-Hua Dai, and Chen-Xi Li

2.1 Overview

Fluid flow is the basis for this text book for the following reasons:

- (1) When studying the transport of polluted water or gas, it is necessary to study their flow principles to design pipelines, select transport equipments, and calculate the required power.
- (2) To improve the efficiency of pollutant removal, it is often necessary to provide appropriate or optimal flow conditions to improve the efficiency of equipment or reactors.
- (3) During the process of heat transfer and gas absorption, most of the target objects are in a flow state.
- (4) To understand and control the removal process of pollutants in fluids, a series of parameters, including pressure, flow rate, and flow velocity, need to be measured in pipelines and equipment. The operating principle of some instruments for measuring these parameters is based on the static or flow law of fluids.

In conclusion, the study of fluid flow is of great significance for improving pollutant removal efficiency and reducing costs.

Fluid is a general term for gas and liquid, and it has the following three characteristics:

- (1) Fluidity, i.e., both shear and tension resistance are very low.
- (2) There is no fixed shape, which varies with the shape of the container.
- (3) Relative motion occurs in the fluid under external force.

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To study the principles of fluid motion, it is necessary to use a scientific method. The hypothesis of fluid continuity is to assume that fluid is a continuous medium consisting of a great number of point particles with no voids between them and a fully occupied space. The purpose of the continuity hypothesis is to eliminate complex molecular motion and to study the flow principles of fluid from a macroscopic point of view. From this view, the physical properties and motion parameters of fluid are continuously distributed in space, which can be described by mathematical tools of the continuity function. The investigation of fluid flow studies the change in fluid point particles with space position and time.

2.1.1 *Methods for Investigating Fluid Flow*

There are two different ways to investigate fluid flow:

- (1) The Lagrange method studies the motion of individual fluid point particles, tracks point particles, describes the variation of their motion parameters (displacement, velocity, etc.) with time and obtains the motion law of the whole fluid by studying the motion law of each fluid point particle. This method is also called the point particle system method.

$$\left\{ \begin{array}{l} u_x = \frac{\partial x}{\partial \tau} = \frac{\partial x(a,b,c,\tau)}{\partial \tau} \\ u_y = \frac{\partial y}{\partial \tau} = \frac{\partial y(a,b,c,\tau)}{\partial \tau} \\ u_z = \frac{\partial z}{\partial \tau} = \frac{\partial z(a,b,c,\tau)}{\partial \tau} \end{array} \right\}$$

The basic characteristic of the Lagrange method is to track the motion of a single point particle. It is frequently used to study the motion of a single solid point particle and the trajectory of a fluid point particle.

- (2) The Euler method investigates the movement of different fluid point particles through fixed spatial points to understand the flow situation in the whole flow space (such as the velocity, pressure, density of each point in the space), that is, to study the distribution field of various motion elements. This method is also called the flow field method. In the Euler method, any element of motion in the flow field can be expressed as a function of spatial coordinates and time.

The Euler method is a commonly used method in fluid mechanics. By using this method, the streamline of a fluid (an instantaneous curve in the flow field) can be obtained, and the tangent direction of each point on the curve is the velocity direction of the point. For the steady flow of fluid in a straight pipe, the trajectory of the fluid coincides with the streamline. It is very convenient to describe the flow state of fluid by the Euler method, as it can be applied to study the fluid flow in certain reactor (control body) during the environmental engineering process. In a space rectangular coordinate system,

$$\begin{cases} u_x = u_x(x, y, z, \tau) \\ u_y = u_y(x, y, z, \tau) \\ u_z = u_z(x, y, z, \tau) \end{cases} \quad (2.1)$$

If points in space are fixed, i.e., x , y and z are constant, and τ is a variable, then the variation in velocity at different times can be obtained at a fixed space point. If τ is a constant and x , y and z are variables, then the velocity distribution of liquid particles (i.e., instantaneous velocity field) at different points can be obtained at the same time.

According to the derivative method of the composite function, the time derivative of Eq. (2.1) can be obtained. The fluid particle acceleration component passing through any point in the flow field in the x -, y - and z -axes is as follows:

$$\begin{cases} a_{jx} = \frac{du_x}{d\tau} = \frac{\partial u_x}{\partial \tau} + \frac{\partial u_x}{\partial x} \frac{dx}{d\tau} + \frac{\partial u_x}{\partial y} \frac{dy}{d\tau} + \frac{\partial u_x}{\partial z} \frac{dz}{d\tau} \\ a_{jy} = \frac{du_y}{d\tau} = \frac{\partial u_y}{\partial \tau} + \frac{\partial u_y}{\partial x} \frac{dx}{d\tau} + \frac{\partial u_y}{\partial y} \frac{dy}{d\tau} + \frac{\partial u_y}{\partial z} \frac{dz}{d\tau} \\ a_{jz} = \frac{du_z}{d\tau} = \frac{\partial u_z}{\partial \tau} + \frac{\partial u_z}{\partial x} \frac{dx}{d\tau} + \frac{\partial u_z}{\partial y} \frac{dy}{d\tau} + \frac{\partial u_z}{\partial z} \frac{dz}{d\tau} \end{cases}$$

Because $\frac{dx}{d\tau} = u_x$, $\frac{dy}{d\tau} = u_y$, and $\frac{dz}{d\tau} = u_z$, the above equation can be substituted:

$$\begin{cases} a_{jx} = \frac{du_x}{d\tau} = \frac{\partial u_x}{\partial \tau} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \\ a_{jy} = \frac{du_y}{d\tau} = \frac{\partial u_y}{\partial \tau} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \\ a_{jz} = \frac{du_z}{d\tau} = \frac{\partial u_z}{\partial \tau} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} \end{cases}$$

The first term on the right side of the equation above ($\frac{\partial u_x}{\partial \tau}$, $\frac{\partial u_y}{\partial \tau}$, $\frac{\partial u_z}{\partial \tau}$) represents the rate of velocity change of fluid particles in three coordinate directions due to time variation at a fixed point in space, which is called local acceleration. The sum of the last three terms on the right side of the equation indicates acceleration resulting from the change of the position of fluid particles, which is called migration acceleration, that is, the acceleration caused by the change of space position occupied by fluid particles during the flow process. When the Euler method is used to describe fluid motion, the acceleration of the fluid particle is the sum of the local acceleration and migration acceleration.

When describing fluid motion by the Euler method, fluid flow can be divided into steady flow and unsteady flow. If all the elements of motion at any point in the flow field do not vary with time, the flow is called steady flow. In this case, for any fixed point in space in the flow field, regardless of which fluid point particle passes through the field, the motion elements are the same. That is, the motion elements are independent of time and are only functions of the space coordinates.

According to the definition of steady flow, the velocities u_x , u_y , and u_z , the pressure p , and the density ρ of a flow should satisfy the following conditions:

$$\left\{ \begin{array}{l} \frac{\partial u_x}{\partial \tau} = \frac{\partial u_y}{\partial \tau} = \frac{\partial u_z}{\partial \tau} = 0 \\ \frac{\partial \tau}{\partial \rho} = 0 \\ \frac{\partial \rho}{\partial \tau} = 0 \end{array} \right.$$

Since the elements of motion are independent of time, Eq. (2.1) becomes

$$\left\{ \begin{array}{l} u_x = u_x(x, y, z) \\ u_y = u_y(x, y, z) \\ u_z = u_z(x, y, z) \end{array} \right.$$

If any element of motion at any point in the flow field changes with time, the fluid flow will be unsteady.

In short, the Lagrange method describes the state of the same point particle at different times, while the Euler method describes the state of individual points in the flowing space and their relationship with time. In fluid flow, because there are numerous fluid particles involved, the Euler method is more suitable to describe fluid flow, especially for steady flow.

2.1.2 Basic Concepts of Fluid Flow

(1) Density, specific volume, and specific gravity of fluid

Fluid density (ρ): The mass of a fluid per unit volume, $\rho = m/V$, kg/m³.

Specific volume (Δ): The volume of a unit mass is called the specific volume of a fluid, which is the reciprocal of density, m³/kg.

Specific gravity: The ratio of the density (or gravity) of the material to the density (or gravity) of pure water at 277 K (4 °C).

(2) Surface force and pressure

Surface force (F): The force acting perpendicularly on the surface of any fluid element is called as surface force, kg·m/s², N.

Pressure: The force on the unit surface area of the fluid is called the pressure, kg/(m·s²), Pa, N/m².

1 atm = 1.0133 × 10⁵ Pa = 1.0133 bar = 760 mmHg = 10.33 mH₂O = 1.033 kgf/cm².

1 kgf = 1 kg × 9.807 m/s² = 9.807 N.

Figure 2.1 shows the relationships among absolute pressure, gauge pressure and vacuum.

Gauge pressure = absolute pressure – local atmospheric pressure.

Vacuum = local atmospheric pressure – absolute pressure.

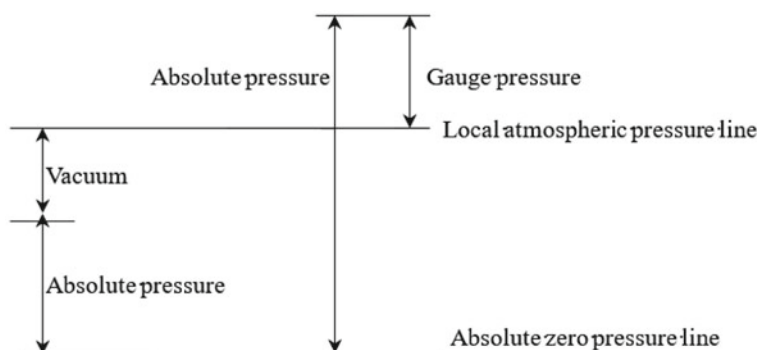


Fig. 2.1 The relationships among absolute pressure, gauge pressure and vacuum

(3) Trace line and streamline

In a flow field, the trace line is the trajectory left by a fluid point particle in space in continuous time. The trace line corresponds to the Lagrange method.

In a flow field, the streamline is an instantaneous curve that is tangent to a series of particle velocity vectors. The streamline corresponds to the Euler method.

A trace line consists of one particle, and a streamline consists of an infinite number of particles. In general, a trace line and a streamline do not coincide. In the case of steady flow, because the velocity of the fluid is independent of time, the streamline in the flow field is constant, and the fluid particle will move along the streamline, the trajectory of a fluid particle coincides with the streamline.

Figures 2.2 and 2.3 show the velocity vector distribution and streamline distribution, respectively, in a horizontal sedimentation tank of a municipal wastewater treatment plant.

(4) Cross section

The section orthogonal to the streamline in the flow field is the cross section. The cross section is generally not a plane; only when the streamlines are parallel, the cross section is a plane, as shown in Fig. 2.4.

(5) Flow rate and flow velocity

Flow rate refers to the quantity of fluid per unit time through any cross section. It can be divided into volume flow rate (q_v , m^3/s) and mass flow rate (q_m , kg/s). The relationship between them is as follows:

$$q_m = \rho q_v$$

where

q_m —Mass flow rate, kg/s

ρ —Fluid density, kg/m^3

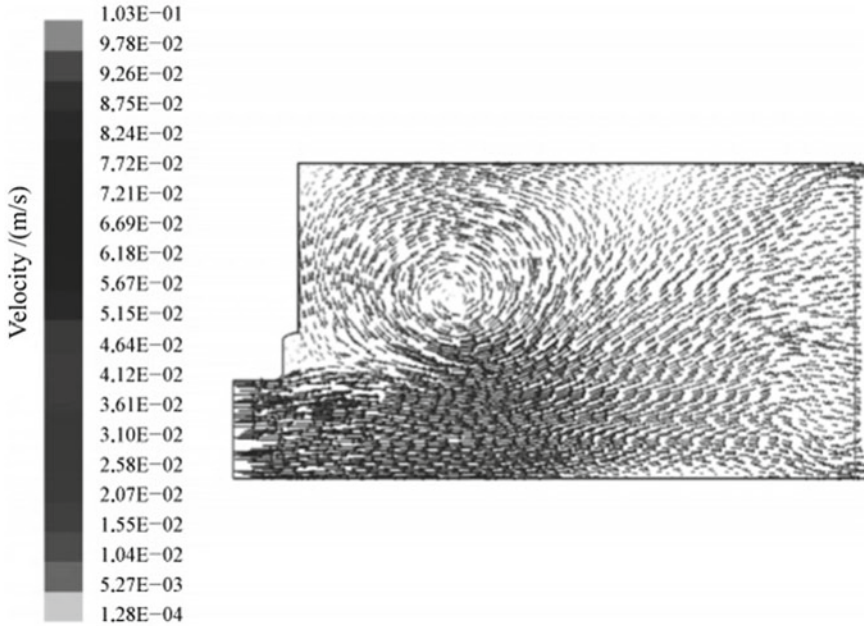


Fig. 2.2 Vector distribution of flow velocity in horizontal sedimentation tank of a municipal wastewater treatment plant

q_V —Volume flow rate, m^3/s .

Flow velocity (m/s) is the distance of fluid passed in the direction of flow per unit of time. Usually, the flow velocity is different at different points on the cross section. For example, in pipe flow, the flow velocity is zero at the pipe wall and the maximum on the axis of the pipe center. For the convenience of analysis and calculation in engineering, the average flow velocity or the average velocity (u , m/s), i.e., the average flow velocity or the average velocity of a cross section, is often used. Its definition is as follows:

$$u = q_V / A$$

where

q_V —Volume flow rate, m^3/s

u —Average flow velocity or average velocity, m/s

A —Cross section, m^2 .

Usually, in environmental engineering facility pipelines, the economic flow velocity of liquid is 1 m/s , and that of gas is 10 m/s .

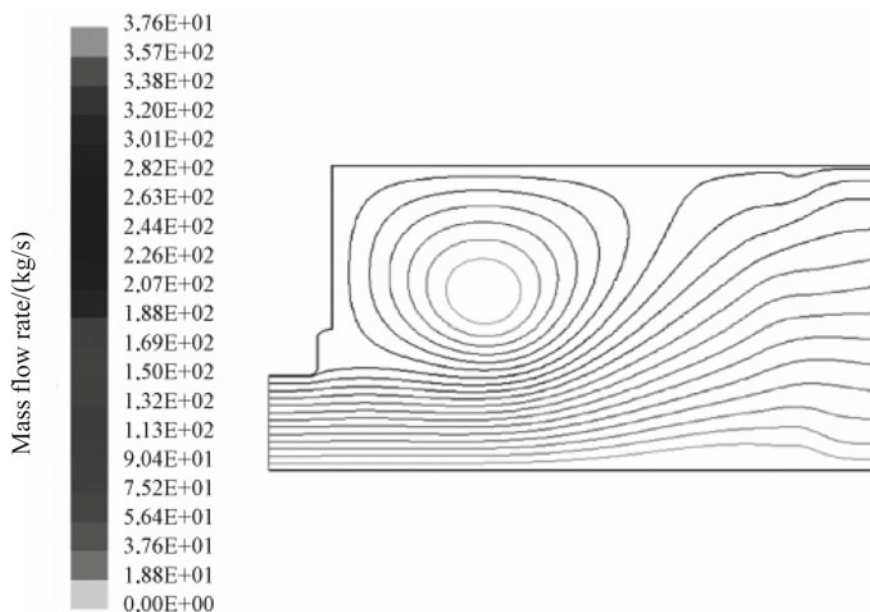


Fig. 2.3 Streamline diagram of horizontal sedimentation tank in a municipal wastewater treatment plant

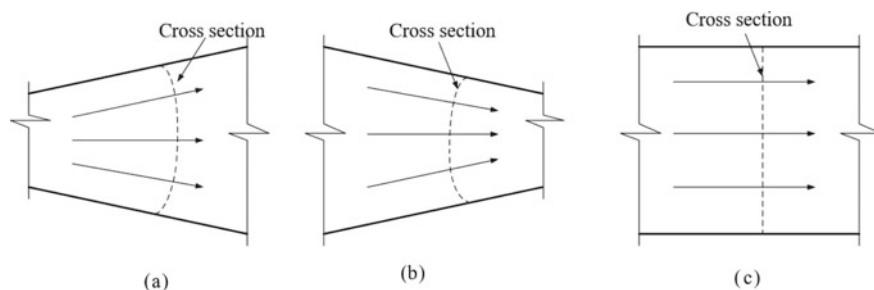


Fig. 2.4 Cross section

(6) Steady and unsteady flow

In a flow field, the motion parameters at any spatial position do not change with time; that is, the partial derivative of time equals zero; for example, $\frac{\partial u}{\partial \tau} = 0$, $\frac{\partial p}{\partial \tau} = 0$, etc. is called a steady-state flow. In steady-state flows, velocity and other motion parameters are only functions of position coordinates. For example, there is no local acceleration, for acceleration.

In a flow field, as long as there is a certain motion parameter at any spatial position that is a function of time, the partial derivative of time is not equal to zero. This flow is called an unsteady flow.

(7) Uniform flow and nonuniform flow

In a flow field, the flow velocity of a fluid point particle does not vary with the change in space position at any time along the flow direction, and then the flow is called uniform flow. Otherwise, it is nonuniform flow. Uniform flow must be a steady flow, and nonuniform flow can be either a steady flow or an unsteady flow.

(8) Pipeline flow, open channel flow, pressurized flow, and nonpressurized flow

A fluid flow without a free surface is pipeline flow, which is a pressurized flow. Fluid flow with free surface is called open channel flow and is nonpressurized.

2.1.3 Internal Friction in Fluid Flow

Because of the adhesion of fluid to the wall of a circular pipe, a static fluid film will adhere to the wall. At the same time, due to the attraction between molecules and the thermal motion of molecules in the fluid, the static fluid film on the wall will retard the flow of adjacent fluid layers, which slows down the flow velocity. This force gradually decreases with increasing distance from the wall. That is, the farther away from the wall, the faster the fluid flow velocity. The flow velocity at the center of the pipe is the fastest. Because of the internal force of the fluid, when the liquid flows in a circular pipe, it is actually divided into numerous concentric cylinder layers. One layer is covered with another layer, and each layer moves forward at different flow velocities, as shown in Fig. 2.5a.

Because of the different flow velocities of each layer, relative motion occurs between layers. The fast fluid layer produces a force to drive the adjacent slower fluid layer towards the moving direction. At the same time, the slower fluid layer also acts on the adjacent fast fluid layer with a force of equal size and opposite direction, which hinders the faster fluid layer from moving forward. The interaction force between two adjacent fluid layers in the moving fluid is called the internal friction force of the fluid, which is an expression of fluid viscosity, so it is also called viscous force or viscous friction force. When fluid flows, it loses some mechanical energy to heat energy because of overcoming friction.

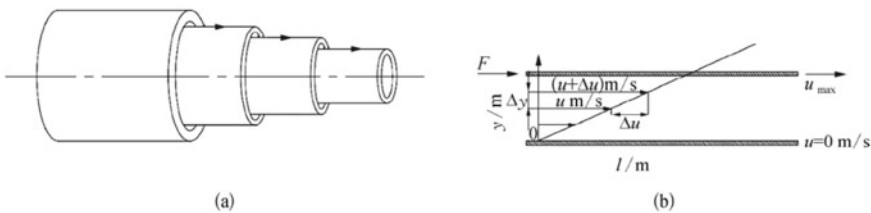


Fig. 2.5 Internal friction in fluid flow

Experiments illustrate that for certain liquids, as shown in Fig. 2.5b, the internal friction force F_n is proportional to the flow velocity difference Δu between the two layers, inversely proportional to the vertical distance Δy between the two layers, and proportional to the contact area A_j between the two layers.

$$F_n \propto \frac{\Delta u}{\Delta y} A_j$$

The above relation can be written as an equation to introduce the proportional coefficient μ :

$$F_n = \mu \frac{\Delta u}{\Delta y} A_j$$

The internal friction force per unit area is called shear stress, which is expressed as τ' . When fluid flows in a pipe, the change in radial flow velocity is not linear.

$$\tau' = \frac{F_n}{A_j} = \mu \frac{du}{dy} \quad (2.2)$$

where

τ' —Shear stress, Pa

$\frac{du}{dy}$ —Normal velocity gradient, which is the change of the flow velocity in the y direction perpendicular to the flow direction, s^{-1}

μ —Proportional coefficient, known as viscous coefficient or dynamic viscosity, referred to as viscosity, Pa·s

A_j —Contact area between two layers, m^2

F_n —Internal friction force, N or $kg \cdot m/s^2$.

The relationship shown in Eq. (2.2) is called Newton's law of viscosity. The shear stress of the fluid is proportional to the normal velocity gradient and has nothing to do with force or pressure.

The viscosity coefficient or viscosity is a physical property of fluid. If the fluid viscosity is high, the normal velocity gradient can only be smaller under the same shear stress. Equation (2.2) shows that the viscosity of the fluid represents the internal friction or shear stress caused by the fluid viscosity when the normal velocity gradient on the unit contact surface area is 1.

The viscosity of fluids is an important physical property affecting fluid flow. The viscosities of many fluids are measured experimentally and can be found in relevant manuals (see Table 2.1).

In addition, the level of fluid viscosity can also be expressed by the ratio of viscosity μ to density ρ , which is called kinematic viscosity, expressed by symbol ν :

Table 2.1 Viscosities of selected fluids (Cardarelli 2018)

Fluid	$t\text{ }^{\circ}\text{C}$	$\mu\text{ Pa}\cdot\text{s}$	Fluid	$t\text{ }^{\circ}\text{C}$	$\mu\text{ Pa}\cdot\text{s}$
Water	0	1.8×10^{-3}	Hydrogen	−1	8.3×10^{-6}
	100	0.3×10^{-3}		250	13×10^{-6}
Mercury	0	1.7×10^{-3}	Carbon dioxide	0	14×10^{-6}
	100	1.0×10^{-3}		302	27×10^{-6}
Castor oil	17.5	2300×10^{-3}	Air	0	18×10^{-6}
	50	1225×10^{-3}		671	42×10^{-6}

Source of Table 2.1: Reprinted from Tables 19.21, 19.23 and 20.12 in book “*Materials Handbook*” with kind permission from Springer Nature

$$\nu = \frac{\mu}{\rho}$$

The legal unit of measurement of kinematic viscosity is m^2/s , and the nonlegal unit is St, which is called a Stokes value.

$$1\text{St} = 10^{-4}\text{m}^2/\text{s}$$

The viscosities of gases are approximately two orders of magnitude smaller than those of liquids, and the viscosities of gases increase with temperature, while the viscosities of liquids decrease with increasing temperature.

The effect of pressure on liquid viscosity can be neglected. For gases, this effect is considered only at very high or very low pressures.

In engineering, it is often necessary to know the viscosities of mixed liquids and gases. If there are no experimental data, the corresponding value can be estimated according to the viscosities of pure materials. For nonassociative mixed liquids, the following equation can be used:

$$\log \mu_{\text{mix}} = \sum_{i=1}^n x_i \log \mu_i$$

where

μ_{mix} —Mixed liquid viscosity, Pa·s

x_i —Mole fraction of i in mixed liquids

μ_i —The viscosity of component i in mixed liquids, Pa·s.

For mixed gases at low pressure:

$$\mu_{\text{mix}} = \frac{\sum_{i=1}^n y_i \mu_i M_i^{1/2}}{\sum_{i=1}^n y_i M_i^{1/2}}$$

where

μ_{mix} —Mixed gas viscosity, $\text{Pa} \cdot \text{s}$

y_i —Mole fraction of i component in mixed gas

μ_i —The viscosity of component i in mixed gases, $\text{Pa} \cdot \text{s}$

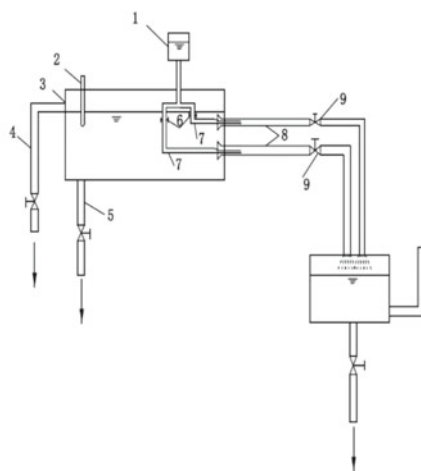
M_i —Molar mass of component i in mixed gases, g/mol .

2.1.4 Types of Fluid Flow and Its Discriminant Method

1) Types of fluid flow

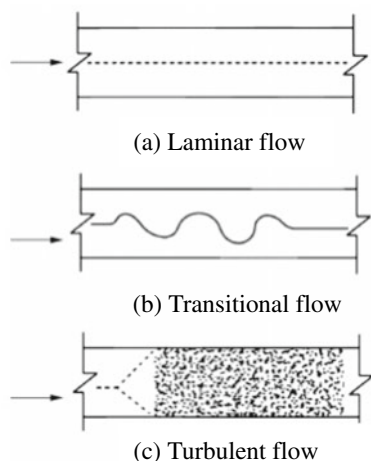
To observe the movement of particles and the influence of various factors on the flow condition directly, the experiment shown in Fig. 2.6 can be arranged, which is called the Reynolds experiment. It reveals two distinct patterns of flow. Inside a water tank, a glass tube with a bell-shaped inlet is installed below the water surface. A valve is installed downstream of the pipe to regulate the flow by adjusting the valve. There is a needle-shaped tube in the center of the bell-shaped inlet. A ray of colored water flows out of the tube, and its density is almost the same as that of water.

As shown in Fig. 2.6, an overflow device is installed in the tank to keep the water level constant. The lower part of the water tank is equipped with horizontal glass tubes, and the flow is regulated by valves. A thin tube is inserted at the entrance of



1-Colored solution; 2-Thermometer; 3-Water tank; 4-Downspout;
5-Emptying tube; 6-Valve; 7-Acicular tubule; 8-Glass tube; 9-Valve

Fig. 2.6 Diagram of Reynolds experimental equipment

Fig. 2.7 Flow patterns

the glass tube. The upper part of the thin tube is connected to a container containing colored liquid.

In the experiment, under the condition of overflow, valve 9 is opened slightly to make the water in the glass tube flow at low velocity, valve 6 is opened, and colored liquid is introduced into the glass tube. The colored liquid flows smoothly through the whole glass tube in a straight line and does not mix with the water in the tube, as shown in Fig. 2.7a. This shows that the fluid particles in the pipe flow regularly in parallel, and the particles do not interfere and mix with each other. This flow pattern is called stagnation or laminar flow.

When the flow of colored liquid is unchanged, the regulating valve 9 increases the flow velocity. When the flow velocity increases to a certain value, the flow line of the colored liquid exhibits an irregular wave shape, as shown in Fig. 2.7b. If the flow velocity continues to increase to a certain critical value, the colored streamline will disappear. At this time, the water in the whole downstream pipe presents a uniform color, as shown in Fig. 2.7c. This shows that in addition to moving forward along the pipeline, there are irregular radial motions of particles, which collide and mix with each other. This flow pattern is called turbulent flow. The flow state between the above two conditions is called transitional flow.

2) Flow pattern criteria—Reynolds number Re

Different flow types have different effects on mass and heat transfer in fluids. Therefore, the flow type should be determined in advance for environmental engineering design. For in-pipe flow, experimental results show that the geometrical size (diameter d), average velocity u and fluid properties (density ρ and viscosity μ) of the flow have an effect on the flow pattern transition. Reynolds, a British physicist, found that these factors can be combined into a dimensionless group, $\frac{du\rho}{\mu}$, which is called the Reynolds number (Re). The symbol Re represents the ratio of inertia force to viscous force and reflects the flow state and turbulence degree of the fluid.

$$Re = \frac{du\rho}{\mu} \quad (2.3)$$

where

d —Inner diameter of tube or pipe, m

u —Main body velocity or average velocity, m/s

ρ —Fluid density, kg/m³

μ —The dynamic viscosity of the fluid, kg/(m·s), Pa·s, N·s/m².

$$Re = \frac{(\text{m}) (\text{m/s}) (\text{kg/m}^3)}{\text{kg/(m} \cdot \text{s)}}$$

Reynolds Number—Criteria to determine flow patterns of fluids:

- (1) When $Re \leq 2000$, laminar flow must occur. This is a laminar flow zone.
- (2) When $2000 < Re < 4000$, laminar flow will sometimes occur, but turbulence will also sometimes occur, depending on the environment. This is a transitional zone.
- (3) When $Re \geq 4000$, turbulence generally occurs. This is a turbulent region.

When $Re \leq 2000$, laminar flow occurs generally, and any disturbance can only temporarily make the flow deviate from a laminar flow. Once the disturbance disappears, the laminar flow state will be restored. Therefore, when $Re \leq 2000$, the laminar flow is stable and is a laminar flow zone.

When the Re number exceeds 2000 and is less than 4000, laminar flow is no longer stable, but whether turbulence occurs depends on external disturbances. If the disturbance is small enough to change the flow pattern, laminar flow can still exist as a transition zone.

When $Re \geq 4000$, turbulence always occurs in general. This is a turbulent region.

According to the Re value, the fluid flow can be divided into three zones: a laminar zone, a transitional zone, and a turbulent zone, but there are only two flow patterns. The transitional zone is not a transitional flow pattern; it only means that there may be laminar flow or turbulence in this region, depending on external disturbances.

The Reynolds number Re not only serves as a basis for distinguishing a fluid flow pattern but also reflects the turbulence degree of liquid particles in the flow. The larger the Re value is, the more turbulent the particles in the fluid and the more violent the collision and mixing of particles in the flow. In practical production, to improve the heat or mass transfer rate of a fluid, the flow pattern of the fluid generally requires turbulence.

[Notes]

(1) **d —Pipe internal diameter**

If the fluid flows in a noncircular pipe (such as a square pipe) or a noncircular cross-sectional pipe (such as a casing annulus), d in Re should be replaced by an equivalent diameter d_e .

$$d_e = \frac{4 \times \text{Circulation cross sectional area}}{\text{Wetted perimeter}}$$

The wetted perimeter is a short name for the wetted perimeter length which does not include the water surface.

For example, for a rectangular cross section Fig. 2.8, the equivalent diameter is

$$d_e = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$$

For pipes with annular cross sections (Fig. 5.4b), then

$$d_e = \frac{4 \times \frac{\pi}{4}(D^2 - d^2)}{\pi(D+d)} = D - d$$

(2) **u —Average flow velocity**

Regardless of laminar or turbulent flow, the particle flow velocity of the fluid on the cross section of the pipeline is regularly distributed (Fig. 2.9). At the pipe wall, the flow velocity is zero, and at the center of the pipe, the flow velocity is the highest. In laminar flow, the flow velocity of fluid in a straight pipe is parabolic along the diameter of the straight pipe. The average flow velocity is 1/2 of the flow velocity in the center of the pipe. The flow velocity distribution in the turbulent flow is slightly

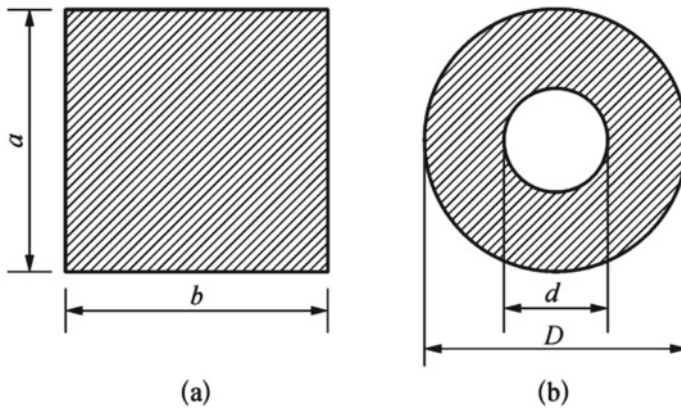


Fig. 2.8 Noncircular cross section

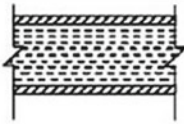
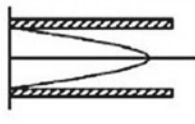
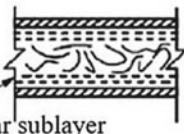
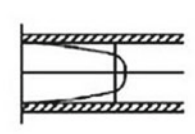
Item	Physical image	Velocity distribution	Average velocity
Laminar Flow			$u = 0.5 u_{\max}$
Turbulent Flow			$u = 0.8 u_{\max}$

Fig. 2.9 Velocity distribution and average velocity

wider at the top, which is caused by the turbulence and mixing of the fluid. The higher the turbulence degree is, the flatter the top of the curve. The average flow velocity in turbulence is approximately 0.8 times that in the center of the pipe.

(3) ρ —Fluid density

For gases, when the pressure does not notably change, the average density is used instead.

Example 2.1 The process of flue gas denitrification in a power plant is studied in a laboratory. It is known that the actual operating pressure of flue gas is atmospheric pressure, the temperature is $400\text{ }^{\circ}\text{C}$, the flow velocity is 2 m/s , and the diameter of the flue gas pipe is 1.5 m . The flue gas at atmospheric pressure and $90\text{ }^{\circ}\text{C}$ is simulated in the laboratory, and its flow velocity is 10 m/s . It is known that the flue gas viscosities at $90\text{ }^{\circ}\text{C}$ and $400\text{ }^{\circ}\text{C}$ are $19.94 \times 10^{-6}\text{ Pa} \cdot \text{s}$ and $31.70 \times 10^{-6}\text{ Pa} \cdot \text{s}$, respectively. Try to calculate:

- (1) The diameter of laboratory simulation equipment;
- (2) The flue gas flow rate in the power plant and the experimental equipment.

The condition of the equipment in the power plant is indicated by subscript 1, and the condition of the experimental equipment is indicated by subscript 2.

Known: $T_1 = 400\text{ }^{\circ}\text{C} = 400 + 273 = 673\text{ K}$, $u_1 = 2\text{ m/s}$, $\mu_1 = 31.70 \times 10^{-6}\text{ Pa} \cdot \text{s}$, $d_1 = 1.5\text{ m}$

$$T_2 = 90\text{ }^{\circ}\text{C} = 363\text{ K}, u_2 = 10\text{ m/s}, \mu_2 = 19.94 \times 10^{-6}\text{ Pa} \cdot \text{s}$$

$$p_1 = p_2 = \text{Atmospheric pressure}$$

Ask: d_2 , q_{V1} , q_{V2} .

Solution:

- (1) To maintain the hydrodynamic similarity between the experimental equipment and the power plant equipment, the Re values of the experimental equipment and the power plant equipment must be equal, that is:

$$\frac{d_1 u_1 \rho_1}{\mu_1} = \frac{d_2 u_2 \rho_2}{\mu_2}$$

$$d_2 = d_1 \left(\frac{u_1}{u_2} \right) \left(\frac{\rho_1}{\rho_2} \right) \left(\frac{\mu_2}{\mu_1} \right)$$

$$pV = nRT$$

$$\frac{n}{V} = \frac{p}{RT}$$

$$\rho = \frac{m}{V} = \frac{nM}{V} = \frac{p}{RT} M$$

$\therefore p_1 = p_2, M_1 = M_2$, and R is a constant.

$$\therefore \frac{\rho_1}{\rho_2} = \frac{T_2}{T_1}$$

$$d_2 = d_1 \left(\frac{u_1}{u_2} \right) \left(\frac{T_2}{T_1} \right) \left(\frac{\mu_2}{\mu_1} \right) = 1.5 \times \frac{2}{10} \times \frac{363}{673} \times \frac{19.94 \times 10^{-6}}{31.70 \times 10^{-6}}$$

$$= 0.10 \text{ m} = 10 \text{ cm}$$

- (2) Flue gas flow rate of power plant equipment:

$$q_{V1} = \frac{\pi d_1^2}{4} u_1 = \frac{3.14 \times 1.5^2 \times 2}{4} = 3.53 \text{ m}^3/\text{s}$$

Flue gas flow rate of experimental equipment:

$$q_{V2} = \frac{\pi d_2^2}{4} u_2 = \frac{3.14 \times 0.10^2 \times 10}{4} = 0.0785 \text{ m}^3/\text{s}$$

Answer: The diameter of the experimental equipment is 10 cm, the flue gas flow rate of the power plant equipment is 3.53 m³/s, and the flue gas flow rate of the experimental equipment is 0.0785 m³/s.

2.1.5 Flow Boundary Layer

Although the Reynolds number Re can be used to determine whether the whole flow state of the fluid is laminar or turbulent, it has been found that even turbulent fluid flows in the region near a solid interface, such as a pipe wall or flat plate, the flow state is still laminar, the resistance in this region is great, and the normal velocity decays fast. This region is called the laminar sublayer, viscous sublayer, or inner flow layer. Prandtl proposed the boundary layer theory; that is, when actual fluid flows along a solid wall, there are two regions in the fluid: the boundary layer with a significant velocity gradient and the mainstream region with almost no velocity gradient. In the boundary layer region, the velocity of fluid near the wall is zero, and because there is a significant velocity gradient du/dy , even if the viscosity is very low, there is also a great internal friction stress τ' , so the friction resistance is very great. In the mainstream region, $du/dy \approx 0$; therefore, $\tau' \approx 0$, so the frictional resistance of the fluid flow in the mainstream region tends to zero, which can be regarded as an ideal fluid. The influence of fluid viscosity is limited within the boundary layer by the theory, which greatly simplifies the flow problem for actual fluid.

As shown in Fig. 2.10, when a fluid flows by a flat-plane at a uniform velocity of u_0 , a static fluid layer will adhere to the wall. This layer of fluid will generate internal friction between the adjacent fluid layer and reduce its velocity. This internal friction and velocity decrease and then transfer to the normal direction of the fluid away from the wall layer by layer and gradually weaken. At a certain distance from the wall, the influence on velocity can be neglected. Usually, the velocity here is $0.99 u_0$, and the distance between here and the wall is the boundary layer. The area outside the boundary layer is called the mainstream area.

The boundary layer can be divided into a laminar boundary layer and a turbulent boundary layer according to the flow state of the fluid in the boundary layer. At the front of the plate, the boundary layer is thin, and the velocity of the fluid is low. The flow state of the fluid in the whole boundary layer is laminar flow, which is

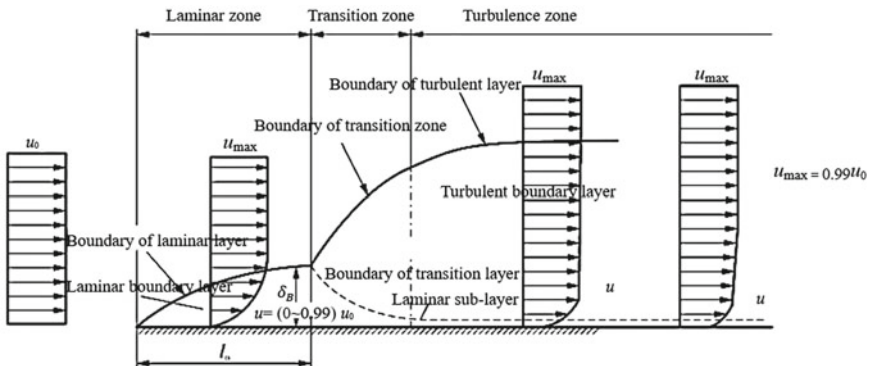


Fig. 2.10 Formation of boundary layer on a flat plate

called the laminar boundary layer. With the increase in the distance l from the front edge of the plate, the boundary layer gradually thickens. When the distance increases to the critical value l_0 , the flow of fluid in the boundary layer sometimes changes from laminar flow to turbulent flow. When turbulence occurs, the boundary layer suddenly thickens, which is called the turbulent boundary layer. However, turbulence sometimes occurs after l_0 . Sometimes it requires additional distance along the flow direction to occur. The distance and the occurrence are uncertain. This unstable region is called the transition zone. The boundary layer after the transition zone is called the turbulent boundary layer. In the turbulent boundary layer, there is still a laminar flow near the wall, that is, the laminar sublayer, with a large friction resistance and velocity gradient.

The boundary layer formed by fluid flowing over a flat plate is related to many factors, such as the physical properties of the fluid, the flow velocity, the distance from the front of the wall, the shape of the front of the wall and the roughness of the wall. The boundary Reynolds number Re_l is typically used to describe the boundary layer of a fluid.

$$Re_l = \frac{\rho u_0 l}{\mu} \quad (2.4)$$

where

Re_l —Boundary Reynolds number, dimensionless

u_0 —Inflow velocity, m/s

ρ —Fluid density, kg/m³

μ —Dynamic viscosity of the fluid, kg/(m·s), Pa · s, N · s/m²

l —Distance from the leading edge of the plate, m.

For a smooth plate, the boundary layer is laminar when $Re_l < 2 \times 10^5$, and turbulent when $Re_l \geq 3 \times 10^6$. Usually, $Re_l = 5 \times 10^5$ is taken as the turning point of boundary layer flow from laminar flow to turbulent flow (l_0 point).

When a fluid flows on a flat plate, the thickness of the boundary layer δ_B can be calculated according to the following equation:

For the laminar boundary layer ($Re_l < 2 \times 10^5$)

$$\frac{\delta_B}{l} = \frac{4.64}{Re_l^{0.5}} \quad (2.5)$$

For the turbulent boundary layer ($5 \times 10^5 < Re_l < 10^7$)

$$\frac{\delta_B}{l} = \frac{0.376}{Re_l^{0.2}} \quad (2.6)$$

When the fluid flows in a circular straight pipe, the boundary layer also experiences similar changes: when the fluid enters the entrance, the boundary layer is very thin, and the thickness of the boundary layer gradually increases with the flow of the fluid, forming an annular boundary layer along the inner wall of the pipe. If the fluid in the pipe is in a state of laminar flow, i.e., $Re \leq 2\,000$, when the thickness of the boundary layer increases to the radius of the pipe, the boundary layer converges at the center of the pipe. Thereafter, the boundary layer occupies the whole pipe. The thickness of the boundary layer equals the radius of the pipe and does not change. The flow state and velocity distribution are shown in Fig. 2.11a and b. If the pipe fluid is turbulent, i.e., $Re \geq 4000$, from the entrance, the fluid will go through the laminar flow zone and then to the turbulent zone, and there is still an unstable transition zone in the middle. In the turbulent region, near the inner wall of the pipe, there is still an annular laminar sublayer, and the frictional resistance and velocity variations are concentrated again, as shown in Fig. 2.11c and d. In turbulence, the thickness of the annular laminar sublayer δ_{Bh} can be estimated by an empirical equation. When the average velocity in the pipe is 0.8 times the maximum velocity, the following equation can be used:

$$\delta_{Bh} = 61.5 \frac{d}{Re^{0.875}} \quad (2.7)$$

where

δ_{Bh} —In a turbulent state, the thickness of the annular laminar sublayer in a circular pipe, m

d —Inner diameter of a circular pipe, m

Re —Reynolds number in the main body of a circular pipe, dimensionless.

The distance from the inlet to the cross section which the boundary layer develops to the whole cross section of the pipe is called the length of the inlet cross section l_j . For a turbulent flow, l_j is approximate $(40 \sim 50) d$. For the laminar flow l_j can be calculated by the following equation:

$$\frac{l_j}{d} = 0.0575 Re \quad (2.8)$$

where

l_j —Length of the inlet cross section, m

d —Inner diameter of a circular pipe, m

Re —Reynolds number in the main body of a circular pipe, dimensionless.

When the fluid flows steadily in the pipe, the boundary layer is close to the wall of the pipe. When the fluid flows by the solid surface of a sphere, cylinder, or other shape or the pipe diameter suddenly changes, another remarkable feature of the boundary layer will appear, the phenomenon of the boundary layer flowing away from the solid wall, called the separation of the boundary layer. Once boundary layer separation

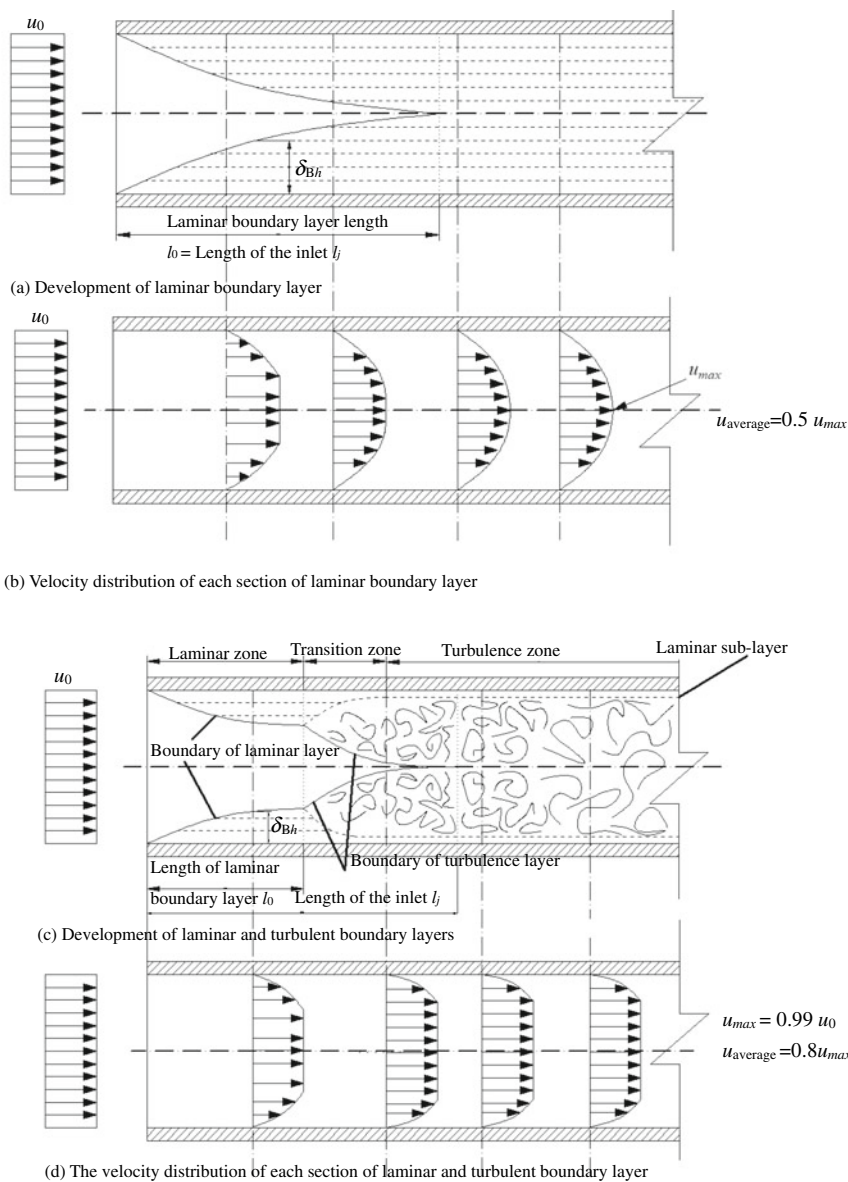


Fig. 2.11 Distribution of the boundary layer at the entrance of a circular pipe

occurs, there will be a blank area between the wall and the fluid, and the downstream fluid will flow back under the action of a pressure gradient, forming a whirlpool. The mechanical energy loss is caused by the strong collision and mixing of fluid particles in the whirlpool. The resistance corresponding to this energy loss is called vortex resistance. This resistance is caused by the separation of the boundary layer, which is caused by the shape of the solid wall surface, so it is called body resistance.

Figure 2.12 shows the boundary layer separation phenomena of fluid flow under several common conditions, such as a sudden pipe enlargement, a sudden pipe reduction, a right-angle elbow, flow around a sphere, and the boundary layer separation phenomena of water flows around obstacles in nature. Although the separation of the boundary layer results in the energy loss, it is hoped that this phenomenon occurs in some cases, such as accelerating heat transfer and mixing materials, because it increases the turbulence of the fluid.

2.2 Friction Loss of Fluid Flow in a System

The friction loss of fluid flowing in a system can be divided into straight pipe friction loss and local loss. The straight pipe friction loss is the energy loss caused by the internal friction of the fluid when the fluid flows in a straight pipe. Local loss is the energy loss caused by the separation of the boundary layer resulted from the change in flow direction or cross section through pipe fittings, valves, sudden expansion or reduction of a cross section, other devices, etc. The friction loss of fluid flowing in a system (or total friction loss) is the sum of all straight pipe friction losses and all local losses, that is:

$$\sum h_{tf} = \sum h_f + \sum h'_f \quad (2.9)$$

Where

h_f —Straight pipe friction loss, pipe head loss, major loss, Moody-type friction loss, J/kg

h'_f —Local loss, minor loss, J/kg

$\sum h_f$ —The sum of all straight pipe friction losses, J/kg

$\sum h'_f$ —The sum of all local losses, J/kg

$\sum h_{tf}$ —Total friction loss, friction loss, the sum of all straight pipe friction losses and all local losses, J/kg.

It is important to determine the friction loss of fluid flow in environmental engineering. First, the friction loss is related to the power required for fluid transport. Second, the change in the friction loss during fluid flow is often used to determine the fluid flow state in a reactor. Third, the friction loss of fluid flow can also be used to analyze the movement of particles in a continuous fluid.

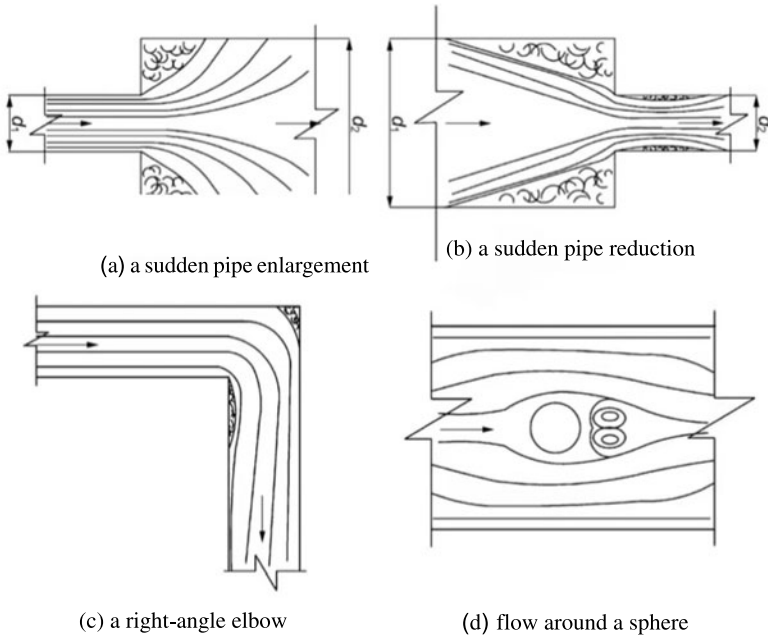


Fig. 2.12 Examples of boundary layer separation

2.2.1 Factors Affecting Friction Loss

The level of fluid friction loss is related to the physical properties of the fluid itself, the flow condition, the distance of the fluid flowing and the shape of the wall. The main manifestations are as follows:

- (1) Because the fluid is viscous, there is a shear stress between the particles of the fluid when it flows. The shear stress is the macroscopic expression of the internal friction produced by the momentum exchange of the random motion of the fluid molecules between the layers. The friction and collision of the molecules will consume the energy of the fluid. In a turbulent case, in addition to the energy consumed by the random movement of molecules, the high-frequency fluctuations of the fluid particles and mixing at the macroscale generate much greater turbulent stress than the former case, which consumes more fluid energy. These two factors are the origin of friction resistance, or the straight pipe friction loss.
- (2) When fluid flows, the boundary layer is separated because of the change in flow direction or channel cross section. As a result of counterpressure, the fluid will flow backwards to form a wake vortex. In the wake region, the particles of the fluid collide and mix strongly and consume energy. The energy loss caused by the backward flow and wake vortices, as well as the uneven distribution of pressure, is called body resistance, or local loss.
- (3) The Reynolds number determines the flow state of the fluid, and the friction loss is not same under different flow states.
- (4) The friction resistance of a rough surface is greater when fluid runs against the smooth pipe wall.

2.2.2 Straight Pipe Friction Loss and Dimensional Analysis

1) Straight pipe friction loss in a laminar fully developed circular straight pipe flow

Laminar flow is the flow of fluid through one layer over another layer, and the flow resistance is mainly the internal friction of the fluid. The resistance of flow conforms to Newton's viscous law of Eq. (2.2).

As shown in Fig. 2.13, when a fluid flows in a horizontal pipe, the contact area A_j between two layers of fluid slides is $2\pi rl$ when the differential distance dr is taken at any r from the center of the pipe to the wall and the length of the pipe is l .

$$F_n = -A_j \mu \frac{du}{dy} = -2\pi rl \mu \frac{du}{dr}$$

where

r —Radius of any point between the center of the pipe and the wall of the pipe, m

l —Pipe length, m

A_j —Force area equal to the contact area of the fluid between the adjacent two layers in the pipe at that time, which is $A_j = 2\pi rl$, m²

$\frac{du}{dy}$ —Normal velocity gradient, which is the change rate of the fluid velocity in the y direction perpendicular to the flow direction, s⁻¹

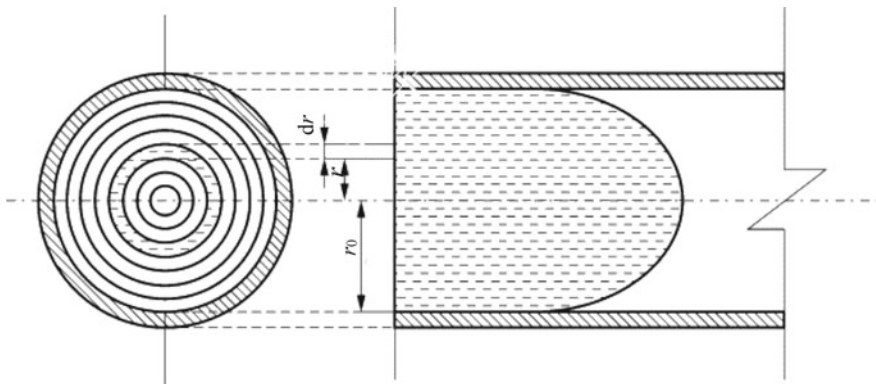


Fig. 2.13 Friction resistance in laminar flow

F_n —Internal friction resistance, which is equal to the magnitude of the force applied but opposite to the direction, N or kg·m/s².

To overcome the internal friction resistance, there must be a certain pressure difference between the two ends of the fluid flow in the pipe, and the pressure difference between the two ends of the fluid is:

$$F'_n = \Delta p A = \Delta p \pi r^2$$

where

A —Force area, which is equal to the cross-sectional area of the fluid perpendicular to the flow direction in the pipe at that time, $A = \pi r^2$, m²

Δp —Pressure difference between the two ends of the fluid flow in the pipe, Pa

F'_n —Force difference between the two ends of the fluid flow in the pipe, N or kg · m/s².

$$F_n = F'_n, \text{ so}$$

$$\Delta p \pi r^2 = -2\pi r l \mu \frac{du}{dr}$$

When $r = 0$ (pipe center), $u = u_{\max}$; and when $r = r_0$ (pipe wall), $u = 0$, and following equation will be obtained:

$$\int_0^{r_0} -\Delta p r dr = \int_{u_{\max}}^0 2\mu l du$$

$$\Delta p \frac{r_0^2}{2} = 2\mu l u_{\max}$$

Because $d = 2r_0$, $u_{\max} = 2u$ in laminar flow; by substituting the upper equation and re-sorting, following equation will be obtained:

$$\Delta p = 32\mu ul/d^2$$

The identical deformation is:

$$\Delta p = \frac{64}{\frac{du\rho}{\mu}} \cdot \frac{l}{d} \cdot \frac{u^2\rho}{2}$$

This can also be written as follows:

$$\Delta p_f = \frac{64}{Re} \cdot \frac{l}{d} \cdot \frac{u^2\rho}{2} \quad \text{Pa} \quad (2.10)$$

Equation (2.10) is then divided by ρ :

$$h_f = \frac{64}{Re} \cdot \frac{l}{d} \cdot \frac{u^2}{2} \quad \text{J/kg} \quad (2.11)$$

Equation (2.11) is then divided by g :

$$H_f = \frac{64}{Re} \cdot \frac{l}{d} \cdot \frac{u^2}{2g} \quad \text{m Liquid column} \quad (2.12)$$

Equations (2.10), (2.11) and (2.12) are equations used for calculating the straight pipe friction loss of the fluid in a laminar fully developed circular straight pipe flow.

Let $\lambda = 64/Re$; then, (2.10) is:

$$\Delta p_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2\rho}{2} \quad \text{Pa} \quad (2.13a)$$

Equation (2.11) is:

$$h_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2}{2} \quad \text{J/kg} \quad (2.13b)$$

Equation (2.12) is:

$$H_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2}{2g} \quad \text{m Liquid column} \quad (2.13c)$$

Equations (2.13a), (2.13b) and (2.13c) are general equations used for calculating the straight pipe friction loss of the fluid in a laminar fully developed circular straight pipe flow or the straight pipe friction loss of fluid in laminar flow.

where

l —Pipe length, m

d —Inner diameter of the pipe, m

u —Average velocity, m/s

Δp_f —The straight pipe friction loss, mechanical energy consumed for fluid flow to overcome resistance, Pa

h_f —The straight pipe friction loss, mechanical energy consumed for fluid flow to overcome resistance, J/kg

H_f —The straight pipe friction loss, mechanical energy consumed for fluid flow to overcome resistance, m Liquid column

λ —Friction coefficient, dimensionless.

The physical meaning of λ represents the ratio of the friction loss over kinetic energy, in which the friction loss is along the flow path per unit mass of fluid flowing through a section of pipe at the same distance as the diameter of the pipe.

2) Straight pipe friction loss in turbulent flow and dimensional analysis

Turbulent flow is a very complex phenomenon. There are many factors affecting the friction loss. It is difficult to derive the relationship between the straight pipe friction loss under a condition of turbulent flow from theoretical analysis. To solve this kind of engineering problem, experimental methods are often used, and a set of theories is used to guide the experiment, data collation, and the use of results. This approach is called dimensional analysis.

The main basis of dimensional analysis is the principle of dimensional consistency and the π theorem.

The principle of dimensional consistency refers to the fact that all the physical quantity equations derived from the basic physical laws must have the same dimension; that is, the dimension on the left side of the physical quantity equation is the same as that on the right side.

The π theorem (Buckingham P_i Theorem) refers to any dimensionally consistent physical equation that can be expressed as a set of zero functions $f(\pi_1, \pi_2, \dots, \pi_n) = 0$ of a group of dimensionless numbers. The number of dimensionless groups is equal to the number of physical quantities affecting process n minus the number of basic dimensions m used to represent these physical quantities. In the following analysis of flow resistance, the total number of physical quantities affecting the process is 7, which means that if the basic dimension of these physical quantities is 3, then the number of dimensionless groups N is:

$$N = n - m = 7 - 3 = 4$$

Any physical equation consists of a set of physical quantities, and any physical quantity has a certain dimension. There are two kinds of dimensions: basic dimensions, which are independent of each other, cannot be derived from each other, and must be set artificially; and derived dimensions, which are derived from basic dimensions. For example, mass, length and time are basic dimensions in SI system. The dimension of velocity can be defined as length and time. Its dimension is length/time (L/τ). Thus, weight is typically expressed by ML/τ^2 .

Next, the dimensionless group that affects the straight pipe friction loss in the turbulent flow will be found and determined by the dimensionless analysis method.

Based on the analysis of the fluid resistance in a turbulent flow and the systematic analysis of the experiments, it can be concluded that the main factors affecting the straight pipe friction loss in the turbulent flow are the diameter (d), length (l), average velocity (u), fluid density (ρ), viscosity (μ), and absolute roughness of the pipe wall (ε); thus,

$$\Delta p_f = F(d, l, u, \rho, \mu, \varepsilon)$$

In the equation, ε is called absolute roughness, which represents the average height of the protruding part of the wall surface with the unit of meter.

If this relation is expressed in the form of a power function, it can be written as follows:

$$\Delta p_f = K d^a l^b u^c \rho^e \mu^f g^g \quad (2.14)$$

The constant K and exponents a, b, c, e, f and g in the equation are undetermined values. The dimensions of each physical quantity in the above equation are expressed as basic dimensions: mass (M), length (L), and time (τ):

$$[p] = ML^{-1}\tau^{-2}$$

$$[d] = [l] = L$$

$$[u] = L\tau^{-1}$$

$$[\rho] = ML^{-3}$$

$$[\mu] = ML^{-1}\tau^{-1}$$

$$[\varepsilon] = L$$

The dimension of each physical quantity is substituted into Eq. (2.14) to obtain:

$$ML^{-1}\tau^{-2} = K[L]^a[L]^b[L\tau^{-1}]^c[ML^{-3}]^e[ML^{-1}\tau^{-1}]^f[L]^g$$

Thus, $ML^{-1}\tau^{-2} = K[M]^{e+f}[L]^{a+b+c-3e-f+g}[\tau]^{-c-f}$.

According to the principle of dimensional consistency, the exponents of the basic dimensions on both sides of the equation should be equal.

$$\begin{cases} e + f = 1 & \text{for } M \text{ (mass)} \\ a + b + c - 3e - f - g = -1 & \text{for } L \text{ (length)} \\ -c - f = -2 & \text{for } \tau \text{ (time)} \end{cases}$$

There are six unknowns in this system of equations, but only three equations, so the values of each unknowns cannot be solved simultaneously. However, three of them can be expressed by other three unknowns:

$$\begin{cases} a = -b - f - g \\ c = 2 - f \\ e = 1 - f \end{cases}$$

By substituting this relation into Eq. (2.14):

$$\Delta p_f = K d^{b-f-g} l^b u^{2-f} \rho^{1-f} \mu^f g^g$$

The physical quantities with the same exponent are grouped together:

$$\frac{\Delta p_f}{\rho u^2} = K \left(\frac{l}{d} \right)^b \left(\frac{du\rho}{\mu} \right)^{-f} \left(\frac{\varepsilon}{d} \right)^g \quad (2.15)$$

Equation (2.15) can be rewritten into a general form:

$$\frac{\Delta p_f}{\rho u^2} = f \left(\frac{l}{d}, \frac{du\rho}{\mu}, \frac{\varepsilon}{d} \right) \quad (2.15a)$$

Equation (2.15a) includes four dimensionless groups, while Eq. (2.14) involves seven physical parameters. Obviously, the number of variables can be reduced by dimensional analysis. It is much simpler to conduct experiments according to Eq. (2.15) than (2.14).

In addition, according to the test of the π theorem, the total number of physical parameters involved in the process is 7, and the number of basic dimensions is 3 (length, mass and time), so there are 4 dimensionless groups.

In the 4 group examples obtained in the above deduction, the length-diameter ratio of pipe (l/d) reflects geometric size characteristic; $du\rho/\mu$ refers to Re , which represents the ratio of inertia force to viscous force and reflects the flow characteristics; ε/d , which is the ratio of absolute roughness to diameter, becomes the relative roughness; $\Delta p'_f/\rho u^2$, which represents the ratio of the pressure drop to the inertia force caused by friction resistance along the path, is called the Euler number and is recorded as Eu .

It is worth noting that the acquisition of dimensionless groups is related to solving simultaneous equations. For example, if b, f , and g are not used to express a, c , and e , the other three dimensionless groups will be obtained. Therefore, it is necessary to pay attention to the physical significance of selecting the obtained dimensionless groups.

Dimensional analysis only considers problems from the dimension of physical quantities and does not involve the essence of the process. It cannot determine which factors have an impact on the process. If extra factors are selected or necessary factors are omitted, the expected results will not be achieved. After dimensionless analysis, the power functions of dimensionless groups can be obtained, and the specific power functional relationship can only be determined by experiments.

3) Straight pipe friction loss in turbulent flow in a circular straight pipe

When turbulent flow occurs, fluid particles are irregularly disturbed and collide with each other, with viscous resistance and body resistance. The situation is very complex. Currently, the equation for calculating friction loss cannot be obtained by theoretical deduction. The relation of the dimensionless groups affecting the turbulent friction loss can be found by the previous dimensional analysis of Eq. (2.15):

$$\Delta p_f = K \left(\frac{l}{d} \right)^b (Re)^{-f} \left(\frac{\varepsilon}{d} \right)^g u^2 \rho \quad (2.15b)$$

$$\Delta p_f = 2K (Re)^{-f} \left(\frac{\varepsilon}{d} \right)^g \left(\frac{l}{d} \right)^b \frac{u^2 \rho}{2} \quad (2.15c)$$

$$\text{Set } \lambda = 2K (Re)^{-f} \left(\frac{\varepsilon}{d} \right)^g$$

Then,

$$\Delta p_f = \lambda \left(\frac{l}{d} \right)^b \frac{u^2 \rho}{2} \quad (2.15d)$$

According to many experiments and proper data processing, $b = 1$. Equation (2.15d) can obtain the following results, which are similar to Eq. (2.13a).

$$\Delta p_f = \lambda \frac{l}{d} \frac{u^2 \rho}{2} \text{ Pa} \quad (2.16)$$

$$h_f = \lambda \frac{l}{d} \frac{u^2}{2} \text{ J/kg} \quad (2.17)$$

$$H_f = \lambda \frac{l}{d} \frac{u^2}{2g} \text{ m Liquid column} \quad (2.18)$$

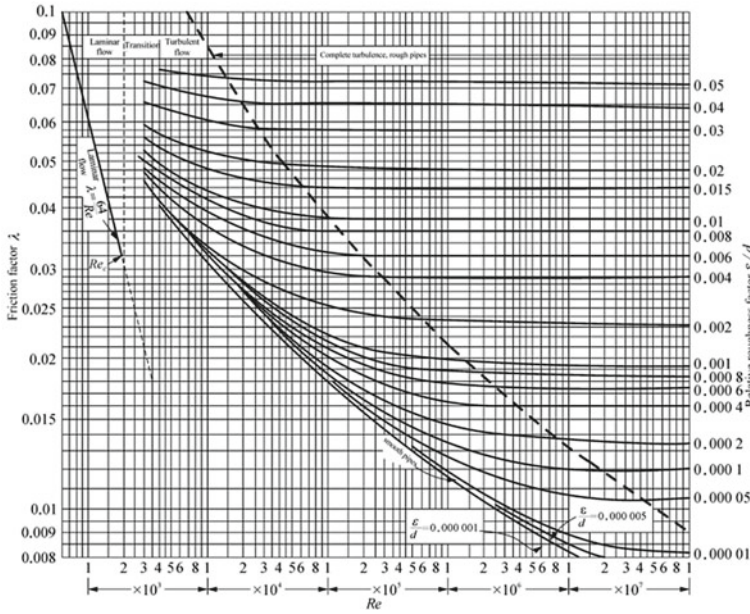


Fig. 2.14 Moody friction coefficient diagram (Moody, 1944). Source of Fig. 2.14: Reprinted from book “Friction Factors for Pipe Flow” with kind permission from American Society of Mechanical Engineers

λ in the equation is also called the frictional coefficient, which is related to Re and the relative roughness of the pipe wall (ϵ/d). Its value can be obtained from the Moody frictional coefficient diagram according to Re and ϵ/d , as shown in Fig. 2.14.

According to Re , Fig. 2.14 can be divided into four regions:

- (1) In the laminar flow region ($Re \leq 2000$), λ is independent of ϵ/d and has a linear relationship with Re , i.e., $\lambda = \frac{64}{Re}$. At this time, $h_f \propto u$, i.e., h_f is proportional to the first power of u .
- (2) In the transition zone ($2000 < Re < 4000$), both the $\lambda \sim Re$ curves of laminar or turbulent flow can be applied. For the friction loss calculation, it is better to estimate a greater value. Generally, the curves of turbulent flow are extended to find the value of λ .
- (3) In the turbulent region ($Re \geq 4000$ and below the dashed line), λ is related to Re and ϵ/d . When ϵ/d is constant, λ decreases with increasing Re . When Re increases to a certain value, λ decreases slowly. When Re is constant, λ increases with ϵ/d .
- (4) In the complete turbulent region (above the dashed line), the curves in this region tend to be horizontal; that is, λ is independent of Re but only related to ϵ/d . For a specific pipe with a specific ϵ/d , λ is constant. Therefore, according to the general equation of straight pipe friction loss, $h_f \propto u^2$, so this region is also called the friction loss square region. Figure 2.14 (Moody, 1944) also

shows that a greater relative roughness ε/d leads to a lower Re value when reaching the friction loss square region.

For the frictional coefficient λ in a turbulent flow, some empirical equations can be used to calculate it, except for the Moody frictional coefficient diagram. For smooth pipes (copper pipes, lead pipes, plastic pipes and glass pipes, the inner surface is very smooth, and the influence of pipe wall roughness can be neglected), when $Re = 3 \times 10^3 \sim 1 \times 10^5$, the relation of λ is as follows:

$$\lambda = 0.3164/Re^{0.25} \quad (2.19)$$

This equation is called the Blasius equation.

Example 2.2 Water at 20 °C flows at a velocity of 0.4 m/s in a pipe with a specification of $\Phi 105 \text{ mm} \times 2.5 \text{ mm}$. It is known that the viscosity of water is $1.0050 \times 10^{-3} \text{ Pa}\cdot\text{s}$ and the absolute roughness is 0.02 mm at 20 °C. Please calculate the straight pipe friction loss of water flowing through the length of pipe per meter.

Known: $T = 20 \text{ }^\circ\text{C}$, $u = 0.4 \text{ m/s}$, $\mu = 1.0050 \times 10^{-3} \text{ Pa}\cdot\text{s}$, $d = 105 - 2 \times 2.5 = 100 \text{ mm} = 0.1 \text{ m}$.

Ask: h_f , H_f , Δp_f .

Solution: When the flow velocity is 0.4 m/s, the solution is obtained.

$$Re = \frac{du\rho}{\mu} = \frac{0.1 \times 0.4 \times 1000}{1.0050 \times 10^{-3}} = 39801 = 3.98 \times 10^4 \geq 4000$$

Therefore, the flow pattern is turbulent at

$$\frac{\varepsilon}{d} = \frac{0.02}{100} = 0.0002,$$

Look up the Moody friction coefficient diagram of Fig. 2.14, $\lambda = 0.023$.

The straight pipe friction loss per meter of pipe is

$$h_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2}{2}$$

$$\Delta p_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2 \rho}{2}$$

$$H_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2}{2g}$$

Then,

$$h_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2}{2} = 0.023 \times \frac{1}{0.1} \times \frac{0.4^2}{2} = 0.0184 \text{ J/kg}$$

$$\Delta p_f = 0.023 \times \frac{1}{0.1} \times \frac{0.4^2 \times 1000}{2} = 18.4 \text{ Pa}$$

$$H_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2}{2g} = 0.023 \times \frac{1}{0.1} \times \frac{0.4^2}{2 \times 9.81}$$

$$= 1.88 \times 10^{-3} \text{ m H}_2\text{O column} = 1.88 \text{ mm H}_2\text{O column}$$

Answer: The friction resistance over the length of the pipe is 0.0184 J/kg, 18.4 Pa, or 1.88 mm H₂O column.

2.2.3 Local Loss

The calculation of local loss is a very complicated problem. Because there are many kinds and specifications of fittings, valves and other devices, etc., their local losses are difficult to calculate accurately. Usually, the following approximation methods are used.

1) Local loss coefficient method

It is approximately considered that the local loss conforms to the square law, that is:

$$h'_f = \zeta \cdot \frac{u^2}{2} \text{ J/kg} \quad (2.20a)$$

$$\Delta p'_f = \zeta \cdot \frac{u^2 \rho}{2} \text{ Pa} \quad (2.20b)$$

$$H'_f = \zeta \cdot \frac{u^2}{2g} \quad \text{m Liquid column} \quad (2.20c)$$

where

ζ —Local loss coefficient, dimensionless, measured experimentally, as listed in Table 2.2.

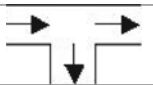
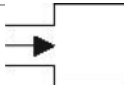
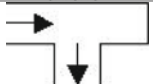

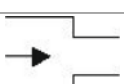

2) Equivalent length method

It is approximately considered that the local loss is equivalent to the friction loss of a straight pipe of a certain length.

$$h'_f = \lambda \cdot \frac{l_e}{d} \cdot \frac{u^2}{2} \text{ J/kg} \quad (2.21a)$$

$$\Delta p'_f = \lambda \cdot \frac{l_e}{d} \cdot \frac{u^2 \rho}{2} \text{ Pa} \quad (2.21b)$$

Table 2.2 Local resistance coefficient ζ (Hu et al., 2015; He and Li, 2014)

Name		Z	Name	ζ	
45° regular elbow		0.35	90° angle valve	5	
90° regular elbow		0.75	Full open gate valve	0.17	
180° return elbow		1.5	Full open diaphragm valve	2.3	
Woggle joint		0.4	Faucet 20°	1.56	
Water meter (disc)		7	Full open globe valve	6.4	
Bottom valve		2	Half open globe valve	9.5	
Water filter		2	Swing-check valve	2	
Tees		0.4	Sudden enlargement		1
		1.3			
		1.5	Abrupt contraction		0.5
		1.0			

Source of Table 2.2: Reprinted from Table 3.3.4 in book “*Principles of Environmental Engineering*” with kind permission from Prof. Hongying Hu, as well as Table 3 in book “*Principles of Environmental Engineering*” with kind permission from Prof. Wenzhi He

$$H'_f = \lambda \cdot \frac{l_e}{d} \cdot \frac{u^2}{2g} \quad \text{m Liquid column} \quad (2.21c)$$

where l_e is the equivalent length of fittings or valves, m. The values are determined by experiments and are usually expressed in terms of their ratio to diameter, as shown in Table 2.3.

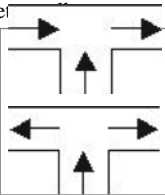
The advantage of using the equivalent length method is that it is easy to calculate the total loss of a pipe system of the straight pipe by combining the friction loss of the straight pipe with the local loss. When d is unchanged, the equation is as follows:

$$h_{tf} = \lambda \cdot \frac{l + \sum l_e}{d} \cdot \frac{u^2}{2} \text{ J/kg} \quad (2.22)$$

$$\Delta p_{tf} = \lambda \cdot \frac{l + \sum l_e}{d} \cdot \frac{u^2 \rho}{2} \text{ Pa} \quad (2.23)$$

$$H_{tf} = \lambda \cdot \frac{l + \sum l_e}{d} \cdot \frac{u^2}{2g} \text{ m Liquid column} \quad (2.24)$$

Table 2.3 Ratio of equivalent length to diameter of various fittings, valves, and flowmeters (Hu et al. 2015; He and Li 2014)

Name	l_e/d	Name	l_e/d
45° regular elbow	15	Full open globe valve	300
90° regular elbow	30 ~ 40	Full open angle valve	145
90° square elbow	60	Full open gate valve	7
180° ref	50 ~ 75	Full open swing-check valve	135
<div><div>Tees</div></div>	40 60	Full open bottom valves with water filters	420
		Disc flowmeter (water meter)	400
		Venturi flow meter	12
		Rotameter	200 ~ 300

Source of Table 2.3: Reprinted from Table 3.4.5 in book “Principles of Environmental Engineering” with kind permission from Prof. Hongying Hu, as well as Table 3 in book “Principles of Environmental Engineering” with kind permission from Prof. Wenzhi He

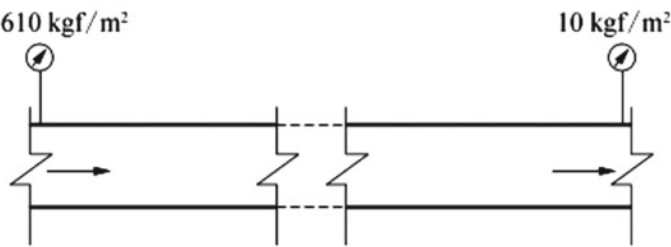


Fig. 2.15 Diagram of Example 2.3

Example 2.3 Figure 2.15 shows that the wastewater with $\mu = 7.2 \times 10^{-2} \text{ Pa} \cdot \text{s}$ is transported by a $\Phi 108 \text{ mm} \times 4 \text{ mm}$ steel pipe with the flow rate of 15 T/h, and a pressure of 610 kgf/m² (gauge pressure) at the outlet of the pump at the starting station and 10 kgf/m² (gauge pressure) at the end. If the pipe is installed horizontally, there are five 90° regular elbows, three 45° regular elbows, one water meter (disc type) and two gate valves (fully open). What is the length of the pipe from the pump outlet to the end cross section of the pipe?

Known: $d = 108 - 2 \times 4 = 100 \text{ mm} = 0.1 \text{ m}$
 $\rho = 1000 \text{ kg/m}^3$
 $p_1 = 610 \text{ kgf/m}^2 = 5978 \text{ Pa}$
 $p_2 = 10 \text{ kgf/m}^2 = 98 \text{ Pa}$
 $q_V = 15 \text{ T/h} = 15 \text{ m}^3/\text{h} = 4.17 \times 10^{-3} \text{ m}^3/\text{s}$
 $\mu = 7.2 \times 10^{-2} \text{ Pa} \cdot \text{s}$
 $\zeta_{\text{total}} = 5\zeta_{90^\circ \text{elbow}} + 3\zeta_{45^\circ \text{elbow}} + \zeta_{\text{watermeter}} + 2\zeta_{\text{valve}} = 5 \times 0.75 + 3 \times 0.35 + 7 + 2 \times 0.17 = 12.14$
Request: l
Solution:

$$u = \frac{q_v}{\frac{\pi}{4}d^2} = \frac{4.17 \times 10^{-3}}{\frac{3.14}{4} \times 0.1^2} = 0.53 \text{ m/s}$$

$$Re = \frac{dup}{\mu} = \frac{0.1 \times 0.53 \times 1000}{7.2 \times 10^{-2}} = 736 < 2000$$

Thus, the wastewater in this pipe flows in a laminar flow.

The total friction loss of the pipe is equal to the pressure drop from the starting cross section to the end cross section when it is placed horizontally.

$$\Delta p_f + \Delta p'_f = p_1 - p_2$$

$$\frac{64}{Re} \times \frac{l}{d} \times \frac{u^2 \rho}{2} + \xi_{\text{total}} \times \frac{u^2 \rho}{2} = p_1 - p_2$$

$$\frac{64}{736} \times \frac{l}{0.1} \times \frac{0.53^2 \times 1000}{2} + 12.14 \times \frac{0.53^2 \times 1000}{2} = 5978 - 98$$

$$122.1l + 1705 = 5880$$

Solution:

$$l = 34.2 \text{ m}$$

Answer: The length of the water pipe from the pump outlet to the terminal cross section is 34.2 m.

2.3 Balance Equation for a Steady Flow System and Its Application

The method of system balance in a steady flow of fluid balances the mass, energy and momentum of the process by the principles of mass conservation, energy conservation (the first law of thermodynamics) and momentum conservation (Newton's second law) to obtain the internal relationship and variation law between physical quantities.

In the accounting process, it is necessary to specify the spatial scope of accounting, i.e., the "control body", and call the closed boundary enclosing the control body the "control surface". The control body can be selected according to the actual needs. It can choose either a macroscale range for general or macroscale balance or a particle or microcluster of a moving fluid for differential balance. The total balance is based on the changes in the external (import, export and environment) physical quantities of the macroscale control body to examine the average changes in the internal physical quantities of the control body. Total balance can solve many practical problems in environmental engineering, such as material balance, energy conversion and

consumption, and equipment stress. The differential equation of process change is established by differential equilibrium basing on the study of the relationship of the physical quantities on the fluid particle in time and space, and then the motion law of the whole field is obtained by integration.

2.3.1 Mass Balance Equation

Mass balance is usually called material balance, which reflects the relationship between various materials in the production process, such as raw materials, products, and byproducts. The principle of mass balance can also be used to track the migration and transformation of pollutants in the environment.

In practical engineering in environmental fields, continuous steady operation is used in most cases; that is, steady flow is the main fluid flow. Therefore, only the mass balance equation in a steady flow system is discussed in this section.

Pipelines with different diameters are shown in Fig. 2.16. Mass balance is conducted between the cross sections 1–1' and 2–2'. The fluid is regarded as a continuous medium, that is, the fluid fills the pipeline and continuously flows out from cross section 1–1' to cross section 2–2'. For a steady flow system, the basic relationship of material balance is that input equals output. The material balance equation is $q_{m1} = q_{m2}$. Because $q_m = uA\rho$, the above equation can be written as follows:

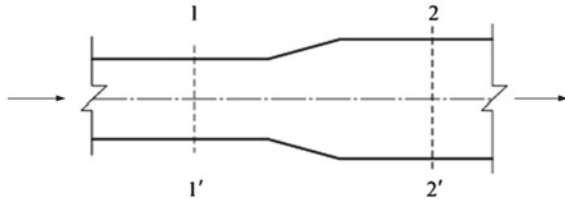
$$u_1 A_1 \rho_1 = u_2 A_2 \rho_2 \quad (2.25)$$

The above equation can be extended to any cross section, that is:

$$q_m = u_1 A_1 \rho_1 = u_2 A_2 \rho_2 = u_3 A_3 \rho_3 = \cdots = uA\rho = \text{constant} \quad (2.25a)$$

Equation (2.25a) denotes that in a steady flow system, the mass flow rate of the fluid through each cross section remains unchanged, while the velocity u varies with the cross-sectional area A of the pipeline and the density of the fluid. If the fluid can be regarded as an incompressible fluid, i.e., $\rho = \text{constant}$, then Eq. (2.25a) can be rewritten as follows:

Fig. 2.16 Mass balance of pipes with different diameters



$$q_V = u_1 A_1 = u_2 A_2 = u_3 A_3 = \dots = u A = \text{constant} \quad (2.25b)$$

Equations (2.25) ~ (2.25b) are called as continuity equations for steady flows in pipeline. They reflect the law of flow velocity change on each cross section of the pipeline when the flow rate is constant in a steady flow system, which is independent of the shape of the pipeline, fittings, valves or conveying equipment in the pipeline.

Equations (2.25) ~ (2.25b) are established for the same total flow rate without inflow rate or outflow rate between two cross sections. If there are inflow rate or outflow rate between two cross sections, the continuity equation should be changed accordingly. When there is an inflow rate addition as shown in Fig. 2.17, the continuity equation is as follows:

$$q_{m1} + q_{m2} = q_{m3} \quad (2.26)$$

When ρ is constant,

$$q_{V1} + q_{V2} = q_{V3}$$

The continuity equation is shown in Fig. 2.18 when there is an outflow rate.

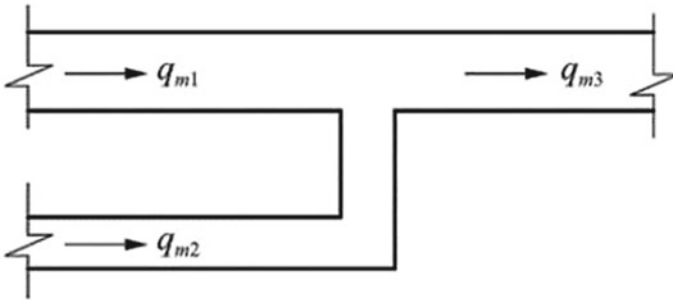


Fig. 2.17 Inflow

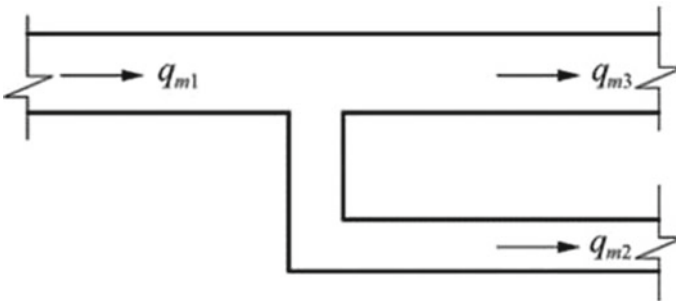


Fig. 2.18 Outflow

$$q_{m1} = q_{m2} + q_{m3}$$

When ρ is constant,

$$q_{V1} = q_{V2} + q_{V3}$$

Example 2.4 Biological filter technology has been widely used in wastewater treatment. Assume a circular biological filter with a diameter of 10 m. A rotating water distributor is installed in the center. The water distributor is 4.5 m in length, and some small holes with a diameter of 3 mm are opened at the bottom of the pipe. Water enters the distribution pipe at a flow rate of 108 m³/h, and the flow velocity is 0.3 m/s. Let all the water passing through the orifice have the same velocity of 5 m/s. Determine the following:

- (1) The number of small holes at the bottom of the distribution pipe.
- (2) The diameter of distribution pipe.

Known: $q_{V1} = 108 \text{ m}^3/\text{h}$, $u_1 = 0.3 \text{ m/s}$, $u_2 = 5 \text{ m/s}$, $d_2 = 3 \text{ mm}$.

Requirement: Number of orifices n , d_1 .

Solution: Assuming that the cross-sectional areas of the pipes and holes are A_1 and A_2 , respectively, according to the continuity equation of incompressible fluid,

$$u_1 A_1 = n u_2 A_2$$

$$n = \frac{u_1 A_1}{u_2 A_2} = \frac{q_{V1}}{u_2 A_2} = \frac{108/3600}{5 \times \pi \times 0.003^2/4} = 849.3 \approx 849$$

$$u_1 \cdot \frac{\pi d_1^2}{4} = q_{V1}$$

$$d_1 = \sqrt{\frac{4q_{V1}}{\pi u_1}} = \sqrt{\frac{4 \times 108/3600}{\pi \times 0.3}} = 0.36 \text{ m} = 360 \text{ mm}$$

Answer: The number of orifices is 849, and the diameter of the distribution pipe is 360 mm.

2.3.2 Energy Balance and Bernoulli Equation

In the field of environmental engineering, there are many processes involved in systemic energy changes, such as flue gas cooling, heat dissipation from equipment and pipelines, energy conversion during the fluid transportation process, and mechanical effects on fluids. Through energy balancing, the amount of heating required by the heating system, the power of the fluid transport machine, the diameter of the pipe,

and the flow rate of fluids can be determined, and the energy change in water bodies such as rivers and lakes can also be analyzed.

The necessary conditions for fluid flow are the pressure differences between both ends of the system. For example, compressed air can be used to convert static pressure energy to kinetic energy; high-level tank can be used to transfer potential energy into kinetic energy. Under the equilibrium conditions of gravity and pressure, the hydrostatic equation can be expressed as:

$$gz_h + \frac{p}{\rho} = \text{constant}$$

The energy conversion law and fluid flow resistance law are discussed, and these laws are applied to solve practical problems. In the steady flow system shown in Fig. 2.19, a fluid flow is assumed to flow from cross section 1-1' and through pipes with different diameters, pump, heat exchanger to 2-2'. Pump and heat exchanger are connected in series in the pipeline. The pipes and equipment between cross section 1-1' and 2-2' comprise a delimited system. Assuming that under steady flow conditions, the flow with a mass of m (kg) per unit time enters the delimitation system from cross section 1-1', the flow of m (kg) per unit time must flow out from cross section 2-2'. Since the fluid itself has certain energy, it carries energy into or out of the delimited system at the same time. The input or output energy of the fluid for the delimited system include the following items:

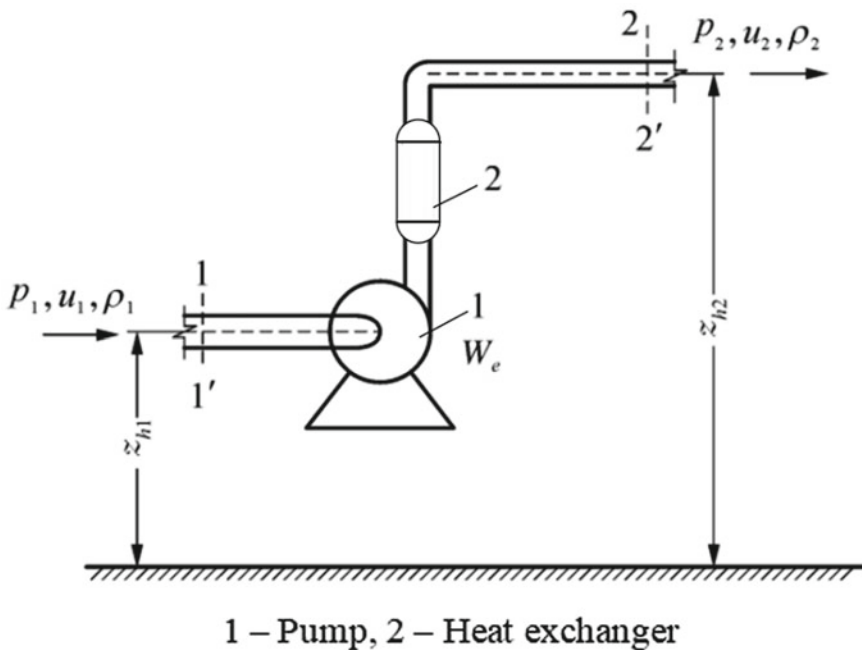


Fig. 2.19 Diagram of energy balance and derivation of Bernoulli equation

1) Potential energy

Fluids have different potential energies at different heights due to the action of gravity. The potential energy is relative and related to the selected reference plane. Its value can be positive, negative or zero. The potential energies at the 1-1' and 2-2' cross sections are mgz_{h1} and mgz_{h2} , respectively, in J.

2) Kinetic energy

When the fluid flows at a certain velocity, it has certain kinetic energy. The kinetic energies at cross sections 1-1' and 2-2' are $\frac{1}{2}mu_1^2$ and $\frac{1}{2}mu_2^2$ respectively, in J.

3) Static pressure energy (pressure energy)

Similar to stationary fluids, there is a certain static pressure at any position in the flowing fluid. When the volume of the fluid is constant, the work needs to be done by introducing the fluid into the pressure system. If the mass of the fluid is m (kg), its volume is V , the static pressure at a cross section is p , and the cross-sectional area is A , pressing the fluid with a mass of m (kg) into the delimited system needs to overcome the resistance of pA with the flowing distance of $\frac{V}{A}$. Then the work is as follows:

$$(pA) \frac{V}{A} = pV$$

Therefore, the static pressure energies at the 1-1' and 2-2' cross sections are p_1V_1 and p_2V_2 , respectively, in J.

4) Internal energy

Also known as thermodynamic energy, internal energy is the sum of the internal kinetic energy and the internal potential energy resulted from the interaction force between molecules formed by a great number of molecular movements in the fluid. The internal potential energy is only for compressible gases. The numerical value varies with the temperature and specific volume of the fluid. The internal energy of the fluid at cross sections 1-1' and 2-2' is expressed as mU_1 and mU_2 , respectively, in J.

5) Work

In the pipeline, the pump and other fluid transport equipment in the delimited system work to the fluid and input external energy into the delimited system, or the fluid works to the outside through the hydraulic machinery and outputs energy. If the work received by a fluid of unit mass in the process of delimited the system is W_e (when the fluid receives external work, W_e is positive. The opposite is negative), then the work received by a fluid of mass m (kg) is mW_e , in J.

6) Heat

Fluid heat is absorbed or released through a heat exchanger. If the heat exchanged per unit mass of fluid in and out of the delimited system is Q_e , then the heat absorbed by the fluid with m (kg) mass is mQ_e , in J.

According to the first law of thermodynamics, for any system, the heat transferred from outside inward is Q , and the system changes from an initial state of internal energy E_{n1} to a final state of internal energy E_{n2} . At the same time, the system does work to outside is W .

$$Q = E_{n2} - E_{n1} + W$$

where

Q —Total heat absorbed by the system from the outside is positive. The opposite is negative, J. $Q = m Q_e$, Q_e represents the heat absorbed by a unit mass fluid, J/kg

E_{n1} —Total internal energy of the initial state of the system, J

E_{n2} —Total internal energy of the final state of the system, J

W —The system receiving external work is negative, and the opposite is positive, J

$W = -mW_e$, where W_e represents the work received by a unit mass fluid during the process of passing through the system, which is positive and the opposite is negative, J/kg.

The specific expression of the first law of thermodynamics for the system shown in Fig. 2.19 is as follows:

$$\begin{aligned} mgz_{h1} + \frac{1}{2}mu_1^2 + mU_1 + p_1V_1 + mQ_e + mW_e \\ = mgz_{h2} + \frac{1}{2}mu_2^2 + mU_2 + p_2V_2 \end{aligned} \quad (2.28)$$

By dividing the items in Eq. (2.28) by mass (m) and substituting a specific volume $\Lambda = V/m$, following Eq. (2.29) will be obtained

$$g\Delta z_h + \frac{\Delta(u^2)}{2} + \Delta U + \Delta(p\Lambda) = Q_e + W_e \quad (2.29)$$

Since heat and internal energy can not be directly converted into mechanical energy for fluid transportation, it is necessary to treat heat and internal energy properly and remove them from Eq. (2.29) when considering the energy needed for fluid transportation and the energy transformation and consumption during the conveying process to obtain an equation of mechanical energy balance for the fluid transportation system.

According to the first law of thermodynamics:

$$\Delta U = Q'_e - \int_{\Lambda_1}^{\Lambda_2} p d\Lambda \quad (2.30)$$

where

Q'_e —Energy obtained by 1 kg fluid when it flows from 1–1' to 2–2', J/kg

Λ —Specific volume, i.e., the volume occupied by a material of unit mass, m³/kg

$\int_{\Lambda_1}^{\Lambda_2} p d\Lambda$ —Work done by volume expansion caused by heating when 1 kg fluid flows from 1–1' to 2–2', J/kg.

Q'_e consists of two parts. One part is the heat directly obtained by the fluid through the environment, as shown by the heat exchanger. The other part is the work done by the resistance of the fluid when it flows from cross section 1–1' to cross section 2–2', thus consuming mechanical energy and converting it into heat. If the isothermal flow of the fluid is considered, this portion of heat is deemed to be lost outside the flow system. This part of the energy is usually the energy loss caused by flow resistance, referred to as resistance loss or friction loss. Let 1 kg of fluid flow in the delimited system, and the energy lost by overcoming the flow resistance is measured in J/kg; then,

$$Q'_e = Q_e + \sum h_{tf} \quad (2.31)$$

Equations (2.30) and (2.31) are substituted into Eq. (2.29):

$$g\Delta z_h + \frac{\Delta(u^2)}{2} + \Delta(p\Lambda) - \int_{\Lambda_1}^{\Lambda_2} p d\Lambda = W_e - \sum h_{tf} \quad (2.32)$$

Since

$$\Delta(p\Lambda) = \int_{\Lambda_1}^{\Lambda_2} p d\Lambda + \int_{p_1}^{p_2} \Lambda dp$$

Equation (2.32) is revised as follows:

$$g\Delta z + \frac{\Delta(u^2)}{2} + \int_{p_1}^{p_2} \Lambda dp = W_e - \sum h_{tf} \quad (2.33)$$

Equation (2.33) is the equation of mechanical energy balance for the steady flow of fluid, which is applicable to both incompressible and compressible fluids. For compressible fluids, $\int_{p_1}^{p_2} \Lambda dp$ should be calculated according to the relationship between Λ and p according to different processes (isothermal, adiabatic or multivariate processes).

For incompressible fluids, the specific volume Λ or density ρ is a constant, and the integral term of Eq. (2.33) can be further simplified to obtain:

$$g\Delta z_h + \frac{\Delta(u^2)}{2} + \Lambda\Delta p = W_e - \sum h_{tf}$$

$$gz_{h1} + \frac{u_1^2}{2} + \frac{p_1}{\rho} + W_e = gz_{h2} + \frac{u_2^2}{2} + \frac{p_2}{\rho} + \sum h_{tf} \text{ J/kg} \quad (2.34a)$$

If the fluid is an ideal fluid, there is no flow resistance, and then the energy loss of the fluid flow is zero, that is $\Sigma h_{tf} = 0$. In addition, if there is no work input or output ($W_e = 0$) in delimited system at same time, Eq.(2.34a) can be simplified as follows:

$$gz_{h1} + \frac{u_1^2}{2} + \frac{p_1}{\rho} = gz_{h2} + \frac{u_2^2}{2} + \frac{p_2}{\rho} \quad \text{J/kg} \quad (2.34)$$

where

z_h —Vertical distance between the fluid and the datum, m

u —Velocity of flow, the distance over which a fluid travels in the direction of flow in a unit time, m/s

p —Pressure, pressure force per unit surface area of fluid, kg/(m·s²) or Pa

W_e —Work received by a unit mass fluid in the process of the delimited system, which is positive, the opposite is negative, J/kg

Σh_{tf} —Total friction loss caused by the flow of a unit mass fluid in the delimited system due to overcoming flow resistance, J/kg

g —Gravity acceleration, 9.81 m/s²

ρ —Density, kg/m³.

Equation (2.34) is called the Bernoulli equation. Equation (2.34a) is an extension of the Bernoulli equation and is also called the Bernoulli equation.

The Bernoulli Equation (2.34) can only be applied to cases where an incompressible ideal fluid flows steadily without work input or output. This indicates that the sum of potential energy, kinetic energy and static pressure energy per unit mass fluid at any cross section is a constant. This sum is called total mechanical energy, in J/kg. The total mechanical energy of an ideal fluid of 1 kg is the same on any cross section, but the mechanical energy of various forms is not necessarily equal and can be converted to each other.

When the fluid is stationary, i.e., u is zero, the Bernoulli equation becomes a hydrostatic equation.

The Bernoulli equation is derived from the mechanical energy balance equation of the flow system. If different benchmarks are used in the calculation, several different forms of the Bernoulli equation can be obtained.

Equation (2.34) is a relationship based on the unit mass fluid, in which the unit is J/kg.

If the unit volume is used as the basis of calculation, then:

$$\rho gz_{h1} + \frac{\rho u_1^2}{2} + p_1 + W_{e-pa} = \rho gz_{h2} + \frac{\rho u_2^2}{2} + p_2 + \Delta p_{tf} \quad \text{Pa} \quad (2.34b)$$

The units in the equation are all Pa for different items.

If the unit weight fluid is used as the basis of calculation, then:

$$z_{h1} + \frac{u_1^2}{2g} + \frac{p_1}{\rho g} + H_e = z_{h2} + \frac{u_2^2}{2g} + \frac{p_2}{\rho g} + \sum H_{tf} \quad \text{m Liquid column} \quad (2.34c)$$

where $H_e = W_e/g$, $\sum H_{tf} = \sum h_{tf}/g$, and each unit is an m Liquid column. z_h , $\frac{u^2}{2g}$ and $\frac{p}{\rho g}$ are the potential head, dynamic head, and static head, respectively, H_e is the lift head, and $\sum H_{tf}$ is the total head loss.

The application of the Bernoulli equation should focus on the following points:

- (1) When determining the accounting scope, a schematic diagram of the flow system should be drawn to indicate the flow direction of the fluid. The upstream and downstream cross sections are determined to define the accounting range of the flow system.
- (2) The selection of cross sections should be perpendicular to the flow direction, and the fluid between the two cross sections must be continuous. The cross section with a rapid change of current flow is not allowed as a cross section, but it is allowed between the two selected cross sections. The unknown parameter required should be on or between the two selected cross sections, and the related physical quantities, such as z_h , u , and p , on the cross section should be known or calculated through other relations except the unknown quantity required. u , p , z_h and the total friction loss between the two cross sections should have corresponding units. If the surface of the container is large, the velocity inside the container is generally too slow relative to the velocity inside the pipe, which can be neglected; then, $u_{\text{surface}} \approx 0$.
- (3) The purpose of selecting the datum level is to determine the potential energy of the fluid. In fact, the value of the potential energy difference ($\Delta z = z_{h2} - z_{h1}$) is reflected in the Bernoulli equation. Therefore, the datum level can be chosen arbitrarily, but it must be parallel to the ground. The z_h value refers to the vertical distance between the center point of the cross section and the datum horizontal plane. For convenience of calculation, the datum level is usually taken the one of the two cross sections within the calculation range. If the cross section is parallel to the ground, the datum level coincides with the cross section, where $z_h = 0$. If the balance system is a horizontal pipeline, the datum level passes through the central line of the pipe, where $z_h = 0$.
- (4) The units must be consistent. Before using the Bernoulli equation, the physical quantities concerned should be converted into uniform units (J/kg, Pa, m liquid column) and then calculated. The pressure of two cross sections requires not only the same unit but also the same method of expression. From the derivation process of the Bernoulli equation, it is known that the pressure of the two cross sections in the equation is absolute pressure, but because the pressure difference ($\Delta p = p_2 - p_1$) is reflected in the equation and the absolute pressure is atmospheric pressure + gauge pressure, the pressure of the two cross sections can also be expressed by gauge pressure at the same time.

The external work W_e and total friction loss $\sum h_{tf}$ are the energy obtained or consumed during the fluid flow process. W_e is the work done by the transport equipment to the fluid of unit mass. The effective power of the work done by the transport equipment to the fluid is expressed by P_e .

$$P_e = W_e q_m = W_e q_V \rho = H_e q_V \rho g \quad (2.35)$$

$$P_a = P_e / \eta \quad (2.35a)$$

$$P_d = P_a / \eta' \quad (2.35b)$$

where

P_e —Effective power of a transport equipment (such as a pump) to work on a fluid, W or $\text{kg}\cdot\text{m}^2/\text{s}^3$

P_a —Power of a transport equipment, such as a pump, W or $\text{kg}\cdot\text{m}^2/\text{s}^3$

η —Efficiency of transport equipment (e.g., pumps), %

P_d —Motor power matching transport equipment (such as pump), W or $\text{kg}\cdot\text{m}^2/\text{s}^3$

η' —Efficiency of motor transmission power, %

W_e —Work received by a unit mass fluid in the process of passing through a delimited system, J/kg or m^2/s^2

q_m —Mass flow, kg/s

q_V —Volume flow, m^3/s

ρ —Fluid density, kg/m^3

H_e —Lift head, m liquid column

g —Gravity acceleration, $9.8 \text{ m}/\text{s}^2$.

Example 2.5 Clear water (density $1000 \text{ kg}/\text{m}^3$) at 20°C is pumped from an open storage tank to a closed container with a gauge pressure of $1.5 \times 10^5 \text{ Pa}$ by a centrifugal pump. The water levels of both the storage tank and the container keep unchanged, and the relative position of each part is shown in Fig. 2.20. The pipes are all $\Phi 108 \text{ mm} \times 4 \text{ mm}$ seamless steel pipes. The length of the suction pipe before pumping is 30 m, and the length of the discharge pipe after pumping is 80 m (the length of each section includes the equivalent length of all straight pipe friction losses and local losses). When the valve is 3/4 open, the vacuum meter reading at the pump inlet is 42500 Pa, and the vertical distance between the front and back pressure measuring ports of the pump is 1 m. The friction coefficient is 0.02, and the local atmospheric pressure is $8.5 \times 10^4 \text{ Pa}$, ignoring the friction losses between the two pressure measuring ports. Determine the following:

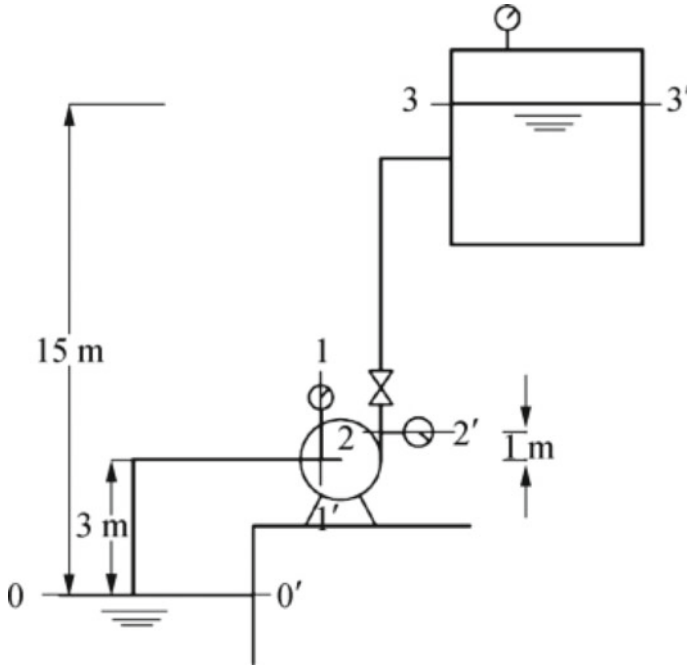


Fig. 2.20 Schematic diagram of Example 2.5

- (1) Flow rate of pipe at 3/4 opening of valve (m^3/h)
- (2) Pump outlet pressure meter reading (Pa)

Solution:

- (1) Apply Bernoulli equation between 0-0' surface liquid level of the tank and 1-1' of cross section at vacuum gauge, with 0-0' as datum level.

Known: $z_{h0} = 0 \text{ m}$, $p_0 = 8.5 \times 10^4 \text{ Pa}$, $u_0 \approx 0$, $z_{h1} = 3 \text{ m}$, $p_1 = p_0 - 42500 \text{ Pa} = 85000 - 42500 = 42500 \text{ Pa}$, $\rho = 1000 \text{ kg/m}^3$,

$d_i = 108 - 2 \times 4 = 100 \text{ mm} = 0.1 \text{ m}$, $l_{0-1} + \Sigma l_{e,0-1} = 30 \text{ m}$, $\lambda = 0.02$, $W_{e,0-1} = 0$.

Requirement: Pipe flow rate at valve 3/4 open.

Solution: The Bernoulli equation is listed.

$$gz_{h0} + \frac{u_0^2}{2} + \frac{p_0}{\rho} + W_{e,0-1} = gz_{h1} + \frac{u_1^2}{2} + \frac{p_1}{\rho} + \Sigma h_{tf,0-1}$$

$$\Sigma h_{tf,0-1} = \lambda \cdot \frac{l_{0-1} + \Sigma l_{e,0-1}}{d_i} \cdot \frac{u_1^2}{2} = 0.02 \times \frac{30}{0.1} \times \frac{u_1^2}{2} = 3u_1^2 \text{ J/kg}$$

Substituted data:

$$0 + 0 + \frac{8.5 \times 10^4}{\rho} + 0 = 9.81 \times 3 + \frac{u_1^2}{2} + \frac{4.25 \times 10^4}{\rho} + 3u_1^2$$

$$0 = 9.81 \times 3 + \frac{u_1^2}{2} + \frac{4.25 \times 10^4}{1000} + 3u_1^2$$

Solution $u_1 = 1.93$ m/s.

Therefore,

$$q_v = \frac{\pi}{4} d^2 u_1 = 0.785 \times 0.1^2 \times 1.93 \times 3600 = 54.57 \text{ m}^3/\text{h}$$

- (2) Apply Bernoulli equation between the central 2–2' cross section at the pressure gauge of the pump outlet and the 3–3' liquid surface level in the container, still based on the 0–0' level as the datum level.

Known: $\rho = 1000 \text{ kg/m}^3$, $d_i = 108 - 2 \times 4 = 100 \text{ mm} = 0.1 \text{ m}$, $l_{2-3} + \Sigma l_{e,2-3} = 80 \text{ m}$, $\lambda = 0.02$, $z_{h2} = 4 \text{ m}$, $z_{h3} = 15 \text{ m}$, $u_1 = u_2 = 1.93 \text{ m/s}$, $u_3 = 0 \text{ m/s}$, $p_3 = 1.5 \times 10^5 \text{ Pa}$, $W_{e,2-3} = 0$.

Requirement: Pump outlet pressure reading p_2 .

Solution: Apply Bernoulli equation.

$$gz_{h2} + \frac{u_2^2}{2} + \frac{p_2}{\rho} + W_{e,2-3} = gz_{h3} + \frac{u_3^2}{2} + \frac{p_3}{\rho} + \Sigma h_{tf,2-3}$$

$$\Sigma h_{tf,2-3} = \lambda \cdot \frac{l_{2-3} + \Sigma l_{e,2-3}}{d_i} \cdot \frac{u_2^2}{2} = 0.02 \times \frac{80}{0.1} \times \frac{1.93^2}{2} = 29.8 \text{ J/kg}$$

$$9.81 \times 4 + \frac{1.93^2}{2} + \frac{p_2}{1000} + 0 = 9.81 \times 15 + \frac{1.5 \times 10^5}{1000} + \frac{0}{2} + 29.8$$

Solution $p_2 = 2.86 \times 10^5 \text{ Pa}$.

Answer: When the valve is 3/4 open, the flow rate of the pipe is $54.57 \text{ m}^3/\text{h}$, and the pump outlet gauge pressure is $2.86 \times 10^5 \text{ Pa}$.

Example 2.6 As shown in the Fig. 2.21, in a wastewater treatment plant, the wastewater from a catchment well is lifted by a pump and then enters the aerated sand settler. It is known that the wastewater temperature is constant at 20°C , the wastewater surface level of the catchment well before the pump is -5 m , the wastewater surface level of the aerated sand settler is 3 m , and the height of the inlet and outlet of the pump is -7 m . The pipes are $\Phi 106 \text{ mm} \times 3 \text{ mm}$, and the friction coefficient λ is 0.0182 . The length of the suction pipe of the pump is 5 m , and the length of the outlet pipe of the pump is 45 m . There are two 90-degree elbows, two gate valves (all open) and one swing-check valve. The gauge pressure at the outlet of the pump is 149 kPa , neglecting the resistance loss of the pump. The efficiency of the pump is 75% .

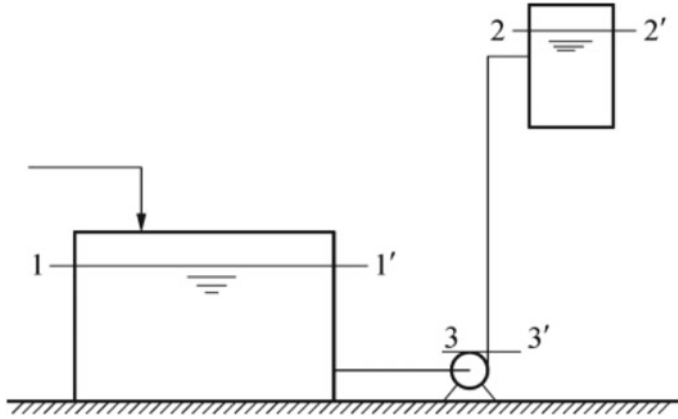


Fig. 2.21 Schematic diagram of Example 2.6

Determine the following:

- (1) Flow velocity in the pipe
- (2) Pump output power

Known:

If the wastewater surface of the catchment well before the pump is 1–1′, the wastewater surface of the aerated settler sink is 2–2′, the wastewater outlet after the pump is 3–3′. If the pump outlet is set as datum level, then $z_{h3} = 0$ m, $z_{h1} = 2$ m, $z_{h2} = 10$ m, $p_1 = p_2 = 0$ kPa, $p_3 = 149$ kPa = 149×10^3 Pa, $u_1 = u_2 = 0$ m/s, $d = 106 - 2 \times 3 = 100$ mm = 0.1 m, $\lambda = 0.0182$, $l_{1-3} = 5$ m, $l_{3-2} = 45$ m, $\zeta_{\text{shrink}} = 0.5$, $\zeta_{90^\circ} = 0.75$, $\zeta_{\text{check valve}} = 2$, $\zeta_{\text{expansion}} = 1.0$, $\zeta_{\text{gate valve}} = 0.17$. The water temperature is 20 °C and from the table: $\rho = 998.2$ kg/m³, $\mu = 1.005 \times 10^{-3}$ Pa · s.

Determine the following:

- (1) Flow velocity in pipe u_3
- (2) Pump power P_a

Solution:

- (1) Establish the Bernoulli equation between the cross section of pump outlet 3–3′ and the wastewater surface of aerated sand settler 2–2′.

$$gz_{h3} + \frac{u_3^2}{2} + \frac{p_3}{\rho} + W_{e,3-2} = gz_{h2} + \frac{u_2^2}{2} + \frac{p_2}{\rho} + \Sigma h_{tf,3-2}$$

Among them:

$$\Sigma h_{tf,3-2} = \left(\lambda \frac{l_{3-2}}{d} + 2 \times \zeta_{90^\circ} + 2 \times \zeta_{\text{gate valve}} + \zeta_{\text{check valve}} + \zeta_{\text{expansion}} \right) \times \frac{u_3^2}{2}$$

$$= \lambda \frac{45}{0.1} + 2 \times 0.75 + 2 \times 0.17 + 2 + 1 \frac{u_3^2}{2} = (450\lambda + 4.84) \frac{u_3^2}{2}$$

Substitution data are available:

$$9.81 \times 0 + \frac{u_3^2}{2} + \frac{149 \times 10^3}{998.2} + 0 = 9.81 \times 10 + \frac{0^2}{2} + \frac{0}{\rho} + (450\lambda + 4.84) \frac{u_3^2}{2}$$

$$(450\lambda + 3.84)u_3^2 = 102$$

The following equation can be obtained by substituting λ with 0.0182:

$$(450 \times 0.0182 + 3.84)u_3^2 = 102$$

$$u_3 = 2.91 \text{ m/s}$$

- (2) The Bernoulli equation is established on the wastewater surfaces of the catchment well (cross section 1-1') and aerated sand settler (cross section 2-2'):

$$gz_{h1} + \frac{u_1^2}{2} + \frac{p_1}{\rho} + W_e = gz_{h2} + \frac{u_2^2}{2} + \frac{p_2}{\rho} + \Sigma h_{tf,1-2}$$

Among

$$\Sigma h_{tf,1-2} =$$

$$\left(\lambda \frac{l_{1-3} + l_{3-2}}{d} + \zeta_{\text{shrink}} + 2 \times \zeta_{90^\circ} + 2 \times \zeta_{\text{gate valve}} + \zeta_{\text{check valve}} + \zeta_{\text{expansion}} \right) \times \frac{u_3^2}{2}$$

$$= (0.0182 \times \frac{5 + 45}{0.1} + 0.5 + 2 \times 0.75 + 2 \times 0.17 + 2 + 1) \times \frac{2.91^2}{2}$$

$$= (9.1 + 5.34) \times \frac{2.91^2}{2} = 61.14 \text{ J/kg}$$

Substituting into the previous equation:

$$9.81 \times 2 + \frac{0^2}{2} + \frac{0}{\rho} + W_{e,1-2} = 9.81 \times 10 + \frac{0^2}{2} + \frac{0}{\rho} + 61.14$$

$$W_{e,1-2} = 9.81 \times 8 + 61.14 = 139.62 \text{ J/kg}$$

$$P_e = q_m \times W_{e,1-2} = \frac{1}{4} \pi d^2 u_3 \rho \times W_e$$

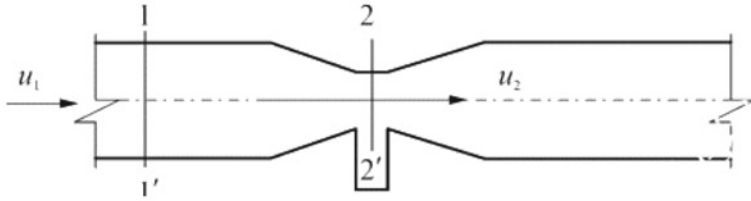


Fig. 2.22 Schematic diagram of Example 2.7

$$= \frac{1}{4} \times 3.14 \times 0.1^2 \times 2.91 \times 998.2 \times 139.62 = 3183 \text{ W} = 3.18 \text{ kW}$$

Output power of pump: $P_a = P_e / \eta = 3.18 / 0.75 = 4.24 \text{ kW}$.

Answer: The flow velocity in the pipe is 2.91 m/s, and the output power of the pump is 4.24 kW.

Example 2.7 A circular pipe is placed horizontally, as shown in Fig. 2.22. Water ($\rho = 1000 \text{ kg/m}^3$) flows from the left of the pipe to the right with a flow rate of $10 \text{ m}^3/\text{h}$.

It is known that the inner diameter of the inlet is 100 mm, the pressure at the inlet is 0.03 MPa (gauge pressure), and there is a small hole in the wall of the middle throat. If air can enter the pipe through the small hole, what conditions for the inner diameter of the throat neck need to be met? (Neglecting all straight pipe friction loss and local losses.)

Known: Inlet diameter $d_1 = 100 \text{ mm} = 0.1 \text{ m}$, flow rate $q_V = 10 \text{ m}^3/\text{h}$, inlet pressure $p_1 = 0.03 \text{ MPa} = 3 \times 10^4 \text{ Pa}$, $\rho = 1000 \text{ kg/m}^3$, $z_{h1} = z_{h2} = 0$, $W_{e,1-2} = 0$, $\Sigma h_{tf,1-2} = 0$.

Requirement: Internal diameter of throat neck d_2 .

Solution: Cross section 1-1' and cross section 2-2' are taken as the entrance and throat neck, and the Bernoulli equation is obtained between the two cross sections.

$$gz_{h1} + \frac{u_1^2}{2} + \frac{p_1}{\rho} + W_{e,1-2} = gz_{h2} + \frac{u_2^2}{2} + \frac{p_2}{\rho} + \Sigma h_{tf,1-2}$$

By ignoring all straight pipe friction loss and local losses between cross section 1-1' and cross section 2-2',

$$0 + \frac{u_1^2}{2} + \frac{p_1}{\rho} + 0 = 0 + \frac{u_2^2}{2} + \frac{p_2}{\rho} + 0$$

$$\text{Among them, } u_1 = \frac{q_V}{\frac{\pi d_1^2}{4}} = \frac{2.78 \times 10^{-3}}{\frac{\pi}{4} \times 0.1^2} = 0.35 \text{ m/s}$$

If the atmospheric pressure is p_0 , then if the air can enter the pipe, then $p_2 < p_0$, and the critical condition is $p_2 = p_0$:

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_2^2}{2} + \frac{p_0}{\rho}$$

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_2^2}{2} + \frac{0}{\rho}$$

$$u_2 = \sqrt{u_1^2 + \frac{2 \times p_1}{\rho}} = \sqrt{0.35^2 + \frac{2 \times 3 \times 10^4}{1000}} = 7.75 \text{ m/s}$$

$$u_2 \cdot \frac{\pi d_2^2}{4} = q_V$$

$$d_2 = \sqrt{\frac{4q_V}{\pi u_2}} = \sqrt{\frac{4 \times 2.78 \times 10^{-3}}{3.14 \times 7.75}} = 0.021 \text{ m} = 21 \text{ mm}$$

When $p_2 < p_0$, $u_2 > 7.75 \text{ m/s}$, $d_2 < 21 \text{ mm}$.

Answer: When the inner diameter of the throat neck is less than 21 mm, air can enter the pipe.

2.3.3 The Calculation of Pipeline System

Pipeline calculation applies the continuity equation, Bernoulli equation and resistance loss calculation equation. Pipelines can be divided into simple pipes and complex pipes according to their configuration. The following are introduced separately.

2.3.3.1 Pipeline system

A simple pipeline system usually indicates a flow of fluid from the inlet to the outlet in a pipeline without branching and junction. The diameter of the whole pipeline can be the same, or it can be composed of pipelines with different diameters in series, as shown in Fig. 2.23.

The calculation principle of a simple pipeline system is as follows:

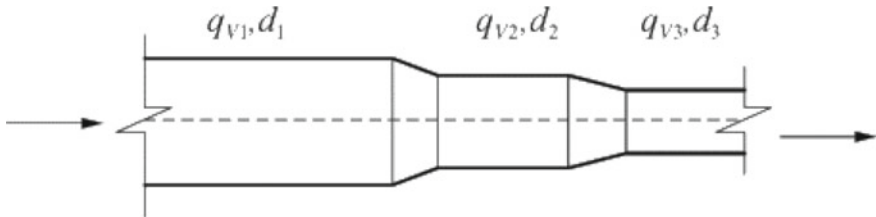


Fig. 2.23 Simple pipeline

- (1) Continuity equation. The mass flow rates of the fluid through each cross section remain unchanged, and the volume flow rates of the incompressible fluid keep unchanged.

$$q_{V1} = q_{V2} = q_{V3} \quad (2.36)$$

- (2) The total energy loss of the whole pipeline is equal to the sum of the energy losses of each section.

$$\sum H_{tf} = H_{tf1} + H_{tf2} + H_{tf3} \quad (2.37)$$

- (3) Bernoulli equation

Because the conditions of both known parameters and unknown parameters are different, the calculation method is also not same. The typical calculation problems of pipelines system can be summarized as follows:

① Given the pipe diameter, length, fittings, valves and the flow rate of fluids, the energy loss of fluid through the pipeline system is calculated to further determine the external work needed by transport equipment, the pressure in the equipment or the relative position between the equipments. This kind of calculation is relatively easy.

② Design calculation: When a pipeline system does not exist, the pipeline system should be designed economically and reasonably, given the transportation task and the length of the pipeline, the equivalent length of the fittings and valves, and the allowable resistance loss.

③ Operational calculation: Once the pipeline system has been determined, that is, the diameter, length, fittings and valves have been set, and the allowable energy loss has been determined, the calculations of the transport capacity or a certain technical parameter are required under another new given condition.

There are selection and optimization for designing calculation. The most economical and reasonable choice of pipe diameter or flow rate should minimize the sum of the annual operating cost and depreciation cost of equipment based on service life.

The operational calculation can present challenges. Because the velocity of flow is unknown, the Re value cannot be calculated, the flow pattern of fluid cannot be judged, and the friction coefficient cannot be determined. In this case, trial and error method and other methods are often used in engineering calculation.

Example 2.8 As shown in Fig. 2.24, the wastewater in the high tank is continuously transported to the low tank during the wastewater treatment process. The wastewater is transported at $23 \text{ m}^3/\text{h}$, the diameter of the pipe is $\Phi 89\text{mm} \times 3.5\text{mm}$, the length of the pipe is 130 m (including the equivalent length of all friction losses), the relative roughness of the pipe wall is 0.0001, the density of the wastewater is 1000 kg/m^3 , and the viscosity is $0.8 \times 10^{-3} \text{ Pa} \cdot \text{s}$. What is the height difference between the wastewater surfaces of the two tanks in meter?

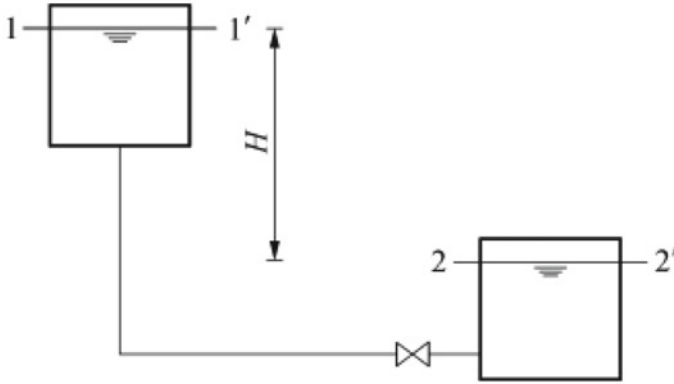


Fig. 2.24 Schematic diagram of Example 2.8

Known: $l_{1-2} + \Sigma l_{e,1-2} = 130$ m, $d = 89 - 2 \times 3.5 = 82$ mm = 0.082 m, $\varepsilon/d = 0.0001$, $q_V = 23$ m³/h, $\rho = 1000$ kg/m³, $\mu = 0.8 \times 10^{-3}$ Pa · s, $W_e = 0$.

Determine: H .

Solution: By taking the wastewater surfaces of the high and low tanks as cross sections 1-1' and 2-2', respectively, and the wastewater surface of the low tank as the datum level, the Bernoulli equation between cross sections 1-1' and 2-2' is obtained.

$$z_{h1}g + \frac{1}{2}u_1^2 + \frac{p_1}{\rho} + W_e = z_{h2}g + \frac{1}{2}u_2^2 + \frac{p_2}{\rho} + \sum h_{tf}$$

Equation: $z_1 = H$, $z_{h2} = 0$, $p_1 = p_2 = 0$ (gauge pressure), $u_1 \approx u_2 = 0$.
Therefore:

$$g(z_{h1} - z_{h2}) + \frac{0}{2} + \frac{0}{\rho} + 0 = \frac{0}{2} + \frac{0}{\rho} + gH = \lambda \cdot \frac{l_{1-2} + \Sigma l_{e,1-2}}{d} \cdot \frac{u^2}{2}$$

Average cross section velocity

$$u = \frac{q_V}{\frac{\pi}{4}d^2} = \frac{23/3600}{\frac{3.14}{4} \times 0.082^2} = 1.21 \text{ m/s}$$

$$Re = \frac{du\rho}{\mu} = \frac{0.082 \times 1.21 \times 10^3}{0.8 \times 10^{-3}} = 1.24 \times 10^5 > 4000 \text{ belongs to the turbulence}$$

According to the frictional coefficient diagrams in Fig. 2.14, $\lambda = 0.018$.

The height difference between the wastewater surfaces of the two tanks is as follows:

$$H = \frac{\sum h_{tf}}{g} = \lambda \cdot \frac{l_{1-2} + \Sigma l_{e,1-2}}{d} \cdot \frac{u^2}{2g} = 0.018 \times \frac{130}{0.082} \times \frac{1.21^2}{2 \times 9.81} = 2.13 \text{ m}$$

Answer: The height difference between the wastewater surfaces of the two tanks is 2.13m.

Example 2.9 Calculation of flow rate by trial-and-error method

Water is drawn from a high water tower by using a $\Phi 100 \text{ mm} \times 4 \text{ mm}$ steel pipe. The length of the steel pipe is 120 m (including the equivalent length of the straight pipe friction loss and local losses but excluding the losses at the inlet and outlet). The water surface in the water tower remains constant, which is 10 m higher than the drainage outlet. The density of water is 1000 kg/m^3 . The viscosity of water at 20°C is $1.005 \times 10^{-3} \text{ Pa} \cdot \text{s}$. The water temperature is 20°C . Please calculate the flow rate of the pipe by trail-and-error method with the requirement of error ≤ 0.0003 .

- (1) If the absolute roughness is equal to 0.2 mm, determine the flow rate q_V .
- (2) If the absolute roughness is equal to 0.002 mm, determine the flow rate q_V .

Known: $d = 100 - 2 \times 4 = 92 \text{ mm} = 0.092 \text{ m}$, $l + \Sigma l'_e = 120 \text{ m}$, $z_{h1} = 10 \text{ m}$, $z_{h2} = 0 \text{ m}$, $T = 20^\circ\text{C}$, $\mu = 1.005 \times 10^{-3} \text{ Pa} \cdot \text{s}$.

- (1) $\varepsilon = 0.2 \text{ mm} = 2 \times 10^{-4} \text{ m}$, $\varepsilon/d = 2.17 \times 10^{-3}$
- (2) $\varepsilon = 0.002 \text{ mm} = 2 \times 10^{-6} \text{ m}$, $\varepsilon/d = 2.17 \times 10^{-5}$

Ask: q_V .

Solution: The Bernoulli equation between the 1–1' water surface of the water tower and the 2–2' cross section inside the outlet of the pipeline is used. The outlet center of the drain pipe is taken as the datum level. Then,

Figure 2.25 is sketch diagram of Example 2.9

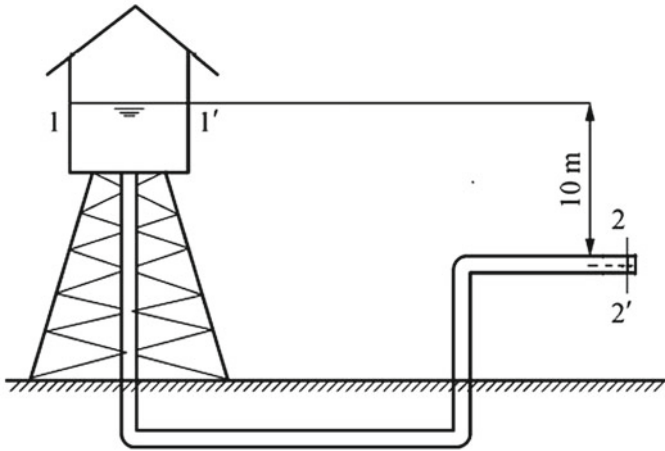


Fig. 2.25 Sketch diagram of Example 2.9

$$z_{h1}g + \frac{1}{2}u_1^2 + \frac{p_1}{\rho} = z_{h2}g + \frac{1}{2}u_2^2 + \frac{p_2}{\rho} + \Sigma h_{tf}$$

where $z_{h1} = 10 \text{ m}$, $z_{h2} = 0$, $p_1 = p_2 = 0$, $u_1 \approx 0$, $u_2 = u$

$$\Sigma h_{tf} = \left(\lambda \frac{l + \Sigma l'_e}{d} + \zeta_{shrink} \right) \frac{u^2}{2} = \left(\lambda \frac{120}{0.092} + 0.5 \right) \frac{u^2}{2}$$

By substituting the above values into the Bernoulli equation, following results are obtained:

$$0 \times g + \frac{0^2}{2} + \frac{0}{\rho} = 0 \times g + \frac{u^2}{2} + \frac{0}{\rho} + \left(\lambda \frac{120}{0.092} + 0.5 \right) \frac{u^2}{2}$$

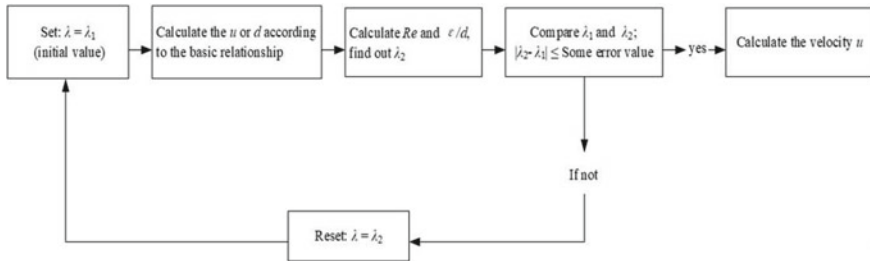
$$u = \sqrt{\frac{2 \times 9.81 \times 10}{\lambda \frac{120}{0.092} + 1.5}} = \sqrt{\frac{196.2}{1304\lambda + 1.5}} \quad (\text{a})$$

In which

$$\lambda = f\left(R_e, \frac{\varepsilon}{d}\right) = \varphi(u) \quad (\text{b})$$

Because u is unknown, it is impossible to obtain the Re value and then also obtain the λ value; u can not be calculated from Eq. (a), so the trial-and-error method can be used to calculate u .

The trial-and-error calculation process is as follows:



Because the variation range of λ is not wide, the friction coefficient λ can be used as the trial difference variable when using the trial-and-error method. It can be assumed that the fluid flow state is the resistance square region, where the value of λ is only related to the value of ε/d , which is convenient for calculation. The specific steps of the trial-and-error method are as follows:

① The value of u can be calculated by substituting an arbitrary value of $\lambda = \lambda_1$ into Eq. (a), and the value of Re can be obtained by using the value of u .

② Based on the calculated Re and ε/d values, the value of λ_2 can be found from the Moody friction coefficient diagram, as shown in Fig. 2.14.

③ If the value of λ_2 found is consistent or close to the hypothesis value λ_1 , that is, $|\lambda_2 - \lambda_1| \leq \text{required error}$, the hypothesis value λ_1 is acceptable. Otherwise, another value of λ_1 should be set, and the above calculation should be repeated until the set value of λ_1 is consistent with or close to the found value of λ_2 , that is, $|\lambda_2 - \lambda_1| \leq \text{required error}$.

If $\lambda_1 = 0.0200$, then (a)

$$u = \sqrt{\frac{196.2}{1304 \times 0.0200 + 1.5}} = 2.67 \text{ m/s}$$

$$Re = \frac{du\rho}{\mu} = \frac{0.092 \times 2.67 \times 1000}{1.005 \times 10^{-3}} = 2.44 \times 10^5$$

$$(1) \quad \varepsilon = 0.2 \text{ mm}, \varepsilon/d = 0.2/92 = 2.17 \times 10^{-3}$$

According to Re and ε/d , $\lambda_2 = 0.0240$, $|\lambda_2 - \lambda_1| = |0.0240 - 0.0200| = 0.0040 > 0.0003$. The requirement of error is not met, so a second trial calculation should be carried out. By resetting $\lambda_1 = 0.0240$ and substituting Eq. (a), the solution $u = 2.45 \text{ m/s}$ is obtained. $Re = 2.24 \times 10^5$ is calculated from the value of u , and $\lambda_2 = 0.0242$ is found from Fig. 2.14 Moody friction coefficient, $|\lambda_2 - \lambda_1| = |0.0242 - 0.0240| = 0.0002 < 0.0003$ meeting the requirement of error, so $u = 2.45 \text{ m/s}$.

The water delivery capacity of the pipe is as follows:

$$q_v = \frac{\pi}{4} d^2 u = \frac{\pi}{4} \times 0.092^2 \times 2.45$$

$$= 0.016 \text{ m}^3/\text{s}$$

$$(2) \quad \varepsilon = 0.002 \text{ mm}, \varepsilon/d = 0.002/92 = 2.17 \times 10^{-5}$$

Let $\lambda_1 = 0.0200$. According to the Re and ε/d values obtained from the first trial calculation in (1), $\lambda_2 = 0.0157$ is obtained using the Moody friction coefficient diagram, $|\lambda_2 - \lambda_1| = |0.0157 - 0.0200| = 0.0043 > 0.0003$. The requirement of error is not met, so a second trial calculation should be carried out. By resetting $\lambda_1 = 0.0157$ and substituting Eq. (a), $u = 2.99 \text{ m/s}$ is obtained. $Re = 2.74 \times 10^5$ is got from the value of u . In Fig. 2.14, $\lambda_2 = 0.0155$ is found, $|\lambda_2 - \lambda_1| = |0.0155 - 0.0157| = 0.0002 < 0.0003$ meeting the requirement of error, so $u = 2.99 \text{ m/s}$.

The water delivery capacity of the pipe is as follows:

$$q_v = \frac{\pi}{4} d^2 u = \frac{\pi}{4} \times 0.092^2 \times 2.99$$

$$= 0.020 \text{ m}^3/\text{s}$$

Answer: If $\varepsilon = 0.2 \text{ mm}$, the flow rate of the pipe is $0.016 \text{ m}^3/\text{s}$. If $\varepsilon = 0.002 \text{ mm}$, flow rate of the pipe is $0.020 \text{ m}^3/\text{s}$.

When using the trial-and-error method to calculate the velocity of flow, the value of u can also be assumed first, and the value of λ can be calculated by Eq. (a). According to Re and ε/d , the value of λ can be found from the figure of the Moody friction coefficient. Compared with the value of λ , the value of u can be judged whether is appropriate.

The operational calculation problem for the pipeline can be solved by a dimensionless group without the velocity of fluid (u).

Example 2.10 It is known that the diameter of a 150 m long water supply pipe including all friction losses is $\Phi 80 \text{ mm} \times 3 \text{ mm}$, the relative roughness of the pipe is 0.0004, and the total friction loss of the pipe is 48 J/kg. Please calculate the water flow rate through the pipe. It is known that the viscosity of water is $0.8 \times 10^{-3} \text{ Pa} \cdot \text{s}$, and the density is 1000 kg/m^3 .

Known: $\Phi 80 \text{ mm} \times 3 \text{ mm}$, $d = 80 - 2 \times 3 = 74 \text{ mm} = 0.074 \text{ m}$, $l + \Sigma l_e = 150 \text{ m}$, $\varepsilon/d = 0.0004$, $\Sigma h_{tf} = 48 \text{ J/kg}$, $\rho = 1000 \text{ kg/m}^3$, $\mu = 0.8 \times 10^{-3} \text{ Pa} \cdot \text{s}$.

Determine: q_V

Solution: since

$$\Sigma h_{tf} = \lambda \cdot \frac{u^2}{2} \cdot \frac{l + \Sigma l_e}{d}$$

$$\lambda = \frac{2d \Sigma h_{tf}}{(l + \Sigma l_e) u^2}$$

$$\text{and } Re^2 = \left(\frac{du\rho}{\mu} \right)^2$$

By multiplying the two equations of λ and Re^2 mentioned above, u can be eliminated, and a dimensionless group independent of u can be obtained.

$$\lambda Re^2 = \frac{2d^3 \rho^2 \Sigma h_{tf}}{(l + \Sigma l_e) \mu^2} \quad (2.38)$$

Because λ is a function of Re and ε/d , λRe^2 is also a function of ε/d and Re . The curves in Fig. 2.26 show the relationship between Re and λRe^2 under different relative roughnesses. When calculating u , the known data can be substituted into Eq. (2.38) to calculate λRe^2 , then the corresponding Re can be determined from Fig. 2.26 according to λRe^2 and ε/d , and then u and q_V can be calculated.

By substituting the data in the question into Eq. (2.38),

$$\lambda Re^2 = \frac{2d^3 \rho^2 \Sigma h_{tf}}{(l + \Sigma l_e) \mu^2} = \frac{2 \times 0.074^3 \times 1000^2 \times 48}{150 \times (0.8 \times 10^{-3})^2} = 4.05 \times 10^8$$

According to the values of λRe^2 and ε/d , $Re = 1.33 \times 10^5$ is found in Fig. 2.26a.

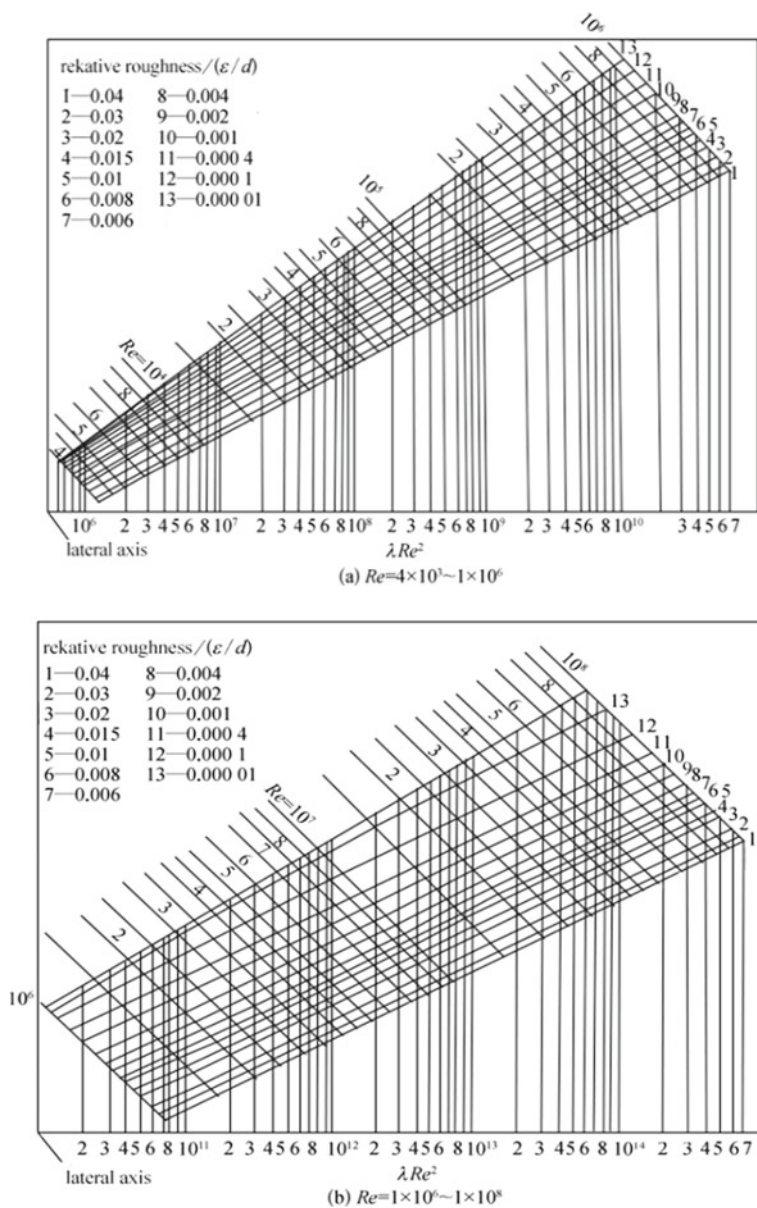


Fig. 2.26 Re and λRe^2 curves with different relative roughness

$$u = \frac{Re\mu}{d\rho} = \frac{1.33 \times 10^5 \times 0.8 \times 10^{-3}}{0.074 \times 1000} = 1.44 \text{ m/s}$$

The flow rate of water is:

$$q_V = \frac{\pi}{4} d^2 u = \frac{\pi}{4} \times (0.074)^2 \times 1.44 = 6.19 \times 10^{-3} \text{ m}^3/\text{s} = 22.3 \text{ m}^3/\text{h}$$

Answer: The flow rate in the water supply pipe is 22.3 m³/h.

2.3.3.2 Complex Pipeline System

1) Parallel pipeline system

A parallel pipeline system is shown in Fig. 2.27. The main pipe flows into several branches at point *A* and then merges into a main pipe at point *B*. The characteristics of such a parallel pipeline system are as follows:

- (1) The mass flow rate in the main pipe is equal to the sum of the mass flow rates of each branch pipe in parallel for incompressible fluids.

$$q_V = q_{V1} + q_{V2} + q_{V3} \quad (2.39)$$

- (2) The pressure drop between cross sections 1–1' and 2–2' in the figure is caused by the fluid overcoming the flow resistances in each branch pipe. Therefore, for the parallel pipeline system, regardless of which branch the unit mass fluid passes through, the friction loss should be equal. The total friction loss for the parallel pipeline system is also equal to the friction losses of each branch pipe.

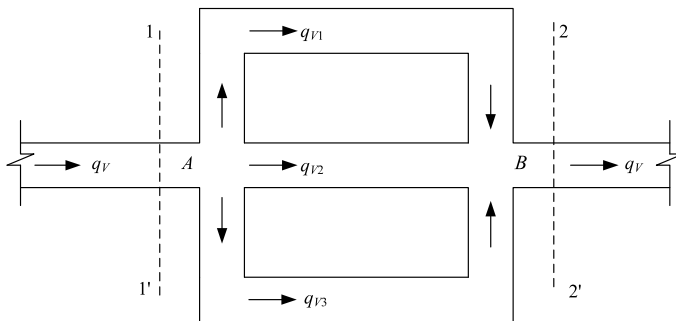


Fig. 2.27 Parallel pipelines

$$\sum h_{tf1} = \sum h_{tf2} = \sum h_{tf3} = \sum h_{tf,1-2} \quad (2.40)$$

Therefore, when calculating the energy loss of the parallel pipeline system, only energy loss of one branch pipe needs to be calculated, and the friction losses of each pipe must not be added as the total friction loss of the parallel pipeline system.

If the local losses at cross sections 1 and 2 are neglected, the friction losses for each pipe can be calculated by the following equation:

$$h_{t fi} = \lambda_i \frac{l_{ti}}{d_i} \frac{u_i^2}{2} \quad (2.41)$$

where l_{ti} is the total length of the branch pipe, including the total equivalent length of the straight pipe friction losses and the local losses $l + \sum l_e$, m.

$$\lambda_1 \frac{l_{t1}}{d_1} \frac{u_1^2}{2} = \lambda_2 \frac{l_{t2}}{d_2} \frac{u_2^2}{2} = \lambda_3 \frac{l_{t3}}{d_3} \frac{u_3^2}{2} \quad (2.41a)$$

In general, the lengths, diameters and roughnesses of each branch pipe are different, but the flow driving force of each branch pipe are the same, so the flow velocities of each branch pipe are also different. $u_i = 4q_{vi} / \pi d_i^2$ is substituted into Eq. (2.41), and the result is obtained after sorting.

$$q_{vi} = \frac{\pi \sqrt{2}}{4} \sqrt{\frac{d_i^5 h_{t fi}}{\lambda_i l_{ti}}} \quad (2.42)$$

From this equation, the flow rate distribution of each branch can be calculated. If there are only three branches,

$$q_{v1} : q_{v2} : q_{v3} = \sqrt{\frac{d_1^5}{\lambda_1 l_{t1}}} : \sqrt{\frac{d_2^5}{\lambda_2 l_{t2}}} : \sqrt{\frac{d_3^5}{\lambda_3 l_{t3}}} \quad (2.43)$$

If the total flow rate q_v , as well as l_{ti} , d_i and λ_i of each branch, are known, the three unknowns of q_{v1} , q_{v2} and q_{v3} can be solved simultaneously by Eqs. (2.39) and (2.43), and the friction loss h_{tf1-2} between points 1 and 2 can be calculated by using a one-pipe Eq. (2.41).

Example 2.11 In the water pipeline system shown in Fig. 2.28, the water flows from cross section 1–1' to cross section 2–2' with the total flow rate of 0.01 m³/s, and the water temperature is 20 °C. The total lengths of each branch (including the equivalent length of all friction losses) are $l_{t1} = 2000$ m, $l_{t2} = 1800$ m, $l_{t3} = 1700$ m, and $l_{t4} = 1900$ m; the diameters are $d_1 = 70$ mm, $d_2 = 53$ mm, $d_3 = 40$ mm, and $d_4 = 70$ mm; and the friction coefficients are $\lambda_1 = 0.038$, $\lambda_2 = 0.044$, $\lambda_3 = 0.049$, and $\lambda_4 = 0.038$.

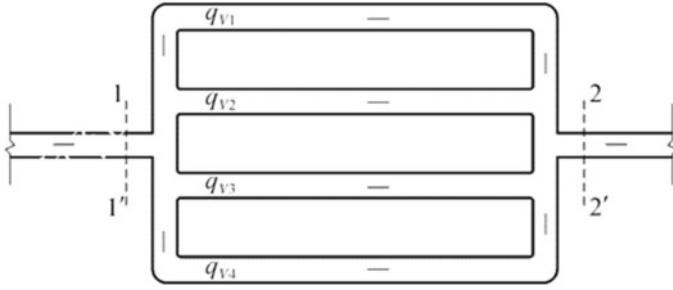


Fig. 2.28 Schematic diagram for Example 2.11

Please calculate the flow rates of each pipe and the friction loss between cross section 1–1' and 2–2'.

Known: $q_V = 0.01 \text{ m}^3/\text{s}$, $T = 20^\circ\text{C}$, $l_{t1} = 2000 \text{ m}$, $l_{t2} = 1800 \text{ m}$, $l_{t3} = 1700 \text{ m}$, $l_{t4} = 1900 \text{ m}$, $d_1 = 70 \text{ mm} = 0.07 \text{ m}$, $d_2 = 53 \text{ mm} = 0.053 \text{ m}$, $d_3 = 40 \text{ mm} = 0.04 \text{ m}$, $d_4 = 70 \text{ mm} = 0.07 \text{ m}$, $\lambda_1 = 0.038$, $\lambda_2 = 0.044$, $\lambda_3 = 0.049$, $\lambda_4 = 0.038$.

Determine the following: q_{V1} , q_{V2} , q_{V3} , q_{V4} , $h_{f,1-2}$.

$$\begin{aligned} \text{Solution: } q_{V1} : q_{V2} : q_{V3} : q_{V4} &= \sqrt{\frac{d_1^5}{\lambda_1 l_{t1}}} : \sqrt{\frac{d_2^5}{\lambda_2 l_{t2}}} : \sqrt{\frac{d_3^5}{\lambda_3 l_{t3}}} : \sqrt{\frac{d_4^5}{\lambda_4 l_{t4}}} \\ &= \sqrt{\frac{0.07^5}{0.038 \times 2000}} : \sqrt{\frac{0.054^5}{0.044 \times 1800}} : \sqrt{\frac{0.04^5}{0.049 \times 1700}} : \sqrt{\frac{0.07^5}{0.038 \times 1900}} \\ &= 1.487 : 0.727 : 0.351 : 1.525 \\ &= 4.236 : 2.071 : 1 : 4.34 \end{aligned}$$

Because $q_V = q_{V1} + q_{V2} + q_{V3} + q_{V4}$

$$q_{V1} = \frac{4.236}{4.236 + 2.071 + 1 + 4.345} \times q_V = \frac{4.236}{11.652} \times 0.01 = 3.63 \times 10^{-3} \text{ m}^3/\text{s}$$

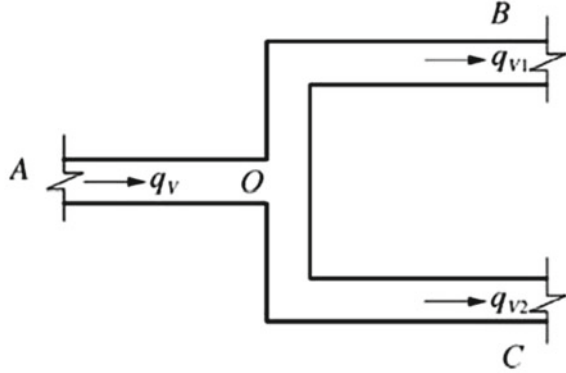
$$q_{V2} = \frac{2.071}{4.236 + 2.071 + 1 + 4.345} \times q_V = \frac{2.071}{11.652} \times 0.01 = 1.78 \times 10^{-3} \text{ m}^3/\text{s}$$

$$q_{V3} = \frac{1}{4.236 + 2.071 + 1 + 4.345} \times q_V = \frac{1}{11.652} \times 0.01 = 0.86 \times 10^{-3} \text{ m}^3/\text{s}$$

$$q_{V4} = \frac{4.345}{4.236 + 2.071 + 1 + 4.345} \times q_V = \frac{4.345}{11.652} \times 0.01 = 3.73 \times 10^{-3} \text{ m}^3/\text{s}$$

$$u_1 = \frac{q_{V1}}{\frac{\pi}{4} d_1^2} = \frac{3.63 \times 10^{-3}}{\frac{\pi}{4} \times 0.07^2} = 0.943 \text{ m/s}$$

Fig. 2.29 Branching pipeline system



$$h_{tf,1-2} = h_{tf1} = \lambda_1 \frac{l_{tf1}}{d_1} \frac{u_1^2}{2} = 0.038 \times \frac{2000}{0.07} \times \frac{0.943^2}{2} = 483 \text{ J/kg}$$

Answer: The flow rates of each pipe are $3.63 \times 10^{-3} \text{ m}^3/\text{s}$, $1.78 \times 10^{-3} \text{ m}^3/\text{s}$, $0.86 \times 10^{-3} \text{ m}^3/\text{s}$ and $3.73 \times 10^{-3} \text{ m}^3/\text{s}$. The friction loss from cross section 1–1' to cross section 2–2' is 483 J/kg.

2) Branching pipeline system and junction pipeline system

Water supply pipes or wastewater pipes are always branched so that fluids can be distributed from one main pipe to several places, as shown in Fig. 2.29. In this case, the flow in each branch affects and restricts the other branches. There are two main flow principles in branching pipeline system:

- (1) The main pipe flow rate is equal to the sum of flow rates of each branch pipe, i.e.,

$$q_{VA} = q_{VB} + q_{VC} \quad (2.44)$$

- (2) Although the lengths and diameters of each branch pipe are different, the total energy at the branch point (O point in Fig. 2.29) is a fixed value. Whichever the fluid at the branch point (O point) flows to any branch pipe, the total energy of each kilogram of fluid must be equal.

$$gz_B + \frac{u_B^2}{2} + \frac{p_B}{\rho} + \Sigma h_{tf1,O-B} = gz_C + \frac{u_C^2}{2} + \frac{p_C}{\rho} + \Sigma h_{tf1,O-C} = gz_O + \frac{u_O^2}{2} + \frac{p_O}{\rho} \quad (2.45)$$

A junction pipeline system refers to a situation in which several branches converge into one main pipe, as shown in Fig. 2.30, which has similar characteristics to branching pipeline system (Fig. 2.29).

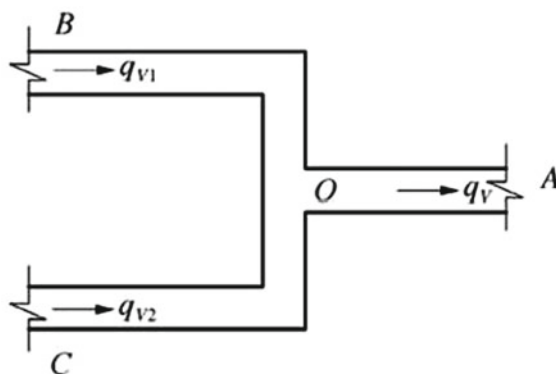


Fig. 2.30 Junction pipeline system

Example 2.12 As shown in the Fig. 2.31, the water to a residential building is supplied by a vertical section of pipe AB connected to the main water pipe. The flow velocity of the main pipe is 1 m/s, and the gauge pressure is 0.4 MPa. The BC and BDE sections of the first and second floors of the residential building are directly supplied by the main vertical pipe, and the third floor and above are supplied by a roof water tank. It is known that the storey height of the residential building is 3 m, the length of section AB is 100 m, the length of section BC is 10 m, the length of section BDE is 20 m, and the inner diameter of the pipe is 20 mm. It is considered that the flow state in each pipe enters the friction loss region, and the friction coefficient is 0.02. C and E are gate valves, and local energy losses at B and D are neglected. The density of water is 1000 kg/m^3 .

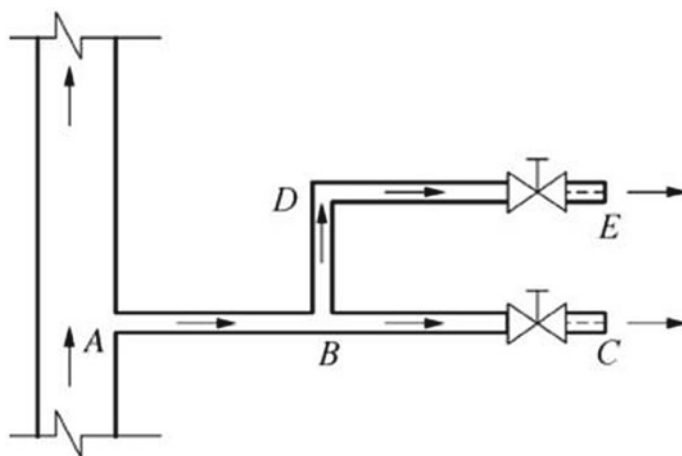


Fig. 2.31 Schematic diagram for Example 2.12

- (1) When valve E is closed and only C is opened, what is the flow rate of outlet C ?
- (2) When valve E is fully opened and valve C is adjusted to lower flow rate, and the flow rate of the second-floor household is the same as that of the first-floor household, please determine the percentage of original flow rate at outlet C .

Known: $u_A = 1.0$ m/s, $u_{C, \text{outside}} = 0$ m/s, $p_A = 0.4$ MPa (Gauge pressure), resident's floor height $h = 3$ m, $l_{AB} = 100$ m, $l_{BC} = 10$ m, $l_{BDE} = 20$ m, $d = 20$ mm = 0.02 m, $\lambda = 0.02$, take $A-C$ as the datum, $z_{hA} = 0$, $z_{hB} = 0$, $z_{hC} = 0$, $z_{hD} = z_{hE} = 3.0$ m, $p_E = p_C = 0$, $\rho = 1000$ kg/m³.

Determine: (1) When valve E is closed and only C is open, determine the outlet flow rate at C .

Solution: The Bernoulli equation is established between cross sections A and C (inside):

$$gz_{hA} + \frac{p_A}{\rho} + \frac{u_A^2}{2} = gz_{hC} + \frac{p_C}{\rho} + \frac{u_{C, \text{inside}}^2}{2} + \sum h_{tf, A-C, \text{inside}}$$

where

$$\begin{aligned} & \sum h_{tf, A-C, \text{inside}} \\ &= \left[\lambda \frac{l_{AB} + l_{BC}}{d} + \zeta_{\text{full open gate valve}} + \zeta_{\text{shrink}} \right] \frac{u_{C, \text{inside}}^2}{2} \\ &= \left[0.02 \times \frac{100 + 10}{0.02} + 0.17 + 0.5 \right] \frac{u_{C, \text{inside}}^2}{2} = 55.34 u_{C, \text{inside}}^2 \end{aligned}$$

Substitute the following data:

$$0 + \frac{0.4 \times 10^6}{1000} + \frac{1^2}{2} = 0 + \frac{0}{1000} + \frac{u_{C, \text{inside}}^2}{2} + 55.34 u_{C, \text{inside}}^2$$

Solve $u_{C, \text{inside}} = 2.67$ m/s

$$q_{VC} = \frac{\pi d^2}{4} u_{C, \text{inside}} = \frac{\pi}{4} \times 0.02^2 \times 2.67 = 8.415 \times 10^{-4} \text{ m}^3/\text{s} = 3.03 \text{ m}^3/\text{h}$$

- (2) When valve E is fully opened and valve C is adjusted to a lower flow rate, and the flow rate of the second-floor household is the same as that of the first-floor household, please determine the percentage of original flow rate at outlet C .

Solution: The Bernoulli equation is established between cross sections A and E (outside).

$$gz_{hA} + \frac{p_A}{\rho} + \frac{u_A^2}{2} = gz_{hE} + \frac{p_E}{\rho} + \frac{u_{E, \text{outside}}^2}{2} + \sum h_{tf, A-E}$$

$u_{AB} = 2u_{E, \text{inside}} = 2u'_{C, \text{inside}}$, $u_{E, \text{outside}} = 0$, neglecting local energy losses at B and D

$$\begin{aligned} \sum h_{tf, A-E} &= \left(\lambda \frac{l_{AB}}{d} + \zeta_{\text{shrink}} \right) \frac{u_{AB}^2}{2} + \left(\lambda \frac{l_{BE}}{d} + \zeta_{\text{full open gate valve}} + \zeta_{\text{expansion}} \right) \frac{u_{E, \text{inside}}^2}{2} \\ &= \left(0.02 \times \frac{100}{0.02} + 0.5 \right) \frac{4u_{E, \text{inside}}^2}{2} + \left(0.02 \times \frac{20}{0.02} + 0.17 + 1 \right) \frac{u_{E, \text{inside}}^2}{2} \\ &= 211.585 \times u_{E, \text{inside}}^2 \end{aligned}$$

Substituting data

$$0 + \frac{0.4 \times 10^6}{1000} + \frac{1^2}{2} = 9.8 \times 3 + \frac{0}{1000} + \frac{0}{2} + 211.585 \times u_{E, \text{inside}}^2$$

Solve $u_{E, \text{inside}} = 1.324 \text{ m/s}$, $u_{C, \text{inside}} = u_{E, \text{inside}} = 1.324 \text{ m/s}$

$$\frac{q'_C}{q_C} = \frac{u'_{C, \text{inside}}}{u_{C, \text{inside}}} = \frac{1.324}{2.67} = 49.6\%$$

Answer: When valves E are closed and only C is opened, the flow rate of outlet C is $3.02 \text{ m}^3/\text{h}$. If valve C is lowered and valve E is fully opened, the flow rate at C should be controlled to 49.6% of the original flow rate for the second-floor households.

Example 2.13 As shown in Fig. 2.32, the water level in the high tank is maintained constant. Water is discharged from the BC and BD pipes. The vertical distance

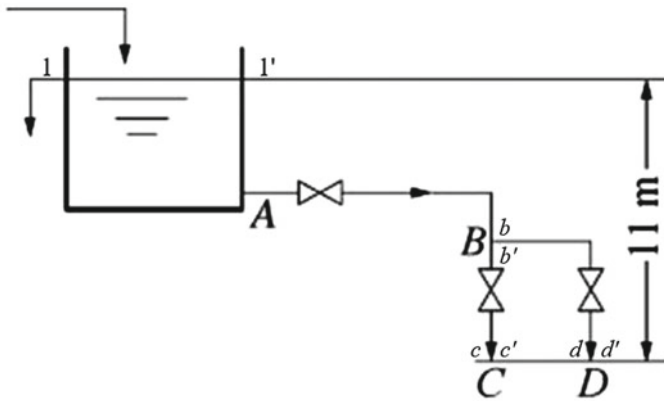


Fig. 2.32 Schematic diagram for Example 2.13

between the liquid level of the high tank and the outlet of the two pipes is 11 m. The AB section has a diameter of 38 mm and a length of 58 m, the BC branch has a diameter of 32 mm and a length of 12.5 m, and the BD branch has a diameter of 26 mm and a length of 14 m. The AB section has a 90-degree bend and a gate valve, the BC cross section has a gate valve, and the BD cross section has a 90-degree regular elbow and a gate valve. There is a tee at BC - BD . The friction coefficients of pipes AB , BC and BD are both 0.03. The density of water is 1000 kg/m^3 . Calculate the following:

- (1) When the valve of the BD branch is closed, what is the maximum flow rate of the BC branch (m^3/h)?
- (2) When all valves are fully open, what are the flow rates of the two branch pipes?

Known: $\Delta z = 11 \text{ m}$, $l_{AB} = 58 \text{ m}$, $d_{AB} = 38 \text{ mm} = 0.038 \text{ m}$, $l_{BC} = 12.5 \text{ m}$, $d_{BC} = 32 \text{ mm} = 0.032 \text{ m}$, $l_{BD} = 14 \text{ m}$, $d_{BD} = 26 \text{ mm} = 0.026 \text{ m}$, $\lambda_{AB} = \lambda_{BC} = \lambda_{BD} = 0.03$, $\rho = 1000 \text{ kg/m}^3$

Determine the following: (1) When the BD valve is closed, q_{VBC} .

(2) When all valves are open, q_{VBC} and q_{VBD} .

Solutions: (1) The Bernoulli equation between liquid level 1-1' of the high tank and inner cross section c - c' of the outlet of the BC branch pipe is obtained, and cross section c - c' is taken as the potential energy datum.

$$gz_{h1} + \frac{p_1}{\rho} + \frac{u_1^2}{2} = gz_{hc} + \frac{p_C}{\rho} + \frac{u_{BC}^2}{2} + \Sigma h_{tf,1-C} \quad (1)$$

where $p_1 = p_C = 0$ (gauge pressure), $u_1 \approx 0 \text{ m/s}$, $z_{h1} = 11 \text{ m}$, $z_{hc} = 0$.

From Table 2.2: $\zeta_{\text{shrink}} = 0.5$, $\zeta_{\text{gate valve}} = 0.17$, $\zeta_{90^\circ} = 0.75$, $\zeta_{\text{tee}} = 1.3$

$$\begin{aligned} \Sigma h_{tf,1-C} &= \lambda_{AB} \frac{l_{AB}}{d_{AB}} \frac{u_{AB}^2}{2} + \lambda_{BC} \frac{l_{BC}}{d_{BC}} \frac{u_{BC}^2}{2} + (\zeta_{\text{shrink}} + \zeta_{\text{gate valve}} + \zeta_{90^\circ}) \frac{u_{AB}^2}{2} \\ &\quad + (\zeta_{\text{tee}} + \zeta_{\text{gate valve}}) \frac{u_{BC}^2}{2} \\ &= 0.03 \times \frac{58}{0.038} \cdot \frac{u_{AB}^2}{2} + 0.03 \times \frac{12.5}{0.032} \cdot \frac{u_{BC}^2}{2} \\ &\quad + (0.5 + 0.17 + 0.75) \cdot \frac{u_{AB}^2}{2} + (1.3 + 0.17) \cdot \frac{u_{BC}^2}{2} \\ &= 47.21 \cdot \frac{u_{AB}^2}{2} + 13.19 \cdot \frac{u_{BC}^2}{2} \end{aligned}$$

The above values are substituted for Eq. ① and sorted out.

$$9.81 \times 11 + \frac{0}{\rho} + \frac{0^2}{2} = 9.81 \times 0 + \frac{0}{\rho} + \frac{u_{BC}^2}{2} + 47.21 \cdot \frac{u_{AB}^2}{2} + 13.19 \cdot \frac{u_{BC}^2}{2}$$

$$107.91 = 23.61u_{AB}^2 + 7.10u_{BC}^2 \quad \text{②}$$

According to the continuity equation $q_{VAB} = q_{VBC}$,

$$u_{AB} \times \frac{\pi}{4}d_{AB}^2 = u_{BC} \times \frac{\pi}{4}d_{BC}^2$$

$$u_{AB} = \frac{d_{BC}^2}{d_{AB}^2} \times u_{BC} = \frac{0.032^2}{0.038^2} u_{BC} = 0.709u_{BC}$$

Substitute Eq. ② obtains

$$107.91 = 23.61 \times 0.709^2 u_{BC}^2 + 7.10u_{BC}^2$$

$$u_{BC} = 2.385 \text{ m/s}$$

$$q_{VBC} = u_{BC} \times \frac{\pi}{4}d_{BC}^2 = 2.385 \times \frac{\pi}{4} \times 0.032^2 = 1.917 \times 10^{-3} \text{ m}^3/\text{s} = 6.902 \text{ m}^3/\text{h}$$

- (2) According to the flow law of branch pipelines, a Bernoulli equation between cross section from $b-b'$ to inside $c-c'$, and cross section from $b-b'$ to inside $d-d'$ is obtained.

$$gz_{hC} + \frac{p_C}{\rho} + \frac{u_{BC}^2}{2} + \Sigma h_{tf,B-C} = gz_{hD} + \frac{p_D}{\rho} + \frac{u_{BD}^2}{2} + \Sigma h_{tf,B-D} \quad \text{③}$$

$p_C = p_D = 0$ (gauge pressure), $z_{hC} = z_{hD} = 0$.

From Table 2.2: $\zeta_{\text{shrink}} = 0.5$, $\zeta_{\text{gate valve}} = 0.17$, $\zeta_{90^\circ} = 0.75$, $\zeta_{\text{tee}} = 0.4$

$$\begin{aligned} \Sigma h_{tf,B-C} &= \lambda \frac{l_{BC}}{d_{BC}} \cdot \frac{u_{BC}^2}{2} + \zeta_{\text{tee}} \cdot \frac{u_{AB}^2}{2} + \zeta_{\text{gate valve}} \cdot \frac{u_{BC}^2}{2} \\ &= 0.03 \times \frac{12.5}{0.032} \cdot \frac{u_{BC}^2}{2} + 0.17 \cdot \frac{u_{BC}^2}{2} \\ &\quad + \zeta_{\text{tee}} \cdot \frac{u_{AB}^2}{2} = 11.89 \times \frac{u_{BC}^2}{2} + \zeta_{\text{tee}} \cdot \frac{u_{AB}^2}{2} \end{aligned}$$

$$\begin{aligned}
\Sigma h_{tf,B-D} &= \lambda \frac{l_{BD}}{d_{BD}} \cdot \frac{u_{BD}^2}{2} + \zeta_{tee} \cdot \frac{u_{AB}^2}{2} + (\zeta_{\text{gate valve}} + \zeta_{90^\circ}) \cdot \frac{u_{BD}^2}{2} \\
&= 0.03 \times \frac{14}{0.026} \cdot \frac{u_{BD}^2}{2} + (0.17 + 0.75) \cdot \frac{u_{BD}^2}{2} + \zeta_{tee} \cdot \frac{u_{AB}^2}{2} \\
&= 17.07 \cdot \frac{u_{BD}^2}{2} + \zeta_{tee} \cdot \frac{u_{AB}^2}{2}
\end{aligned}$$

Substituting data to ③:

$$\begin{aligned}
g \times 0 + \frac{0}{\rho} + \frac{u_{BC}^2}{2} + 11.89 \times \frac{u_{BD}^2}{2} + \zeta_{tee} \cdot \frac{u_{AB}^2}{2} &= g \times 0 + \frac{0}{\rho} + \frac{u_{BD}^2}{2} \\
+ 17.07 \times \frac{u_{BD}^2}{2} + \zeta_{tee} \cdot \frac{u_{AB}^2}{2} & \\
12.89 \cdot \frac{u_{BC}^2}{2} &= 18.07 \cdot \frac{u_{BD}^2}{2}
\end{aligned}$$

$$u_{BC} = 1.18u_{BD} \quad u_{BD} = 0.847u_{BC}$$

According to the pipeline continuity equation:

$$\begin{aligned}
u_{AB} \times \frac{\pi}{4} d_{AB}^2 &= u_{BC} \times \frac{\pi}{4} d_{BC}^2 + u_{BD} \times \frac{\pi}{4} d_{BD}^2 \\
u_{AB} \times \frac{\pi}{4} \times 0.038^2 &= u_{BC} \times \frac{\pi}{4} \times 0.032^2 + u_{BD} \times \frac{\pi}{4} \times 0.026^2 \\
u_{AB} &= 0.709u_{BC} + 0.468u_{BD}
\end{aligned}$$

$$u_{AB} = 0.709u_{BC} + 0.468 \times 0.847u_{BC} = 1.105u_{BC}$$

The Bernoulli equation between 1-1' in the high tank and c-c' of the inner cross section of the exit of the branch pipe BC is

$$gz_{h1} + \frac{p_1}{\rho} + \frac{u_1^2}{2} = gz_{hC} + \frac{p_C}{\rho} + \frac{u_{BC}^2}{2} + \Sigma h_{tf,1-C} \quad (4)$$

$$p_1 = p_C = p_0, \quad u_1 \approx 0 \text{ m/s}, \quad z_{h1} = 11 \text{ m}, \quad z_{hC} = 0 \text{ m}.$$

From Table 2.2: $\zeta_{\text{shrink}} = 0.5$, $\zeta_{\text{gate valve}} = 0.17$, $\zeta_{90^\circ} = 0.75$, $\zeta_{tee} = 0.4$

$$\begin{aligned}
\Sigma h_{tf,1-C} &= \lambda_{AB} \frac{l_{AB}}{d_{AB}} \frac{u_{AB}^2}{2} + \lambda_{BC} \frac{l_{BC}}{d_{BC}} \frac{u_{BC}^2}{2} \\
&+ (\zeta_{\text{shrink}} + \zeta_{\text{gate valve}} + \zeta_{90^\circ} + \zeta_{\text{three-way pipe}}) \frac{u_{AB}^2}{2}
\end{aligned}$$

$$\begin{aligned}
& + \zeta_{\text{gate valve}} \cdot \frac{u_{BC}^2}{2} \\
= & 0.03 \times \frac{58}{0.038} \times \frac{u_{AB}^2}{2} + 0.03 \times \frac{12.5}{0.032} \times \frac{u_{BC}^2}{2} + (0.5 + 0.17 + 0.75 + 0.4) \frac{u_{AB}^2}{2} \\
& + 0.17 \times \frac{u_{BC}^2}{2} \\
= & 47.71 \cdot \frac{u_{AB}^2}{2} + 11.89 \cdot \frac{u_{BC}^2}{2}
\end{aligned}$$

Substitution ④:

$$9.81 \times 11 + \frac{0}{\rho} + \frac{0^2}{2} = 9.81 \times 0 + \frac{0}{\rho} + \frac{u_{BC}^2}{2} + 47.61 \cdot \frac{u_{AB}^2}{2} + 11.89 \cdot \frac{u_{BC}^2}{2}$$

$$107.91 = 47.61 \cdot \frac{u_{AB}^2}{2} + 12.89 \cdot \frac{u_{BC}^2}{2}$$

$$107.91 = 23.805 \times (1.105u_{BC})^2 + 6.445u_{BC}^2$$

$$u_{BC} = 1.74 \text{ m/s}$$

$$u_{BD} = 0.847u_{BC} = 0.847 \times 1.74 = 1.47 \text{ m/s}$$

$$u_{AB} = 1.105u_{BC} = 1.105 \times 1.74 = 1.92 \text{ m/s}$$

$$q_{VBC} = u_{BC} \times \frac{\pi}{4} d_{BC}^2 = 1.74 \times \frac{\pi}{4} \times 0.032^2 = 1.40 \times 10^{-3} \text{ m}^3/\text{s} = 5.04 \text{ m}^3/\text{h}$$

$$q_{VBD} = u_{BD} \times \frac{\pi}{4} d_{BD}^2 = 1.47 \times \frac{\pi}{4} \times 0.026^2 = 7.8 \times 10^{-4} \text{ m}^3/\text{s} = 2.87 \text{ m}^3/\text{h}$$

Answer: (1) When the BD valve is closed, q_{VBC} is $6.90 \text{ m}^3/\text{h}$. (2) When all valves are full open, q_{VBC} is $5.04 \text{ m}^3/\text{h}$, and q_{VBD} is $2.87 \text{ m}^3/\text{h}$.

2.4 Open Channel Uniform Flow and Thin Wall Weir

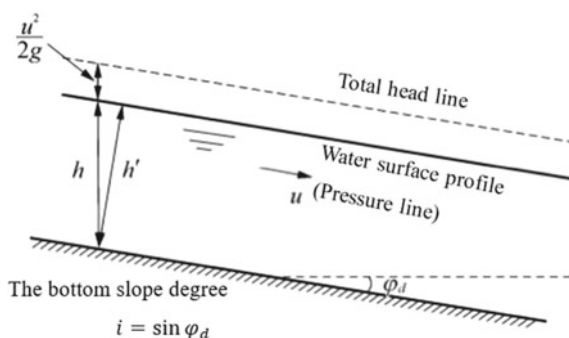
An open channel is a general term for all channels with free surfaces of artificial channels and natural channels, as well as pipes without full flow. The free surface means that the top surface of the fluid in an open channel is exposed to the atmosphere. The flow in an open channel is called an open channel flow. Open channel flow, also

known as pressure-free flow or gravity flow, is a fluid flow with a free surface and relies on the gravity of the liquid itself.

Open channel flow generally occurs in a complete turbulent region, which can be divided into steady flow and unsteady flow. Because of the existence of a free surface, the streamlines of unsteady flow in an open channel cannot be parallel to each other, so the unsteady flow in the open channel cannot be uniform flow but only nonuniform flow. The steady flow in an open channel can be divided into uniform flow and nonuniform flow according to whether the streamlines are parallel or not. Therefore, open channel uniform flow refers to open channel steady uniform flow.

Due to the diversity of open channel boundary conditions, open channel flows generally occur in a nonuniform flow state. The free surface of open channel flow varies under different flow conditions and channel body conditions, forming various flow states and surface morphologies. In practice, it is difficult to form uniform flow in an open channel. However, in practical applications, such as in water supply and the drainage of environmental ecological rivers and ditches in water conservancy projects, the calculation of their drainage or water conveyance capacity is usually based on open channel uniform flow. Open channel uniform flow is the simplest and most basic form of open channel flow. Its basic concepts and calculation principles are also the theoretical basis of open channel nonuniform flow. This chapter discusses the law of uniform flow in open channels. As shown in Fig. 2.33, the water surface line, also known as the piezometric head line, is composed of the sum of potential energy and static pressure energy along the flow direction, and the total head line, also known as the total energy line, is composed of the sum of kinetic energy, potential energy and static pressure energy along the flow direction.

Fig. 2.33 Open channel uniform flow



2.4.1 Formation Conditions and Characteristics of Uniform Flow in Open Channels

Open channel uniform flow is a flow with constant water depth and cross-sectional average velocity. It can only occur under certain conditions. These conditions are as follows:

- (1) The flow rate in an open channel must be constant, with no tributary inflow or outflow and no discharge along the channel.
- (2) The open channel must be a prismatic channel or circular straight pipe. Otherwise, uneven force will be caused at the places where various local resistances occur, such as bends and valves, thus leading to nonuniform flow.
- (3) The roughness coefficient of the open channel must remain unchanged along the channel because the roughness coefficient determines the resistance. If the roughness coefficient changes, this will inevitably cause a change in resistance and become a nonuniform flow.
- (4) The bottom slope of an open channel must be downhill. Uniform flow can only occur in such a long straight section with the above three elements at the same time.

In practical engineering, due to the limitations of various conditions, uniform flow in open channels is often difficult to achieve completely, and there are numerous nonuniform flows in open channels. However, as long as the straight open channel with downhill has enough length, there will always be a trend of uniform flow. This trait is often used in the analysis of nonuniform flow surface curves. Generally, artificial channels are constructed as much as possible to keep the channel line straight and the bottom slope unchanged over a long distance; the same material lining is used to form a regular cross section, which basically ensures the conditions for uniform flow. This is in line with the actual situation according to the theory of open channel uniform flow for designing channels. Natural rivers generally exhibit nonuniform flow. A section with relatively straight and basically the same neat roughness coefficients, and a stable riverbed can also be regarded as a uniform flow section, which maintains a stable relationship between the water level and flow rate.

An open channel uniform flow should have the characteristics of both uniform flow and gravity flow. The streamlines of uniform flow are parallel to each other. All liquid particles move in a uniform straight line in the same direction, and the external force is zero. The gravity flow is driven by the component of the gravity of the liquid itself in the direction of flow. Therefore, the uniform flow in the open channel is the flow in which the gravity force in the direction of flow is balanced with the fluid resistance. It can be inferred that the open channel uniform flow should have the following characteristics:

- (1) The shape, size and depth of the cross section remain unchanged along the flow path.

- (2) The velocity distribution and the average velocity of the cross section on the cross section remain unchanged along the flow path, so the velocity head, i.e., kinetic energy, also remains unchanged along the flow path.
- (3) Because the water depth remains unchanged along the course, the water surface line and the bottom line of the channel are parallel to each other.
- (4) Because the average velocity and kinetic energy of the fluid remain unchanged along the flow path, the piezometric head line (or water surface line) and the total head (or energy) line are parallel to each other.
- (5) Because the water surface line of open channel uniform flow is the piezometric head line, the bottom slope line, the water surface line and the total head line of open channel uniform flow are parallel to each other. Therefore, the bottom slope, the water surface slope and the hydraulic slope of the open channel are equal. This is an important characteristic of open channel uniform flow. This shows that in open channel uniform flow, the kinetic energy of flow remains unchanged along the flow path, and the potential energy along the flow path. At a certain distance, the decreased potential energy because the descent of the elevation at the bottom of the channel is used to overcome head loss (resistance loss or friction loss) and their values are equal, thus ensuring that the kinetic energy remains unchanged along the flow path.
- (6) From the point of view of mechanics, the uniform flow moves in a straight line with the same velocity and without acceleration, and the force acting on the fluid body must be balanced; that is, the uniform flow is the flow produced when the component force of gravity along the flow direction and friction resistance balance each other, which is the mechanical essence of the uniform flow.

2.4.2 Calculation of Uniform Flow in Open Channel

Generally, the open channel flow in practical engineering is in the complete turbulent region or the friction loss square region.

2.4.2.1 Basic Equation

The Chézy Eq. (2.46) is often used in hydraulic calculations of constant and uniform flow in open channels.

$$u = C_c \sqrt{R_h J_s} \quad (2.46)$$

where

u —Average cross-section velocity, m/s

C_c —Chézy coefficient, $\text{m}^{1/2}/\text{s}$

R_h —Hydraulic radius, m

J_s —Hydraulic gradient, $J_s = H_f/l_f$, and J_s represents head loss per unit the length of the flow, dimensionless

H_f —Head loss, m

l_f —Length of the flow, m.

For open channel steady uniform flow, since hydraulic gradient J_s is equal to bottom slope i , the flow equation of open channel uniform flow can be written as follows:

$$u = C_c \sqrt{R_h i} \quad (2.47)$$

or

$$q_v = Au = AC_c \sqrt{R_h i} = K_i \sqrt{i} \quad (2.48)$$

where

K_i —Flow rate modulus, m^3/s , which is the same as the unit of flow. This reflects the influence of the shape, size and roughness of the open channel cross section on the flow rate of the channel

i —Bottom slope or slope, $i = \frac{\Delta z_h}{l_f} = \sin \varphi_d$, dimensionless. Δz_h is the height difference of the open channel between the two points of the bottom slope length l_f . l_f represents the bottom slope length of the corresponding to the height difference of the open channel Δz_h . φ_d is the angle between the bottom slope of the open channel and the horizontal line.

In Eqs. (2.46) and (2.47), the coefficient C_c of the Chézy equation can be calculated by the Manning Eq. (2.49):

$$C_c = \Theta \frac{1}{n_r} R_h^{1/6} \quad (2.49)$$

In this equation, n_r is the rough coefficient (or roughness, or Manning's n_r factor, or resistance factor), dimensionless, which reflects the resistance of the river and channel wall to flow and is one of the main factors in the hydraulic calculation of the open channel. Its influence on the Chézy coefficient C_c is greater than that on the hydraulic radius R_h . Θ is the dimensional conversion factor, $\Theta = 1.0$, $\text{m}^{1/3}/\text{s}$ in SI units. The SI units are adopted during the calculation of uniform flow in open channels, and Θ is omitted and is no longer shown in the following equations.

The Manning equation can be obtained by substituting it into the Chézy equation.

$$u = \frac{1}{n_r} R_h^{2/3} \sqrt{i} \quad (2.50)$$

or

$$q_v = A \frac{1}{n_r} R_h^{\frac{2}{3}} \sqrt{i} \quad (2.51)$$

2.4.2.2 Hydraulic Elements of Water Cross Section

The values of q_v , A , K_i , C_c , and R_h in the basic equation of uniform flow in the open channel are all related to the shape, size and depth of the cross section through which the open channel flows uniformly. Open channel uniform flow depth, commonly known as normal depth, is expressed in h . The cross section shape of the artificial channel is selected according to the use of the channel, the size of the channel, the construction method and the material of the channel. In water and ecological river engineering, trapezoidal cross sections are most suitable for natural soil channels and are the most commonly used cross section shapes. In drainage pipelines, the most commonly used shape is circular. Other cross section shapes, such as rectangular and parabolic shapes, are also used in some cases. Next, the hydraulic elements of trapezoidal and circular cross sections are studied.

As shown in Fig. 2.34, trapezoidal cross section area A is

$$A = (b_w + m_s h) h \quad (2.52)$$

where

b_w —Channel bottom width, m

h —Water depth, m

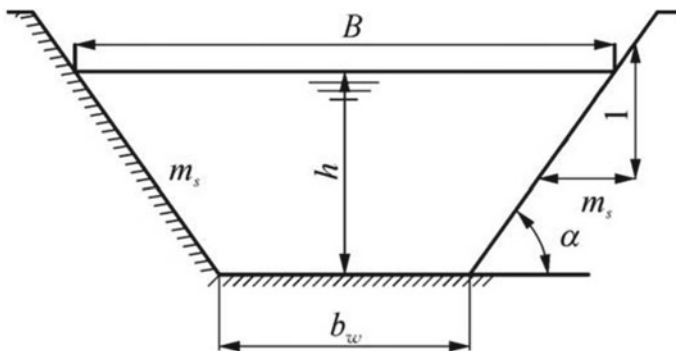


Fig. 2.34 Trapezoidal water-crossing section

m_s —The slope coefficient $m_s = \cot\alpha$, dimensionless.

Water surface width B (m):

$$B = b_w + 2m_s h \quad (2.53)$$

Wetted perimeter χ (m):

$$\chi = b_w + 2h\sqrt{1 + m_s^2} \quad (2.54)$$

Hydraulic radius R_h (m):

$$R_h = \frac{A}{\chi} \quad (2.55)$$

Obviously, in the four equations mentioned above, for a rectangular cross section, the slope coefficient $m_s = 0$, and for a triangular cross section, the bottom width $b_w = 0$.

If the trapezoidal cross section is asymmetrical and the slope coefficients on both sides are not equal ($m_{s1} \neq m_{s2}$),

$$A = \left(b_w + \frac{m_{s1} + m_{s2}}{2} h \right) h \quad (2.56)$$

$$B = b_w + m_{s1} h + m_{s2} h \quad (2.57)$$

$$\chi = b_w + \left(\sqrt{1 + m_{s1}^2} + \sqrt{1 + m_{s2}^2} \right) h \quad (2.58)$$

The bottom width b_w of the drainage open channel and cover channel should be greater than 0.3 m. The slope coefficient m_s can be adopted in accordance with Table 2.4.

Table 2.4 Open channel slope coefficient (m_s) (MOHURD and SAMR, 2021)

Geology	Slope coefficient (m_s)
Paving with brick or concrete block	1:0.75 ~ 1:1
Silt	1:3 ~ 1:3.5
Loose fine sand, medium sand and coarse sand	1:2 ~ 1:2.5
Dense fine sand, medium sand, coarse sand or clayey Silt	1:1.5 ~ 1:2
Silty clay or clay gravel or pebble	1:1.25 ~ 1:1.5
Semi lithologic soil	1:0.5 ~ 1:1
Weathered rock	1:0.25 ~ 1:0.5
Rock	1:0.1 ~ 1:0.25

Drainage pipes and hydraulic tunnels are often constructed by circular pipes because they are not made of soil. When the diameter d , filling depth h and central angle φ_c (Fig. 2.35) are known, the hydraulic elements of the open channel circular pipe cross section can be easily deduced from the geometric relationship.

Wetted cross section area:

$$A = \frac{d^2}{8}(\varphi_c - \sin \varphi_c) \quad (2.59)$$

Wetted perimeter:

$$\chi = \frac{1}{2}\varphi_c d$$

Water surface width:

$$B = d \sin \frac{\varphi_c}{2}$$

Hydraulic radius:

$$R_h = \frac{d}{4} \left(1 - \frac{\sin \varphi_c}{\varphi_c} \right) \quad (2.60)$$

Velocity of flow, according to Chézy's equation:

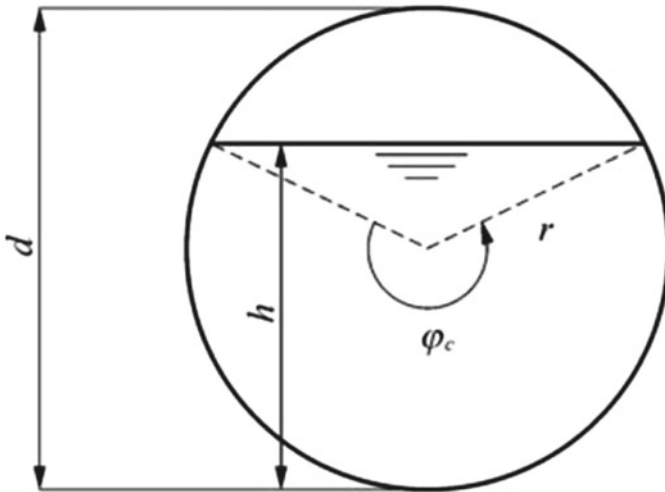


Fig. 2.35 Water cross section of an open channel circular pipe

$$u = \frac{C_c}{2} \sqrt{\left(1 - \frac{\sin \varphi_c}{\varphi_c}\right)} di \quad (2.61)$$

Flow rate:

$$q_v = \frac{C_c (\varphi_c - \sin \varphi_c)^{3/2}}{16 \sqrt{\varphi_c}} d^{5/2} \sqrt{i} \quad (2.62)$$

The relationship between the water filling depth h and central angle φ_c

$$h = \frac{d}{2} \left(1 - \cos \frac{\varphi_c}{2}\right) = d \sin^2 \frac{\varphi_c}{4} \quad (2.63)$$

$$\alpha_f = \frac{h}{d} = \sin^2 \frac{\varphi_c}{4} \quad (2.64)$$

where

d —Pipe diameter, m

h —Depth of water filling in the cross section, m

φ_c —Central angle is expressed in radian, dimensionless

χ —Wetted perimeter (exclusive of water surface), m

C_c —Chézy coefficient, $\text{m}^{1/2}/\text{s}$

α_f —Filling degree, $\frac{h}{d}$, dimensionless.

Let q_{v1} and u_1 be the flow rate and velocity of a full circular pipe, namely, at the filling depth $h = d$. q_v and u are the flow rate and velocity at the filling depth $h < d$. According to different filling degrees $\alpha_f = \frac{h}{d}$, the flow rate ratio $\frac{q_v}{q_{v1}}$ and flow velocity ratio $\frac{u}{u_1}$ can be calculated from the above relationships. With $\frac{h}{d}$ as the ordinate and $\frac{q_v}{q_{v1}}$ and $\frac{u}{u_1}$ as the abscissa, Fig. 2.36 is drawn, which can be used for the hydraulic calculation of open channel circular pipes. Figure 2.36 shows that when $\frac{h}{d} = 0.938$, the flow rate of the open channel circular pipe is the largest, and when $\frac{h}{d} = 0.81$, the flow velocity of the open channel circular pipe is the largest.

In the hydraulic calculation of pressure-free pipelines, the relevant provisions in the book entitled *Standard for Outdoor Drainage Design* issued by the China Ministry of Construction should also be consulted. Among them, the sewage pipeline should be calculated according to a partly full circular pipe flow, its maximum design filling degree should be selected according to Table 2.5, and the rainwater pipeline and combined pipeline should be calculated according to the fully filled pipe flow.

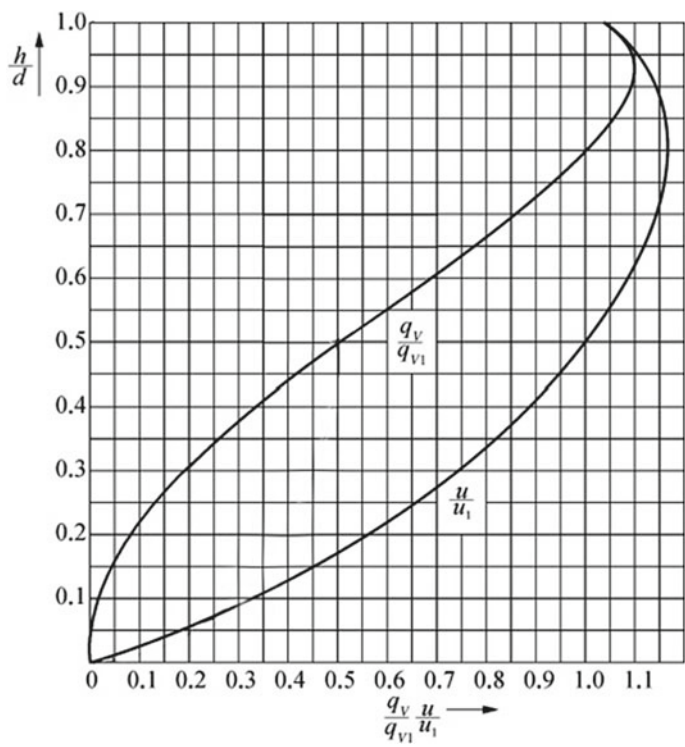


Fig. 2.36 The relationship among $\frac{h}{d}$, $\frac{q_v}{q_{v1}}$ and $\frac{u}{u_1}$

Table 2.5 Maximum design filling degree (Ke et al., 2000; MOHURD and SAMR, 2021)

Pipe diameter (d) or culvert depth (H) (mm)	Maximum design filling degree $\left(\alpha_f = \frac{h}{d} \text{ or } \frac{h}{H}\right)$
200 ~ 300	0.55
350 ~ 450	0.65
500 ~ 900	0.705
≥ 1000	0.75

Source of Table 2.5: Reprinted from Table 6–7 in book “Hydraulics” with kind permission from Prof. Kui Ke

Table 2.6 Rough coefficient n_r values for various pipes and channels (Ke et al., 2000; Gao et al., 2018; MOHURD and SAMR, 2021)

Types and conditions of pipes and channels	Minimum	Normal value	Maximum
(1) Pipe			
① UPVC pipe, PE pipe, glass steel pipe	0.009		0.011
② Asbestos cement pipe, steel pipe		0.012	
③ Ceramic pipe, cast iron pipe		0.013	
④ Concrete pipe, reinforced concrete pipe	0.013		0.014
(2) Lined channel			
① Pure cement surface	0.010	0.011	0.013
② Cement mortar	0.011	0.013	0.015
③ Scraped concrete surface	0.013	0.015	0.016
④ Unscraped concrete surface	0.014	0.017	0.020
⑤ Concrete shotcrete with good surface	0.016	0.019	0.023
⑥ Mortar block stone	0.017	0.025	0.030
⑦ Dry masonry block stone	0.023	0.032	0.035
⑧ Smooth asphalt surface	0.013	0.013	
⑨ Polished wood treated with wood distillate	0.011	0.012	0.015
⑩ Painted smooth steel surface	0.012	0.013	0.017
(3) Unlined channels			
① Clean straight earth channel	0.018	0.022	0.025
② Straight earth channels with weeds	0.022	0.027	0.033
③ Soil channels with bending weeds and varying cross sections	0.025	0.030	0.033
④ Smooth and even stone channel	0.025	0.030	0.040
⑤ Uneven and irregular stone channels	0.035	0.040	0.050
⑥ Channels with dense weeds as deep as water	0.050		

2.4.2.3 Several Problems in Hydraulic Calculation for Open Channels

1) Selection of rough coefficient n_r

According to the Manning equation, the rough coefficient n_r has a great influence on the Chézy coefficient C_c . For the same hydraulic radius, if the selected value n_r is larger, the Chézy coefficient C_c is smaller. According to the basic equation Eq. (2.51) of uniform flow in an open channel, it is necessary to increase the area of the cross section or the bottom slope of the channel to pass the given design flow rate. In this way, on the one hand, the excavation workload is increased; on the other hand, the burial depth of the pipe or channel also becomes greater because of the high bottom slope and the fast water surface drop. If the actual flow velocity after the operation of the channel is too high, this will cause channel scour. In contrast, if the selected n_r value is smaller than the actual value, for the same hydraulic radius, the C_c value is greater, and the flow velocity is larger. To pass through the established design flow,

both bottom slopes and the area of the cross section for channel are designed to be smaller, and the actual roughness coefficient n_r value of the channel after operation is larger than that of the design, which leads to the actual flow velocity of the channel be smaller than the design value, resulting in the sediment and siltation of suspended solids, as well as the low flow rate and insufficient discharge. Therefore, the selection of the rough coefficient n_r is very important in channel design.

Table 2.6 lists the rough coefficient n_r values for various pipes and channels for reference.

The following points should be considered when selecting the roughness coefficient n_r in the design of channel:

- (1) The selection of the n_r value means that the influence of roughness on flow resistance is comprehensively estimated. Therefore, it is necessary to have a correct understanding of the aforementioned influencing factors and general rules of flow resistance and head loss.
- (2) Mature typical rough coefficient data should be referenced.
- (3) The selection of roughness coefficient n_r should be made as practical as possible by referring to the actual measured data and application results of the same type of channel both locally and in the field.
- (4) To ensure that the selected n_r value meets the design requirements, relevant requirements should be put forward in the design documents for the construction quality, operation and maintenance of channels.

2) Optimal hydraulic cross section and practical economic solution

When the bottom slope, roughness coefficient and flow rate of the open channel are fixed, the design of the channel cross section (shape and size) can have a variety of options, which should be compared from the aspects of construction, operation and economy.

From the point of view of hydraulics, when the flow rate, bottom slope and rough coefficient are known, the designed cross section form has the smallest area. Moreover, when the cross section area, bottom slope and rough coefficient are known, the form of the designed cross section can maximize the flow rate through the channel. This cross section is called the optimal hydraulic cross section.

Obviously, the optimal hydraulic cross section should be the one with the lowest flow resistance under given conditions.

From $q_V = AR_h^{2/3}i^{1/2}/n_r$ and $R_h = \frac{A}{\chi}$, obtain:

$$q_V = \frac{A^{5/3}\sqrt{i}}{n_r\chi^{2/3}} \quad (2.65)$$

where

q_V —Flow rate, m^3/s

A —Area of cross section, m^2

χ —Wetted perimeter, m

i —Bottom slope, dimensionless

n_r —Rough coefficient, dimensionless.

Equation (2.65) shows that to maximize the flow rate through a given cross section area, the wetted perimeter of the cross section must be minimal. According to geometry, the cross section that best meets this requirement in all kinds of open channel cross section forms is a semicircular cross section (water surface is not included in the wetted perimeter). Therefore, some artificial channel (such as small concrete aqueducts) cross sections are designed as semicircular or U-shaped cross sections. However, due to geological conditions, construction technology, management and other reasons, channel cross sections often have to be designed into other shapes. Next, the optimal hydraulic conditions of trapezoidal cross sections commonly used in soil channels are discussed.

In the wetted perimeter of trapezoidal cross section $\chi = b_w + 2h\sqrt{1 + m_s^2}$, the slope coefficient m_s is known, area A is given, b_w and h are interrelated, and $b_w = A/h - m_s h$; therefore,

$$\chi = \frac{A}{h} - m_s h + 2h\sqrt{1 + m_s^2}$$

Under the optimal hydraulic conditions,

$$\frac{d\chi}{dh} = -\frac{A}{h^2} - m_s + 2\sqrt{1 + m_s^2} = -\frac{b_w}{h} - 2m_s + 2\sqrt{1 + m_s^2} = 0$$

Thus, the width-depth ratio β_m of the trapezoidal cross section with the optimal hydraulic performance is obtained.

$$\beta_m = \frac{b_w}{h} = 2 \times \left(\sqrt{1 + m_s^2} - m_s \right) \quad (2.66)$$

It can be proven that the three sides of the trapezoid are tangent to the semicircle with radius h and the center of the circle on the water surface (Fig. 2.37). It should be noted here that because the normal water depth changes with the flow, the open channel with the optimal hydraulic cross section under the designed flow rate will no longer satisfy Eq. (2.66) when the flow rate changes.

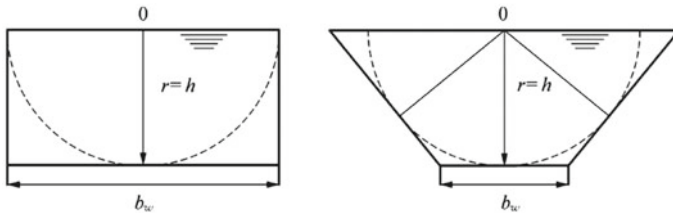


Fig. 2.37 The optimal hydraulic cross sections of rectangular and trapezoidal sections

In a rectangular cross section, as a special case of trapezoidal cross section, $m_s = 0$ is used to calculate, then $\beta_m = 2$ or $b_w = 2h$, so the bottom width of the optimal hydraulic rectangular cross section b_w is twice the depth of water (h). When $m_s > 0$, the value of β_m calculated by Eq. (2.66) decreases with increasing m_s (please see the line of $A = A_m$ in Table 2.7). When $m_s > 0.75$, then $\beta_m < 1$, which is a narrow and deep cross section with a narrower bottom width and deeper water depth.

Although the optimal hydraulic cross section has the smallest cross-sectional area under the same flow rate, it has certain limitations in terms of economy, technology and management. When applied to large-scale channels, because of deep excavation and high filling, the amount and cost of construction excavation are great, and maintenance and management are inconvenient. When the flow rate changes, the water depth changes greatly, which brings inconvenience to the collection of rain water and sewage, as well as irrigation and shipping. In fact, when designing channel cross sections, taking a wider width-depth ratio (β) in a certain range, the cross-sectional area A can still be very close to the optimal hydraulic cross section area A_m . According to Eq. (2.65), under the same flow rate, rough coefficient and bottom slope conditions, there is a relationship between the cross section parameters of the non-optimal hydraulic cross section A and the optimal hydraulic cross section A_m .

$$\left(\frac{A}{A_m}\right)^{5/2} = \frac{\chi}{\chi_m} = \frac{h(\beta + 2\sqrt{1+m_s^2})}{h_m(\beta_m + 2\sqrt{1+m_s^2})}$$

and

$$\frac{A}{A_m} = \frac{h^2(\beta + m_s)}{h_m^2(\beta_m + m_s)}$$

thus

$$\frac{h}{h_m} = \left(\frac{A}{A_m}\right)^{5/2} \left[1 - \sqrt{1 - \left(\frac{A_m}{A}\right)^4}\right] \quad (2.67a)$$

$$\beta = \left(\frac{h_m}{h}\right)^2 \frac{A}{A_m} \left(2\sqrt{1+m_s^2} - m_s\right) - m_s \quad (2.67b)$$

The variables with subscript m is the variables when β is equal to β_m ($\beta = \beta_m$) among above equations.

Lines $A/A_m = 1.01$ and 1.04 in Table 2.7 show that the area of the cross section is only 1 ~ 4% larger than that of the optimal hydraulic cross section, the corresponding width-depth ratio β is much larger than β_m , and the water depth h is much shallower than h_m , which provides designers with considerable room for manoeuvre. This cross section is called the suitable economic cross section.

Table 2.7 Width-depth ratio of the optimal hydraulic cross section ($A/A_m = 1.00$) and suitable economic cross section (Ke et al. 2000)

A/A_m	h/h_m	m_s	0.00	0.50	0.75	1.00	1.50	2.00	2.50	3.00
1.00	1.000	β	2.000	1.236	1.000	0.828	0.608	0.480	0.380	0.320
1.01	0.882		2.992	2.097	1.868	1.734	1.653	1.710	1.808	1.967
1.04	0.683		4.462	3.373	3.154	3.078	3.202	3.533	3.925	4.407

Source of Table 2.7: Reprinted from Table 6-2 in book “*Hydraulics*” with kind permission from Prof. Kui Ke

3) **Permissible flow velocity of channel (or pipe)**

A well-designed channel (or pipe), in addition to considering the above mentioned optimal hydraulic conditions and economic factors, should also make the design flow velocity of the channel (or pipe) not so high as to make the channel bed (or pipe wall) suffer erosion, nor so low as to make the suspended solids in the liquid sink and silt in the channel (or pipe), that is, the flow velocity is that value which protects the channel (or pipe) from nonscouring and nonsilting. Therefore, in the design, the channel (or pipeline) velocity u is required to be within the permissible velocity range for nonscouring and nonsilting, i.e.,

$$u'' < u < u'$$

where

u' —The maximum allowable flow velocity for protection from scouring, referred to as the allowable flow velocity avoiding nonscouring or the maximum design flow velocity, m/s

u'' —The minimum allowable flow velocity without siltation, referred to as the allowable flow velocity without siltation or the minimum design flow velocity, m/s.

Maximum design flow velocity u' : this value depends on ① soil properties, i.e., soil type, particle size and compactness; ② lining material (or inner wall material); ③ flow rate in a channel (or pipeline); ④ the depth of fluid and other factors. The minimum design flow velocity u'' guarantees that the suspended solids carried in the flow are not allowed to deposit in the channel (or pipe) at the lower allowable velocity limit. Details are listed in Tables 2.8 and 2.9.

The design depth of drainage open channels is generally 0.4 ~ 1.0 m. When the water depth is beyond 0.4 ~ 1.0 m, the maximum design velocity is multiplied by the

Table 2.8 Maximum or minimum design flow velocity (m/s) of sewage pipeline (unit: m/s) (Gao et al. 2018)

Maximum design velocity		Minimum design velocity	
Metal pipe	Nonmetal pipe	Design filling degree	Full flow
10	5	0.6	0.75

Source of Table 2.8: Reprinted from Table 2-2 in book “*Water Pollution Control Engineering* (Volume 1)” with kind permission from Prof. Qi Zhou

Table 2.9 Maximum or minimum design flow velocities (m/s) in drainage open channels (Gao et al., 2018; MOHURD and SAMR, 2021)

Open channel material	Maximum design flow velocity (m/s)				Minimum design flow velocity (m/s)
	$0.4 \text{ m} \leq \text{water depth} \leq 1.0 \text{ m}$	Water depth $< 0.4 \text{ m}$	$1.0 \text{ m} < \text{water depth} < 2.0 \text{ m}$	Water depth $\geq 2.0 \text{ m}$	
Coarse sand or low plastic silty clay	0.80	0.68	1.00	1.12	0.4
Silty clay	1.00	0.85	1.25	1.40	
Clay	1.20	1.02	1.50	1.68	
Herbaceous cover	1.60	1.36	2.00	2.24	
Dry masonry block stone	2.00	1.70	2.50	2.80	
Masonry block stone or brick	3.00	2.55	3.75	4.20	
Limestone or medium sandstone	4.00	3.40	5.00	5.60	
Concrete	4.00	3.40	5.00	5.60	

Source of Table 2.9: Reprinted from Table 4-10 in book “*Water Pollution Control Engineering* (Volume 1)” with kind permission from Prof.Qi Zhou

following coefficients: water depth $< 0.4 \text{ m}$, coefficient 0.85; $1.0 \text{ m} < \text{water depth} < 2.0 \text{ m}$, coefficient 1.25; water depth $\geq 2 \text{ m}$, coefficient 1.40. The minimum design velocity is 0.4 m/s , which is independent of both water depth and the material in the open channel.

In practical engineering, trapezoidal cross section channels are one of the most widely used. Taking the trapezoidal channel as an example, the following cross section illustrates several hydraulic calculation methods often encountered.

4) Hydraulic calculation

Equation (2.68) can be obtained according to the basic equations for calculating uniform flow in an open channel and the equations for calculating hydraulic elements in a trapezoidal cross section can be obtained.

$$q_V = AC_c \sqrt{R_h i} = A \frac{1}{n_r} R_h^{2/3} \sqrt{i} = \frac{\sqrt{i}}{n_r} \frac{[(b_w + m_s h)h]^{5/3}}{(b_w + 2h\sqrt{1 + m_s^2})^{2/3}} \quad (2.68)$$

where

q_V —Flow rate, m^3/s

A —Area of cross section, m^2

C_c —Chézy coefficient, $\text{m}^{1/2}/\text{s}$

R_h —Hydraulic radius, m

i —Bottom slope, dimensionless

n_r —Rough coefficient, dimensionless

b_w —Channel bottom width, m

h —Water depth, m

m_s —Slope coefficient, dimensionless.

Equation (2.68) shows that $q_V = f(b_w, h, m_s, n_r, i)$ have 6 variables. If the values of 5 variables are known, the remaining linked variables can be obtained from Eq. (2.68). Sometimes, remaining unknown variables can be obtained directly from Eq. (2.68). Sometimes, this is quite difficult to obtain from Eq. (2.68) due to the need to solve complex higher-order equations. For this reason, two kinds of problems are studied in a unified way from the perspective of computational methods. As long as these methods are mastered, the hydraulic calculation of open channel uniform flow can be carried out smoothly.

(1) Direct Solution Method

If the values of other five variables are known, the flow rate q_V , rough coefficient n_r , and bottom slope i are required respectively, and the solution can be obtained directly by simple algebraic operations using basic Eq. (2.68). An example is given to illustrate the method.

Example 2.14 There is a sewage channel with a rectangular cross section with a bottom width of 1.0 m, a bottom slope of 0.005, a uniform flow water depth of 0.5 m, and a velocity of 2.0 m/s. The rough coefficient of the sewage tank is required.

Known: $b_w = 1.0$ m, $i = 0.005$, $h = 0.5$ m, $u = 2.0$ m/s, the slope coefficient m_s of the rectangular cross section is equal to 0, $m_s = 0$.

Determine: n_r

Solution:

$$q_V = ub_w h = 2.0 \times 1.0 \times 0.5 = 1.0 \text{ m}^3/\text{s}$$

$$q_V = \frac{\sqrt{i}}{n_r} \cdot \frac{(b_w h)^{\frac{5}{3}}}{(b_w + 2h)^{\frac{2}{3}}}$$

$$n_r = \frac{\sqrt{i}}{q_V} \cdot \frac{(b_w h)^{\frac{5}{3}}}{(b_w + 2h)^{\frac{2}{3}}} = \frac{\sqrt{0.005}}{1.0} \times \frac{(1.0 \times 0.5)^{\frac{5}{3}}}{(1.0 + 2 \times 0.5)^{\frac{2}{3}}} = 0.014$$

Answer: The rough coefficient of the sewage channel is 0.014.

Example 2.15 There is a trapezoidal sewage channel with a slope coefficient $m_s = 1.5$, bottom width $b_w = 3.0$ m, water depth $h = 2.84$ m, bottom slope $i = 1/6000$, and rough coefficient $n_r = 0.01$. Determine the flow rate and velocity of the channel.

Known: $m_s = 1.5$, $b_w = 3.0$ m, $h = 2.84$ m, $i = 1/6000$.

Determine: q_V , u

Solution:

$$A = (b_w + m_s h)h = (3 + 1.5 \times 2.84) \times 2.84 = 20.62 \text{ m}^2$$

$$\chi = b_w + 2h\sqrt{1 + m_s^2} = 3 + 2 \times 2.84 \times \sqrt{1 + 1.5^2} = 13.24 \text{ m}$$

$$R_h = \frac{A}{\chi} = \frac{20.62}{13.24} = 1.56 \text{ m}$$

$$u = \frac{1}{n_r} \cdot R_h^{\frac{2}{3}} \sqrt{i} = \frac{1}{0.01} \times 1.56^{\frac{2}{3}} \times \sqrt{\frac{1}{6000}} = 1.74 \text{ m/s}$$

$$q_V = uA = 1.74 \times 20.62 = 35.88 \text{ m}^3/\text{s}$$

Answer: The flow rate of the channel is $35.88 \text{ m}^3/\text{s}$, and the flow velocity is 1.74 m/s .

Example 2.16 There is a circular cement sewage pipe with a diameter of 800 mm and a rough coefficient of 0.012. The bottom slope of the pipe is 0.003. Please calculate the flow velocity and flow rate at the maximum design filling degree.

Known: $d = 800 \text{ mm} = 0.8 \text{ m}$, $n_r = 0.012$, $i = 0.003$.

Determine: u and q_V at maximum design filling degree.

Solution: In Table 2.5, it is seen that the maximum design filling degree α_f of an 800 mm sewage pipe is 0.705. $\alpha_f = \sin^2(\varphi_c/4)$, and the solution of $\varphi_c = 1.27\pi$. Equations (2.59 ~ 2.64) are used to calculate the hydraulic elements of the cross section of the circular pipe

$$A = \frac{d^2}{8}(\varphi_c - \sin \varphi_c) = \frac{0.8^2}{8}(1.27\pi - \sin 1.27\pi) = 0.38 \text{ m}^2$$

$$\chi = \frac{d}{2}\varphi_c = \frac{0.8}{2} \times 1.27\pi = 1.60 \text{ m}$$

$$R_h = \frac{A}{\chi} = \frac{0.38}{1.60} = 0.24 \text{ m}$$

$$\text{Since } C_c = \frac{1}{n_r} R_h^{1/6} = \frac{1}{0.012} \times 0.24^{1/6} = 65.69 \text{ m}^{1/2}/\text{s}$$

$$u = C_c \sqrt{R_h i} = 65.69 \times \sqrt{0.3 \times 0.0024} = 1.76 \text{ m/s}$$

$$q_V = uA = 1.76 \times 0.38 = 0.67 \text{ m}^3/\text{s}$$

Answer: The flow velocity and rate are 1.76 m/s and 0.67 m³/s at the maximum design filling degree.

(2) Trial method

If the values of other five variables values are known, the water depth h or the bottom width b_w needs to be calculated. Because the relationship between b_w and h expressed in the basic Eq. (2.68) is a high-order form, the direct solution method cannot be used; a trial method has to be used.

A. Graphing method

The graphing method is as follows: Suppose several h values are substituted into the basic Eq. (2.68) to calculate the corresponding q_V value. If the calculated q_V value is equal to the known value q_V , the corresponding h value is obtained. In fact, the first and second trial calculations often fail to produce correct results. To reduce the number of trial calculation, three to five h values, namely, $h_1, h_2, h_3, \dots, h_5$, are used, and the corresponding $q_{V1}, q_{V2}, q_{V3} \dots q_{V5}$ values are calculated and plotted as $q_V = f(h)$ curves. Then, h is determined from the known q_V on the curve. If b_w is required, it is the same as the trial algorithm for h . The curve drawn here is $q_V = f(b_w)$.

B. Iterative method

Another trial method is the iterative method, which can be divided into Calculator “Ans” and “SOLVE” method.

(a) Calculator “Ans” method. This method is as follows: the basic Eq. (2.68) is written into an appropriate equivalent equation $h = f(h)$ or $b_w = f(b_w)$ for iterative calculation, and then the value of h or b_w can also be solved. The basic steps are as follows:

① First, change the basic Eq. (2.68) into the form of “ $h = f(h)$ ” or “ $b_w = f(b_w)$ ”,

$$q_V = \frac{\sqrt{i}}{n_r} \frac{[(b_w + m_s h)h]^{5/3}}{(b_w + 2h\sqrt{1 + m_s^2})^{2/3}}$$

Change into the form of “ $h = f(h)$ ” or “ $b_w = f(b_w)$ ”,

$$h = \frac{q_V^{3/5} (\frac{n_r}{\sqrt{i}})^{3/5} (b_w + 2h\sqrt{1 + m_s^2})^{2/5}}{b_w + h m_s}$$

Or

$$b_w = \frac{q_V^{3/5} (\frac{n_r}{\sqrt{i}})^{3/5} (b_w + 2h\sqrt{1 + m_s^2})^{2/5}}{h} - m_s h$$

② Substitute the values of the other five known variables.

③ Use a scientific calculator to input the right part of the equation and use the “Ans” key instead of the unknown variable h or b_w . After the input is completed, press the “=” key repeatedly until the value which appears unchanged, and the value is the required h or b_w .

(b) Calculator “SOLVE” method (Also Known as Newton Iterative Method).

The value of h , b_w or m_s can also be solved according to the basic Eq. (2.68) using “SOLVE” key in calculator. The basic steps are as follows:

$$\textcircled{1} q_V = \frac{\sqrt{i}}{n_r} \frac{[(b_w + m_s h)h]^{5/3}}{(b_w + 2h\sqrt{1 + m_s^2})^{2/3}}$$

② Substitute the values of the other five known variables

$$\textcircled{3} \text{ Use a scientific calculator to input: } q_V = \frac{\sqrt{i}}{n_r} \frac{[(b_w + m_s h)h]^{5/3}}{(b_w + 2h\sqrt{1 + m_s^2})^{2/3}}$$

A use the “x” key instead of the one of unknown number h , b_w or m_s . After the input is completed, press the “SOLVE” key, input a hypothetical value “x” (such as 1) at will, press the “=” key and the shown value is the required h , b_w or m_s .

Example 2.17 There is a sewage channel with a trapezoidal cross section, where the slope coefficient $m_s = 1.5$, rough coefficient $n_r = 0.02$, bottom width $b_w = 4.5$ m and bottom slope $i = 0.0006$. Please calculate the uniform flow depth h when the flow rate is $10 \text{ m}^3/\text{s}$.

Known: $m_s = 1.5$, $n_r = 0.02$, $b_w = 4.5$ m, $i = 0.0006$, $q_V = 10 \text{ m}^3/\text{s}$.

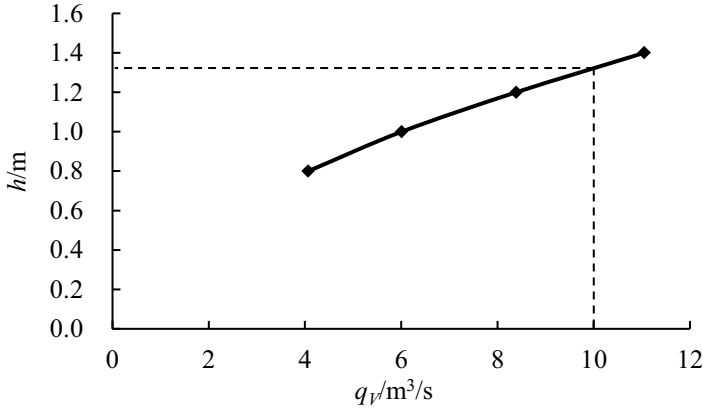
Determine: h

Solution: (1) Graphing method

Suppose several values of h and corresponding q_V values are listed in following table.

b_w	m_s	h	A	X	R_h	$\sqrt{R_h}$	n_r	C_c	\sqrt{i}	q_V
4.5	1.5	0.8	4.56	7.38	0.62	0.79	0.02	46.17	0.0245	4.07
		1.0	6.00	8.11	0.74	0.86		47.55		6.01
		1.2	7.56	8.83	0.86	0.93		48.76		8.39
		1.4	9.24	9.55	0.97	0.98		49.83		11.05

The corresponding values of q_V and h in the table are plotted as $q_V = f(h)$ curves as shown in following figure.



From the curve of $q_v = 10 \text{ m}^3/\text{s}$, the corresponding water depth $h_0 = 1.32 \text{ m}$ is found.

(2) Iterative method

① Calculator “Ans” method

$$h = \frac{q_v^{3/5} \left(\frac{n_r}{\sqrt{i}} \right)^{3/5} \left(b_w + 2h\sqrt{1 + m_s^2} \right)^{2/5}}{b_w + hm_s}$$

Substitute “ $m_s = 1.5$, $n_r = 0.02$, $b_w = 4.5 \text{ m}$, $i = 0.0006$, $q_v = 10 \text{ m}^3/\text{s}$ ” in:

$$h = \frac{10^{3/5} \left(\frac{0.02}{\sqrt{0.0006}} \right)^{3/5} \left(4.5 + 2h\sqrt{1 + 1.5^2} \right)^{2/5}}{4.5 + 1.5h}$$

Using a scientific calculator, enter the right side of the equation, and replace “ h ” with “Ans”,

$$h = \frac{3.98 \times 0.89 \left(4.5 + 2\text{Ans}\sqrt{1 + 1.5^2} \right)^{2/5}}{4.5 + 1.5\text{Ans}}$$

Press the “=” key repeatedly, and the value does not change at 1.32, i.e., the water depth is 1.32 m.

② Calculator “SOLVE” method

$$q_v = \frac{\sqrt{i}}{n_r} \frac{[(b_w + m_s h)h]^{5/3}}{(b_w + 2h\sqrt{1 + m_s^2})^{2/3}}$$

Substitute $m_s = 1.5$, $n_r = 0.02$, $b_w = 4.5 \text{ m}$, $i = 0.0006$, $q_v = 10 \text{ m}^3/\text{s}$,

$$10 = \frac{\sqrt{0.0006}}{0.02} \frac{[(4.5 + 1.5h)h]^{5/3}}{(4.5 + 2h\sqrt{1 + 1.5^2})^{2/3}}$$

Using a scientific calculator, and use the “ x ” key instead of the unknown variable h .

$$10 = \frac{\sqrt{0.0006}}{0.02} \frac{[(4.5 + 1.5x)x]^{5/3}}{(4.5 + 2x\sqrt{1 + 1.5^2})^{2/3}}$$

After the input is completed, press the “SOLVE” key, input a hypothetical value “ x ” (such as 1) at will, press the “=” key and the shown value is the required h , then the water depth is 1.32 m.

Answer: When the flow rate is $10 \text{ m}^3/\text{s}$, the water depth is 1.32 m.

Example 2.18 There is a trapezoidal sewage channel with a flow rate of $8 \text{ m}^3/\text{s}$. The rough coefficient $n_r = 0.033$, the bottom slope $i = 0.0002$, $m_s = 1.5$, and the known uniform flow water depth is 1.8 m. Please calculate the bottom width of the channel b_w .

Known: $q_V = 8 \text{ m}^3/\text{s}$, $n_r = 0.033$, $i = 0.0002$, $m_s = 1.5$, $h = 1.8 \text{ m}$.

Determine: b_w

Solution: (1) Graphing method

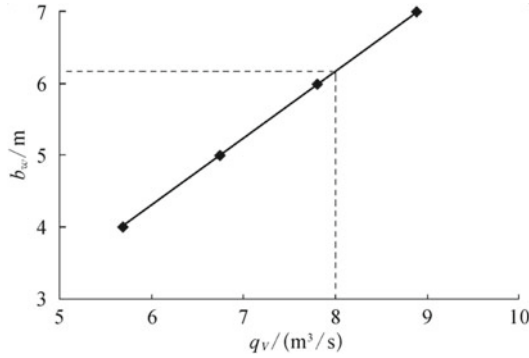
From the basic equation

$$q_V = \frac{\sqrt{i}}{n_r} \cdot \frac{[(b_w + m_s h)h]^{5/3}}{(b_w + 2h\sqrt{1 + m_s^2})^{2/3}} = 0.429 \times \frac{[(b_w + 2.7) \times 1.8]^{5/3}}{(b_w + 6.49)^{2/3}}$$

Assuming $b_w = 4, 5, 6, 7$, the corresponding q_V values are calculated as follows:

$b_w(\text{m})$	4	5	6	7
$q_V(\text{m}^3/\text{s})$	5.68	6.74	7.81	8.89

Draw the $q_V = f(b_w)$ curve according to the results obtained.



From the curve, it is found that $b_w = 6.18$ m when $q_v = 8$ m³/s.

(2) Iterative method

① Calculator “Ans” method

$$b_w = \frac{q_v^{3/5} \left(\frac{n_r}{\sqrt{i}} \right)^{3/5} \left(b_w + 2h\sqrt{1 + m_s^2} \right)^{2/5}}{h} - m_s h$$

Because $q_v = 8$ m³/s, $n_r = 0.033$, $i = 0.0002$, $m_s = 1.5$, $h = 1.8$ m:

$$\begin{aligned} b_w &= \frac{8^{3/5} \left(\frac{0.033}{\sqrt{0.0002}} \right)^{3/5} \left(b_w + 2 \times 1.8\sqrt{1 + 1.5^2} \right)^{2/5}}{1.8} - 1.5 \times 1.8 \\ &= \frac{3.48 \times 1.66 \times (b_w + 6.49)^{2/5}}{1.8} - 2.7 \end{aligned}$$

Using a scientific calculator, enter the right side of the equation, and replace “ b_w ” with “Ans” and obtain:

$$b_w = \frac{8^{3/5} \left(\frac{0.033}{\sqrt{0.0002}} \right)^{3/5} \left(\text{Ans} + 2 \times 1.8\sqrt{1 + 1.5^2} \right)^{2/5}}{1.8} - 1.5 \times 1.8$$

Press the “=” key repeatedly, and the value does not change at 6.18; thus, the water depth is 6.18 m.

② Calculator “SOLVE” method

$$q_v = \frac{\sqrt{i}}{n_r} \frac{[(b_w + m_s h)h]^{5/3}}{(b_w + 2h\sqrt{1 + m_s^2})^{2/3}}$$

Substitute “ $q_v = 8$ m³/s, $n_r = 0.033$, $i = 0.0002$, $m_s = 1.5$, $h = 1.8$ m”:

$$8 = \frac{\sqrt{0.0002}}{0.033} \frac{[(b_w + 1.5 \times 1.8)1.8]^{5/3}}{(b_w + 2 \times 1.8\sqrt{1 + 1.5^2})^{2/3}}$$

Using a scientific calculator, enter the equation, and use the “ x ” key instead of the unknown number b_w .

$$8 = \frac{\sqrt{0.0002}}{0.033} \frac{[(x + 1.5 \times 1.8)1.8]^{5/3}}{(x + 2 \times 1.8\sqrt{1 + 1.5^2})^{2/3}}$$

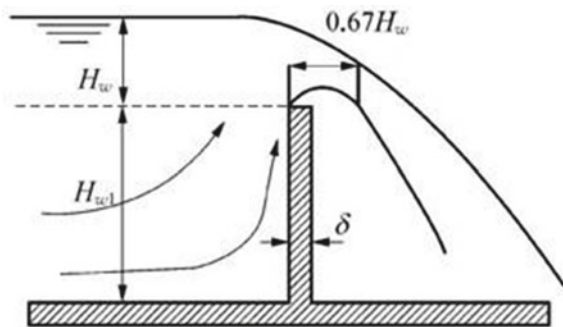
After the input is completed, press the “SOLVE” key, input a hypothetical value “ x ” (such as 1) at will, press the “=” key and the shown value is the required b_w ; thus, the water depth is 6.18 m.

Answer: The width of the channel bottom is 6.18 m.

2.4.3 Thin Wall Weir

The incoming flow in front of a weir is blocked by the weir wall, and the bottom flow bends upward because of inertia. When the water tongue falls back to the crest elevation, it is approximately $0.67H_w$ away from the upstream wall (H_w is the height between the crest of the weir and the surface of upstream flow. H_w is called as head above weir, or blockage height above weir, with the unit of meter (m)). When the crest thickness of the weir is less than $0.67H_w$ ($\delta \leq 0.67H_w$), as shown in Fig. 2.38, the water tongue is not affected by the width of the weir, the lower edge of the water tongue is only in line contact with the crest of the weir, the weir and the flow passing through the weir are only in line contact. The crest thickness of the weir has no effect on the flow, so it is called a thin wall weir. A thin wall weir is also called a sharp edge weir because its top is often made into sharp edges. A thin wall weir is mainly

Fig. 2.38 Thin wall weir



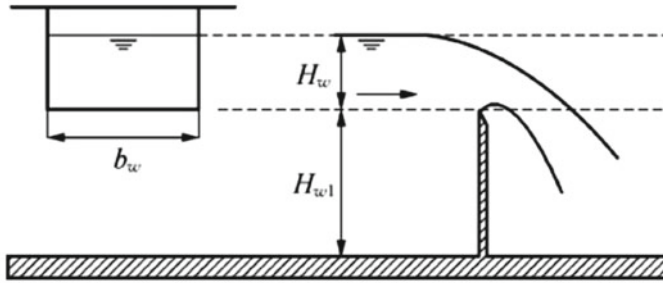


Fig. 2.39 Rectangular thin wall weir

used as flow rate measurement equipment as well as effluent weir of tanks, such as the primary settler, secondary clarifier, thickening tank, ect.

A thin wall weir flow has a stable head-flow rate relationship, so it is often used as flow rate measuring tool in laboratory model tests or field measurements. Curved practical weirs and tunnel entrance curves widely used in engineering practice are often constructed according to the lower edge curve of the flow tongue of thin wall weirs. According to the shape of the outlet, thin wall weir can be divided into rectangular weir, triangular weir and trapezoidal weir, and commonly used weir are rectangular weir and triangular weir.

1) Rectangular thin wall weir

The overflow of a rectangular weir is shown in Fig. 2.39. The basic equation of free overflow is as follows.

$$q_v = K_w b_w \sqrt{2g} H_w^{3/2}$$

If the influence of the near-velocity head $\frac{au_0^2}{2g}$ is included in the flow coefficient, the basic equation is rewritten as follows:

$$q_v = K'_w b_w \sqrt{2g} H_w^{3/2} \quad (2.69)$$

where H_w —Head above weir, or blockage height above weir, m

K_w —Flow rate coefficient, dimensionless

K'_w —Flow rate coefficient taking into account the kinetic energy of upstream velocity. This is determined by Bazan's equation and is dimensionless.

$$K'_w = (0.405 + \frac{0.0027}{H_w}) [1 + 0.55 (\frac{H_w}{H_w + H_{w1}})^2] \quad (2.70)$$

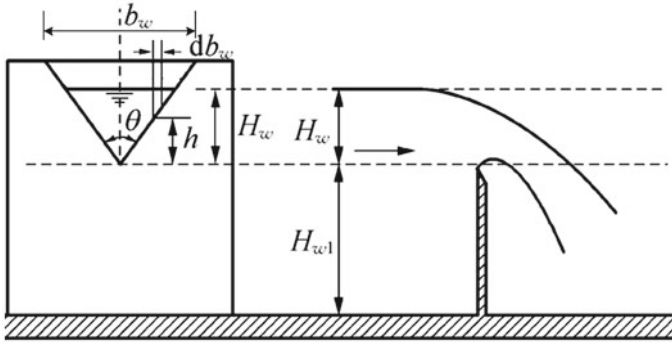


Fig. 2.40 Triangular thin wall weir

In Eq. (2.70), the unit of both H_w and H_{w1} is meter (m). The applicable range of the equation is $H_w \leq 1.24$ m, $H_{w1} \leq 1.13$ m, $b_w \leq 2$ m. H_{w1} is the height of the rectangular thin wall weir.

2) Triangular thin wall weir

The shape of a triangular thin wall weir is an isosceles triangle and is referred to as a triangular weir or V-notch weir, as shown in Fig. 2.40.

A triangular weir is used to measure low flow rate, because a smaller change in flow rate will make the head of the weir change greatly to ensure the accuracy of the measurement. Therefore, a triangular weir is often used to measure low flow rate.

As shown in Fig. 2.40, the angle of the triangular weir is θ , and the head above the weir is H_w . The small width of db_w is considered a thin wall weir flow. The expression of the microflow rate is as follows:

$$dq_V = K'_w \sqrt{2g} h^{3/2} db_w \quad (2.71)$$

In the equation, the head at h is determined by a geometric relation.

$$b_w = (H_w - h) \tan \frac{\theta}{2}, \text{ thus: } db_w = -\tan \frac{\theta}{2} dh$$

Substitute the Eq. (2.71):

$$dq_V = -K'_w \tan \frac{\theta}{2} \sqrt{2g} h^{3/2} dh$$

Overflow rate of weir

$$q_V = -2K'_w \tan \frac{\theta}{2} \sqrt{2g} \int_{H_w}^0 h^{3/2} dh = \frac{4}{5} K'_w \tan \frac{\theta}{2} \sqrt{2g} H_w^{5/2}$$

When $\theta = 90^\circ$, the above equation is as follows:

$$q_v = \frac{4}{5} K'_w \sqrt{2g} H_w^{5/2} = K''_w H_w^{5/2} \quad (2.72)$$

where

K''_w —Flow rate coefficient of triangular thin wall weir, $\text{m}^{1/2}/\text{s}$

H_w —Head above weir, m.

The vertex angle of a triangular thin wall weir is generally made into a 90-degree shape, which is called a right-angle triangular thin wall weir. For a right-angle triangular thin wall weir, the flow rate coefficient K''_w can be calculated according to the following equation:

$$K''_w = 1.354 + \frac{0.004}{H_w} + \left(0.14 \frac{0.2}{\sqrt{H_{w1}}} \right) \left(\frac{H_w}{B} - 0.09 \right)^2 \quad (2.73)$$

where

H_w —Head above weir, m

H_{w1} —Height of the weir, m

B —Width of the upstream diversion channel of the weir, m.

When $0.5 \text{ m} \leq B \leq 1.2 \text{ m}$, $0.1 \text{ m} \leq H_{w1} \leq 0.75 \text{ m}$, $0.07 \text{ m} \leq H_w \leq 0.26 \text{ m}$, and $H_w \leq B/3$, the measurement error for flow rate is less than $\pm 1.4\%$.

Thomson's experiment gave $K''_w = 1.4$, which was approximately adopted. Then, the flow rate equation is:

$$q_v = 1.4 H_w^{5/2} \quad (2.74)$$

A triangular thin wall weir provides a highly accurate flow rate measurement. It mainly uses a V-notch on a vertical thin plate to measure flow rate, as shown in Fig. 2.40. The bisector of the notch top angle should be vertical and equidistant from the sidewalls on both sides of the channel. The range of vertex angle θ is $20^\circ \sim 100^\circ$. It must be processed by a precise method. The triangular weir is only suitable for straight and horizontal rectangular channel cross sections. However, if the gap area is so small as to neglect its effect on the near velocity, the channel shape does not matter. The flow in the approaching channel should be uniform and stable.

Attention should be paid to the installation and use of thin wall weirs:

① The weir plate must be flat and vertical, and the centerline of the weir should coincide with the centerline of the channel.

② The weir slab should be made of a steel plate or wooden board. The weir mouth should be a sharp edge of 45° , with its inclination facing downstream.

③ The height and shoulder width of the triangular weir should be greater than the maximum depth of the weir, and the maximum depth of the rectangular weir

should be less than the height of the weir; otherwise, submerged flow will occur (the downstream water level is higher than the weir).

④ The water depth gauge should be set on the weir slabs on both sides of the notch, as far as possible in the stable position of the inner water level.

⑤ The weir body should be closely intermingled with soil channels to avoid leaking water.

⑥ Weir slabs should be made and installed according to specifications, and the installed cross sections should be used as bottom protection.

2.5 Common Measuring Instruments

There are many common measuring instruments, but in the field of environmental engineering, the main measuring instruments are Venturi flowmeters, triangular weirs and Ubbelohde viscometers.

2.5.1 Venturi Flowmeter

To reduce the energy loss of a fluid flowing through a throttle element, a tapering and expanding tube can be used instead of an orifice plate. The flowmeter thus constituted is called a Venturi flowmeter, as shown in Fig. 2.41. The upstream pressure gauge of the Venturi flowmeter (cross section 1–1') should be at least one-half the diameter of

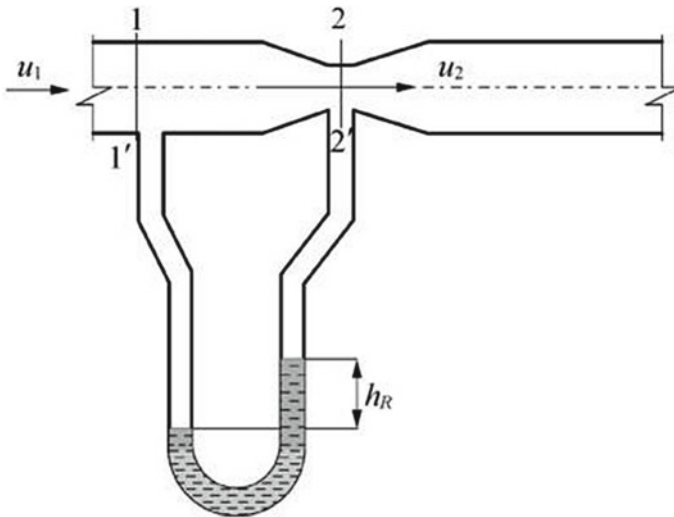


Fig. 2.41 Venturi flowmeter

the tube from the beginning of contraction. The downstream pressure gauge should be located at the minimum flow cross section 2–2' (called the Venturi throat). Because there are gradual shrinkage and gradual expansion cross sections, the flow velocity of the fluid changes gently and the eddy current is lessened, so the energy loss is much less than that of orifice plates.

The flow calculation equation of the Venturi flowmeter is as follows:

$$q_V = K_V A_0 \sqrt{\frac{2\Delta p}{\rho}} = K_V A_0 \sqrt{\frac{2h_R g(\rho_A - \rho)}{\rho}} \quad (2.75)$$

where

q_V —Flow rate of the measured pipe, m^3/s

K_V —Flow rate coefficient, dimensionless, which can be measured experimentally or from the instrument manual and is generally 0.6 ~ 0.7

Δp —Pressure difference between cross sections 1–1' and 2–2', Pa

A_0 —Cross-sectional area of the throat, m^2

ρ_A —Density of indicator liquid, kg/m^3

ρ —Density of the fluid under test, kg/m^3 .

Advantage: Venturi flowmeter has less energy loss

Disadvantage: Each part size requirement is strict and requires fine processing, so the cost is relatively high.

2.5.2 Ubbelohde Viscometer

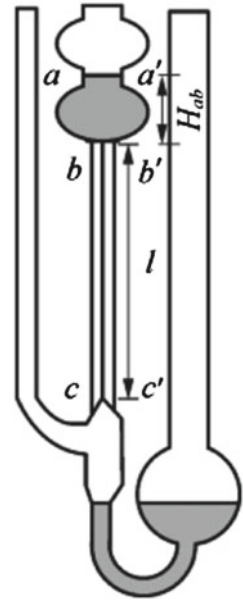
An Ubbelohde viscometer is a common instrument for measuring liquid viscosity, as shown in Fig. 2.42. The viscosities of fluids are calculated by measuring the volume of V (liquids between scales a and b in Fig. 2.42) flowing through a capillary tube of a certain length l . The time required is τ . Its principle is based on the analysis of the resistance of fluid flowing through the capillary. The small tube on the left side of the capillary is a bypass tube that allows point c to pass through the atmosphere. The thick tube on the right is the container tube for storing fluids.

In the capillary, the Bernoulli equation can be obtained between the b - b' cross section and c - c' cross section, and the c - c' cross section is the datum level.

$$gz_{hb} + \frac{p_b}{\rho} + \frac{u_b^2}{2} = gz_{hc} + \frac{p_c}{\rho} + \frac{u_c^2}{2} + h_{tf}$$

$\therefore z_{hc} = 0, z_{hb} = l, u_b = u_c, p_c = 0$ (gauge pressure), ignoring the potential difference between a and b , $H_{ab} \approx 0, p_b = \rho g H_{ab} \approx 0$ (gauge pressure).

Fig. 2.42 Ubbelohde viscometer



Assuming that the flow is laminar, $h_{tf} = \lambda \frac{l}{d} \frac{u^2}{2g} = \frac{64}{\frac{du\rho}{\mu}} \cdot \frac{l}{d} \frac{u^2}{2}$; by substituting the Bernoulli equation, the following results are obtained:

$$\begin{aligned}
 gl + \frac{0}{\rho g} + \frac{u_b^2}{2} &= 0 + \frac{0}{\rho} + \frac{u_c^2}{2} + h_{tf} \\
 gl &= \frac{\frac{64}{\frac{du\rho}{\mu}}}{\mu} \cdot \frac{l}{d} \frac{u^2}{2} \Rightarrow \frac{32\mu u}{d^2 \rho g} = 1 \\
 \therefore \mu &= \frac{d^2 \rho g}{32u}
 \end{aligned} \tag{2.76a}$$

$\therefore u = \frac{V}{\frac{\pi}{4} d^2 \tau}$ can be substituted into Eq. (2.76a); thus:

$$\mu = \frac{d^2 \rho g \times \pi d^2 \tau}{32 \times V \times 4} = \frac{\pi \rho g d^4 \tau}{128V} \tag{2.76b}$$

where

d —Capillary diameter, m

V —Fluid volume, m^3

τ —Time required for a fluid of volume V to flow through the capillary, s.

The unit of the viscosity measured by the equation is $\frac{(\text{kg}/\text{m}^3) \cdot (\text{m}/\text{s}^2) \cdot \text{m}^4 \cdot \text{s}}{\text{m}^3} \Rightarrow \text{kg}/(\text{m} \cdot \text{s})$ or $\text{Pa} \cdot \text{s}$ SI unit.

Equations (2.76a) and (2.76b) are the equations for calculating the viscosity of the Ubbelohde viscometer. The two equations show that the viscosities of liquids can be calculated directly from the average velocity u of liquids in capillaries, the diameter d of capillaries and the density ρ of liquids. In addition, in the same Ubbelohde viscometer, the time required for different liquids to pass through capillaries is proportional to the viscosity of liquids and inversely proportional to the density of liquids.

$$\mu_1 = \frac{d^2 \rho_1 g}{32 u_1} = \frac{\pi \rho_1 g d^4 \tau_1}{128 V}$$

Therefore, it can be obtained that:

$$\frac{\mu_1 / \rho_1}{\mu_2 / \rho_2} = \frac{u_2}{u_1} = \frac{\tau_1}{\tau_2}$$

Thus, by measuring the ratio of time required for two different liquids to pass through the same Ubbelohde viscometer, the viscosity of a liquid can be obtained from another liquid with known viscosity.

Example 2.19 Measuring the viscosity of a glycerol-water mixture solution, when the density of the solution is $1230 \text{ kg}/\text{m}^3$ and the volume of the solution is 5 cm^3 , the time required to flow through a capillary with a diameter of 2 mm is 140 s . What is the viscosity of the solution?

Known: $\rho = 1230 \text{ kg}/\text{m}^3$, $V = 5 \text{ cm}^3 = 5 \times 10^{-6} \text{ m}^3$, $d = 2 \text{ mm} = 0.002 \text{ m}$, $\tau = 140 \text{ s}$.

Determine: μ .

Solution:

$$\mu = \frac{\pi \rho g d^4 \tau}{128 V} = \frac{3.14 \times 1230 \times 9.81 \times 0.002^4 \times 140}{128 \times (5 \times 10^{-6})} = 0.133 \text{ Pa} \cdot \text{s}$$

Check whether Re is in the range of laminar flow.

$$Re = \frac{du\rho}{\mu} = \frac{0.002 \times \frac{5 \times 10^{-6}}{\frac{\pi}{4} \times 0.002^2 \times 140} \times 1230}{0.133} = 0.21 \ll 2000$$

In the laminar flow range, the calculation results are valid.

Answer: The viscosity of the solution is 0.133 Pa·s.

2.6 Two-Phase Flow

Previous sections have mainly elaborated the basic law of fluid flow, discussed the change in pressure and energy loss when a fluid flows in a pipeline, and focused on the influence of a solid boundary on fluid flow. In the field of environmental engineering, the relative movement between particles and fluids often occurs, such as the sedimentation and filtration of particulate pollutants in fluids, the adsorption of pollutants in solid particles or granular layers, ion exchange and so on. Therefore, it is of great significance to understand the flow law between fluids and particles to master many pollution control technologies in environmental engineering.

2.6.1 *Motion Resistance and Resistance Coefficient of Spherical Particles in Fluid*

When there is a relative motion between fluid and solid particles, momentum transfer occurs. The particle surface has resistance to the fluid, and the fluid has a drag force on the particle surface. Resistance and drag forces are paired reaction forces.

The relative motion between fluid and solid particles can be divided into the following three situations:

- (1) The particles are stationary, and the fluid flows around them.
- (2) The fluid is stationary, and the particles move in settlement.
- (3) Both particles and fluids move but maintain a certain relative motion.

In these three cases, as long as the relative velocity between particles and fluids is the same, the drag force of fluid on particles is essentially the same, which is caused by the relative motion between the fluid and particles. Therefore, the resistance of particles relative to fluid can be analyzed by taking case (1) as an example.

The drag force F_d is related to the relative velocity u between a fluid and solid, the density ρ of the fluid, the viscosity μ and flow direction, as well as the size and shape of solid particles. The relationship among them is very complex. Only for solid particles with simple shapes, such as spheres, when the flow velocity of fluid is very low, the theoretical equation for calculating drag force be obtained analytically. The relationship between drag force and various factors can be obtained only through conducting experiments.

When a fluid passes around a particle with a certain shape in a certain direction, the factors affecting the drag force can be expressed as follows:

$$F_d = f(L, u, \rho, \mu).$$

L is the characteristic size of particles, and for smooth spheres, L is the diameter d_p of particles. By using dimensional analysis, an equation similar to the Fanning equation and friction coefficient can be obtained.

$$\frac{F_d}{A_p \cdot \frac{1}{2} \rho u^2} = f \left(\frac{d_p u \rho}{\mu} \right)$$

$$\text{Reynolds number of particles } Re_p = \frac{d_p u \rho}{\mu}.$$

$$\text{Therefore, } F_d = \xi A_p \frac{\rho u^2}{2}.$$

where

F_d —Drag force, action force on surface of particle by fluid caused by the relative motion between fluid and particle, N or kg·m/s²

μ —Dynamic viscosity of the fluid, kg/(m·s), pa·s, N·s/m²

ρ —Fluid density, kg/m³

u —Relative velocity between fluid and particle, m/s

d_p —Diameter of particle, m

A_p —Projection area of spherical particles in the flow direction, $\frac{\pi}{4} d_p^2$, m²

ξ —Drag coefficient, dimensionless.

ξ is called the drag coefficient, dimensionless, and its functional relationship with Re_p varies with particle shape and its relative moving type with fluid flow. Generally, it needs to be measured by experiment. The relationship between spherical particles ξ and Re_p can be roughly divided into the following regions:

(1) Laminar flow region (Stokes law region): $Re_p < 2$

$$\xi = \frac{24}{Re_p}$$

This relation is consistent with the Stokes law. In theory, the Stokes law is correct when Re_p is much less than 1. In fact, when Re_p is less than 2, the error caused by applying the above relation is very small.

It should be noted that this laminar flow region is defined as a $Re_p < 2$ system in this book, but $Re_p < 1$ or 0.3 in some other books. This kind of difference based on the experimental curve area will not have a significant impact on the calculation results in practical applications.

(2) Transitional region: $2 < Re_p < 1000$

$$\xi = \frac{18.5}{Re_p^{0.6}}$$

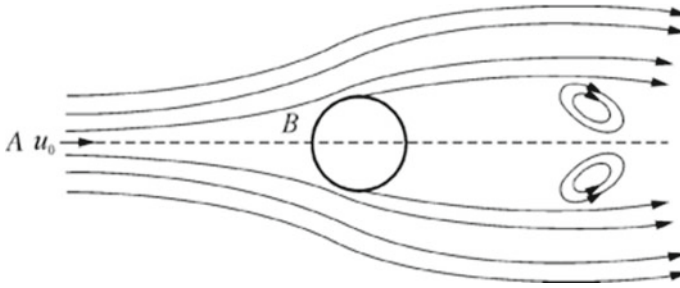


Fig. 2.43 Relative motion of particle and fluid

When Re_p increases beyond the laminar flow region, boundary layer separation occurs at a slight front of the hemispheric line of the particle (Fig. 2.43). Therefore, the space behind the particle is filled with whirlpools, and the whirlpools cause greater friction losses. At the same time, the fluid in the whirlpool has a great angular velocity and rotational function, which reduces its pressure and thus produces a larger drag force on the particles, resulting in an increase in the total drag force on the particles.

(3) Turbulent region: $1000 < Re_p < 2 \times 10^5$

$$\xi = 0.44$$

In this region, due to the increase in velocity, the enhancement of vortices, the increase in the proportion of body drag force and the decrease in the proportion of surface drag force can be neglected. Therefore, the drag force is proportional to the square of the velocity, independent of the fluid viscosity, and the drag coefficient is constant, averaging 0.44.

(4) Turbulent boundary layer region: $Re_p > 2 \times 10^5$

$$\xi = 0.1$$

With the increase in the Reynolds number of particles, the flow in the boundary layer changes from laminar flow to turbulent flow, and the velocity in the boundary layer increases, which makes the separation point of the boundary layer move to the back of the hemispheric line so that the eddy area behind the sphere decreases, and the friction loss and drag force decrease. Therefore, when the Reynolds number of particles reaches approximately 2×10^5 , the drag force coefficient decreases from 0.44 to 0.1 and remains almost constant.

2.6.2 Gravity Settlement

In the field of environmental engineering, particle pollutants or solid particles used to purify fluids are relatively small, and the contact surface between particles and fluids is relatively large, so the drag increases rapidly. Therefore, the resistance can be balanced with the net gravity of particles in a short time, so the acceleration stage can be neglected in the process of gravity settlement.

As shown in Fig. 2.44, a single particle is subjected to three forces:

Gravity F_g : $F_g = mg = \frac{1}{6}\pi d^3 \rho_s g$.

Buoyance F_b : $F_b = mg = \frac{1}{6}\pi d^3 \rho g$.

Drag force F_d : $F_d = \xi A_p \frac{\rho u^2}{2} = \xi \frac{\pi}{4} d_p^2 \frac{\rho u^2}{2}$

$$F_g - F_b - F_d = ma$$

Thus,

$$\frac{\pi}{6} d_p^3 \rho_s g - \frac{\pi}{6} d_p^3 \rho g - \xi \frac{\pi}{4} d_p^2 \frac{\rho u^2}{2} = \frac{\pi}{6} d_p^3 \rho_s a$$

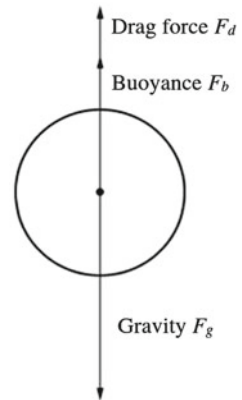
When the particles begin to settle: $u = 0$, so $F_d = 0$, and a (acceleration, m/s^2) is the largest.

$$u \uparrow \quad F_d \uparrow \quad a \downarrow$$

When the settlement distance is long enough, the final $F_g - F_b - F_d = 0$.

Here, $a = 0$, $u = u_\tau$, derivation settlement velocity “Terminal Settling Velocity” u_τ .

Fig. 2.44 Forces on a particle in the process of gravity sedimentation



$$\text{Then, } u_\tau = \sqrt{\frac{4gd_p(\rho_s - \rho)}{3\rho\xi}} \quad (2.77)$$

where

u_τ —Free settling velocity of spherical particle or terminal settling velocity, m/s

d_p —Particle diameter, m

ρ_s —Particle density, kg/m³

ρ —Fluid density, kg/m³

g —Gravity acceleration, m/s²

ξ —Drag coefficient, resistance coefficient, dimensionless,

$$\xi = f(\varphi_s \cdot Re_p)$$

φ_s —Sphericity, dimensionless

a —Acceleration, m/s².

The above equation is the free settlement equation for spherical particles with smooth surfaces in a fluid. ξ is the function of particle Reynolds number Re_p . According to the difference in Re_p , the relationship between ξ and Re_p can be divided into several different regions, so the calculation of settlement velocity should also be carried out according to different regions.

- (1) Laminar flow region: $10^{-4} < Re_p < 2$. The relationship between ξ and Re_p is $\xi = \frac{24}{Re_p}$, which can be obtained in the substitution Eq. (2.77):

$$\begin{aligned} u_\tau^2 \times 3\rho\xi &= 4gd_p(\rho_s - \rho) \\ u_\tau^2 \times \frac{24\mu}{u_\tau\rho d_p} \times 3\rho &= 4gd_p(\rho_s - \rho) \\ u_\tau &= \frac{d_p^2(\rho_s - \rho)g}{18\mu} \end{aligned} \quad (2.78)$$

- (2) Transitional region: $2 < Re_p < 10^3$. The relation between ξ and Re_p is $\xi = \frac{18.5}{Re_p^{0.6}}$. Substituting Eq. (2.77) can obtain the following:

$$u_\tau = 0.27 \sqrt{\frac{d_p(\rho_s - \rho)g}{\rho} Re_p^{0.6}} \quad (2.79)$$

Replacing Re_p with $\frac{d_p u_\tau \rho}{\mu}$, the u_τ can be calculated directly as follows.

$$u_\tau = 0.78 \frac{d_p^{1.143} (\rho_s - \rho)^{0.715}}{\rho^{0.286} \mu^{0.428}} \quad (2.79b)$$

- (3) Turbulence region: $10^3 < Re_p < 2 \times 10^5$ and $\xi = 0.44$. The substituted Eq. (2.77) can be obtained as

$$u_\tau = 1.74 \sqrt{\frac{d_p (\rho_s - \rho) g}{\rho}} \quad (2.80)$$

When calculating u_τ with Eqs. (2.78) ~ (2.80), it should be first known which region the settlement belongs to, that is, which equation is used to calculate ξ . However, it is not known which region the settlement belongs to before u_τ is calculated. Therefore, when calculating terminal settling velocity u_τ with the above equation, the trial-and-error method is needed. The calculation steps are as follows: first, assuming which region the settlement belongs to, calculate u_τ according to the equation of this region, then calculate particle Reynolds number Re_p according to the calculated u_τ to verify whether the original hypothesis is correct or not. The calculated u_τ is the correct result; otherwise, it needs to be recalculated to correct the result.

Two conditions are required for the application of Eqs. (2.78) ~ (2.80):

- (1) The size of the vessel is much larger than that of the particle, for example, more than 100 times, otherwise the wall of the vessel will have a significant retarding effect on the settlement of particles. Moreover, free settlement occurs when the settlement of any particle is not disturbed by the presence of other particles in the fluid. Free settlement occurs when the particles in the fluid are sparse. Otherwise, the particles will interact with each other so that the velocity of settlement is different from that of free settlement. The settlement at this time is called interference settlement, which mostly occurs in the settlement process with high concentrations of suspended solids, such as the secondary clarifier in an activated sludge process.
- (2) Particles should not be too fine; otherwise, a Brownian motion of particles will occur due to the collision of fluid molecules.

Example 2.20 The viscosity of a fluid is measured by the gravity sedimentation. Particles with a diameter of 2 mm and a density of 1.85 g/cm³ settle at a velocity of 0.03 m/s in the fluid, and the density of the fluid is 1 g/cm³. Please calculate the fluid viscosity coefficient.

Known: $d_p = 2 \text{ mm}$, $\rho_s = 1.85 \text{ g/cm}^3 = 1850 \text{ kg/m}^3$, $\rho = 1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$, $u_\tau = 0.03 \text{ m/s}$.

Determine: μ

Solution: Assuming that the settlement belongs to the laminar flow region, the Eq. (2.78) is used.

$$u_{\tau} = \frac{d_p^2(\rho_s - \rho)g}{18\mu},$$

Then,

$$\mu = \frac{d_p^2(\rho_s - \rho)g}{18u_{\tau}} = \frac{0.002^2 \times (1850 - 1000) \times 9.81}{18 \times 0.03} = 0.062 \text{ Pa} \cdot \text{s}$$

$$Re_p = \frac{d_p u_{\tau} \rho}{\mu} = \frac{0.002 \times 0.03 \times 1000}{0.062} = 0.97 < 2$$

This belongs to laminar flow and meets the hypothesis.

Answer: The viscosity of the fluid is 0.062 Pa·s.

Example 2.21 Settlement experiments are conducted with solid particles A and B with a diameter of 0.5 mm in a settling column with a height of 2.0 m. It is known that the liquid level in the settling column is 1.8 m high, the density of solid particle A is 1.5 g/cm³, the density of solid particle B is 1.8 g/cm³, and the viscosity of the liquid is 0.017 Pa·s. The process of particles falling from the initial state to constant velocity is negligible. How long does it take for the two particles to reach the bottom of the settling column?

Known:

$d_{pA} = d_{pB} = d_p = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$, $h = 1.8 \text{ m}$, $\rho_A = 1.5 \text{ g/cm}^3 = 1500 \text{ kg/m}^3$, $\rho_B = 1.8 \text{ g/cm}^3 = 1800 \text{ kg/m}^3$, $\mu = 0.017 \text{ Pa} \cdot \text{s}$.

Determine: The time required for the particles to reach the bottom of the settling column τ_A , τ_B .

Solution: Assuming that the settling of both particles is in the laminar flow region, Eq. (2.78) is used to calculate the settlement of the two particles.

$$u_{\tau} = \frac{d_p^2(\rho_s - \rho)g}{18\mu}$$

$$u_{\tau A} = \frac{d_p^2(\rho_A - \rho)g}{18\mu} = \frac{0.0005^2 \times (1500 - 1000) \times 9.81}{18 \times 0.017} = 4.00 \times 10^{-3} \text{ m/s}$$

$$u_{\tau B} = \frac{d_p^2(\rho_B - \rho)g}{18\mu} = \frac{0.0005^2 \times (1800 - 1000) \times 9.81}{18 \times 0.017} = 6.41 \times 10^{-3} \text{ m/s}$$

$$Re_{pA} = \frac{d_p u_{\tau A} \rho}{\mu} = \frac{0.0005 \times 4.00 \times 10^{-3} \times 1000}{0.017} = 0.12 < 2$$

$$Re_{pB} = \frac{d_p u_{\tau B} \rho}{\mu} = \frac{0.0005 \times 6.41 \times 10^{-3} \times 1000}{0.017} = 0.19 < 2$$

They are all in laminar flow and conform to the hypothesis.

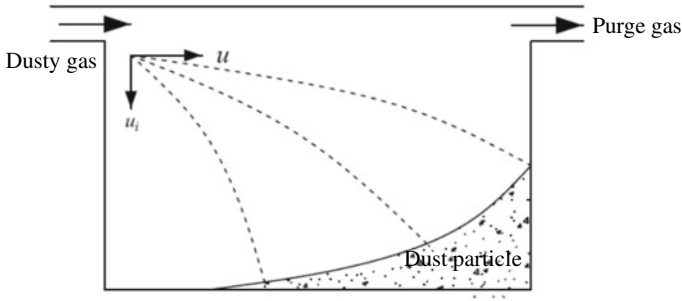


Fig. 2.45 Schematic diagram of a dust chamber

$$\tau_A = \frac{h}{u_{\tau A}} = \frac{1.8}{4.00 \times 10^{-3}} = 450 \text{ s}$$

$$\tau_B = \frac{h}{u_{\tau B}} = \frac{1.8}{6.41 \times 10^{-3}} = 281 \text{ s}$$

Answer: It takes 450 s for solid particle A to reach the bottom of the settling column and 281 s for solid particle B to reach the bottom of the settling column.

Typical Application Examples of Gravity Settlement in Environmental Engineering

Dust chamber: A dust chamber is a device for removing particles from airflow by gravity sedimentation.

A schematic diagram of a dust chamber is shown in Fig. 2.45.

where

l —Dust chamber length, m

H —Dust chamber height, m

b —Dust chamber width, m

u_{τ} —Terminal settling velocity of particles, m/s

u —Velocity of gas passing horizontally in the dust chamber, m/s.

Particle settling time: $\tau_p = \frac{H}{u_{\tau}}$; Gas passage time: $\tau_g = \frac{l}{u}$.

The conditions under which the particles are separated are as follows: $\tau_g \geq \tau_p$,
i.e., $\frac{l}{u} \geq \frac{H}{u_{\tau}}$.

q_v —The volume flow rate of dusty gas in dust chamber, (m^3/s).

Horizontal gas velocity: $u = \frac{q_v}{Hb}$, substituted $\frac{l}{u} \geq \frac{H}{u_{\tau}}$.

Then, $q_v \leq blu_{\tau}$ or $u_{\tau} \geq \frac{q_v}{bl}$.

It can be seen that q_V is only related to settling areas bl and μ_τ but not to the height of the dust chamber. Therefore, the dust chamber should be designed as a flat shape, often with multilayer horizontal partitioning. The height between partitions is generally 40 ~ 100 mm. A multilayer dust chamber can separate fine particles and save space, but it is inconvenient to clean the ash.

2.7 Fluid Transport Equipment

Fluid transport equipment delivers fluid from a low energy level to a high energy level. When conveying liquid, this is generally referred to as a pump. There are water pumps, oil pumps, mud pumps, corrosion-resistant pumps, high-viscosity pumps, high temperature (low temperature) pumps, etc. When delivering gas, pumps can be divided into ventilators, blowers, compressors, vacuum pumps, etc. according to different uses and pressures. The difference lies in different pressures.

Fan: The outlet pressure is less than 15 kPa.

Blower: The outlet pressure ranges from 15 to 35 kPa.

Compressor: The outlet pressure is higher than 35 kPa.

Vacuum pump: The inlet pressure is lower than an atmosphere pressure.

According to the working principle, fluid transport equipment can be categorized as follows:

- (1) Power type (impeller type): includes centrifugal and axial-flow transport machinery, where a high-speed rotating impeller of transport machinery is used to supply energy for fluid.
- (2) Volumetric type (positive displacement type): includes reciprocating and rotating transport machinery, where the fluid is squeezed by a piston or rotor of the transport machinery to obtain energy.
- (3) Other type: refers to other types, such as jet type, which do not belong to the above two categories.

This chapter mainly introduces the basic structure, working principle and characteristics of common fluid transport equipment to select and use fluid transport equipment properly.

Problems to be solved when selecting fluid transport equipment (taking a centrifugal pump as an example):

- (1) Specification

Flow rate and lift head are determined by calculation.

- (2) Type selection

According to the characteristics and working conditions of the fluid, choose the appropriate type of pump, such as a corrosion-resistant pump to transport strong corrosive fluid.

(3) Calculated power

The power of the motor is determined by calculation.

(4) Determining installation height

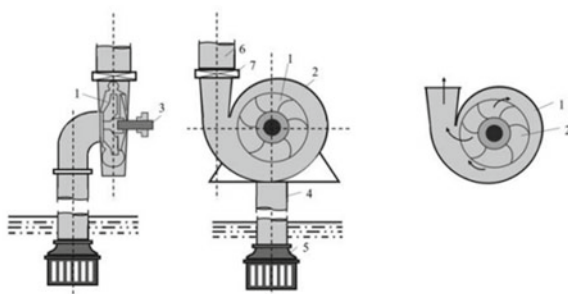
(5) Choosing the right working point. Look for a convenient operation at the work point with high efficiency.

2.7.1 Centrifugal Pump

Centrifugal pumps are commonly used fluid transport equipment in the field of environmental engineering. They can be used for transporting general liquids such as wastewater, surface water, rainwater, tap water, coagulant, flocculant, etc. and for conveying corrosive liquids such as hydrogen peroxide, acid, alkali and other liquids containing solid suspensions such as sludge.

1) Working principle of centrifugal pump

A working schematic diagram of a centrifugal pump is shown in Fig. 2.46a. The liquid enters suction pipe 4 after bottom valve 5, enters the flow passage in the volute pump housing through rotating impeller 1, and then enters the extruding pipe. The main working parts of the centrifugal pump are the rotating impeller and fixed pump shell. As shown in Fig. 2.46b, the impeller directly moves the liquid to enhance its energy. Generally, there are 4 ~ 8 backward-curved blades on the impeller. When the impeller spins, it rotates at 1000 ~ 3000 r/min as driven by the motor, which drives the liquid in the impeller to rotate accordingly. At the same time, under the action of inertial centrifugal force, the liquid moves radially from the center of the impeller to the outer edge. During this process, the liquid obtains energy and enters the volute



(a) Schematic diagram of a centrifugal pump

1 – Impeller, 2 – Pump housing,

3 – Pump shaft, 4 – Suction tube,

5 – Bottom valve, 6 – Discharge pipe, 7 – Outlet valve

(b) Pump casing and guide wheel of a centrifugal pump

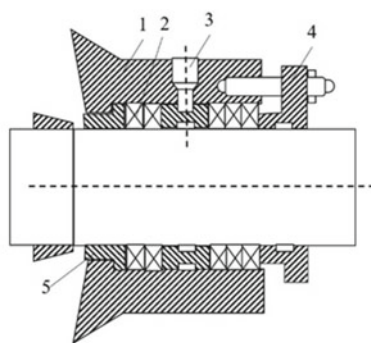
1 – Pump housing, 2 – Impeller,

Fig. 2.46 a Schematic diagram of a centrifugal pump b Pump casing and guide wheel of a centrifugal pump

pump shell at high speed. In the pump case, the cross section of the liquid expands, the flow velocity decreases, the kinetic energy is converted into static energy, and the pressure increases. Finally, the liquid flows into the discharge pressure pipe along the tangential direction and is transported to the destination. When the fluid moves radially under the action of inertial centrifugal force, the pressure at the impeller center decreases, forming a vacuum area. At this time, the pressure at the external liquid level (usually atmospheric pressure) is higher than the pressure inside the pump (usually negative pressure). The external liquid continues to enter the pump through the suction pipe of the pump under the action of a pressure difference. With centrifugal pump impeller rotation, the liquid is continuously sucked into and out of the centrifugal pump. During this process, the transportation of liquid mainly depends on the centrifugal force produced by high-speed rotation of the impeller, so it is called a centrifugal pump.

In summary, if the centrifugal pump is filled with air before start-up, because of its low density and small centrifugal force, it is difficult to form a sufficient vacuum in the pump, and the pressure difference is not enough to suck the liquid into the pump shell. At this time, the centrifugal pump is idle and cannot realize the effective delivery of liquid. This phenomenon is called a “gas binding”. Therefore, before starting the centrifugal pump, the pump shell must be filled with liquid. A check valve is installed at the bottom of the suction pipe to prevent drainage of the liquid. In the case of liquid containing more solids, the end of the suction pipe also needs to install a filter to prevent large solids (such as branches) from entering the pump housing and causing a blockage. Usually, the centrifugal pump is also equipped with a check valve on the discharge pipe, followed by a regulating valve. The check valve prevents the reflux of the high level liquid which has been lifted by pump due to sudden power outages to the damage of pipe fittings and centrifugal pump blades caused by pressure impact, that is, to prevent a “water hammer”. The regulating valve is used to regulate the flow rate and lift head of the output liquid. Figure 2.47 shows the shaft sealing device of the centrifugal pump to avoid liquid leakage of the centrifugal pump.

Fig. 2.47 Shaft sealing device for a centrifugal pump



1 – Packing shell, 2 – Soft packing, 3 – Liquid seal ring,
4 – Packing gland, 5 – Inner bushing

2) Power and efficiency of centrifugal pump

There are many kinds of energy losses in the actual operation of centrifugal pumps; that is, the liquid cannot obtain all the energy provided by the motor to the pump shaft.

The effective power of pump (P_e) can be calculated by the following Eq. (2.81):

$$P_e = q_v H_e \rho g \quad (2.81)$$

where

H_e —Lift head, effective pressure head of the pump, namely, the energy obtained from the pump per unit weight of liquid, m liquid column

q_v —Actual volume flow rate of the pump, m^3/s

ρ —Liquid density, kg/m^3

P_e —Effective power of the pump, that is, the mechanical energy of the liquid obtained from the pump in unit time, W.

The shaft power of the pump generally refers to the power input from the motor to the centrifugal pump, expressed in P_a . The total pump efficiency η is defined as the ratio of effective power to shaft power, as shown in Eq. (2.82). The efficiency of matching motor (η') is defined as the ratio of shaft power of the pump (P_a) to power of the motor (P_d), as shown in Eq. (2.82a).

$$\eta = \frac{P_e}{P_a} \quad (2.82)$$

$$\eta' = \frac{P_a}{P_d} \quad (2.82a)$$

where

η —Efficiency of the pump, %

η' —Efficiency of matching motor, %

P_a —Shaft power of pump, W

P_d —Power of the motor, W.

The efficiency of small centrifugal pumps is generally 50% ~ 70%, while that of large pumps is approximately 90%.

The losses in centrifugal pumps can be divided into three types: volume loss, hydraulic loss and mechanical loss. Volume loss refers to the energy loss caused by the mechanical leakage of high-pressure liquid at the outlet of the impeller returning to the impeller entrance. Among the three types of impellers shown in Fig. 2.48, the volume loss of the open impeller is larger, but the passage of the blade is not easily blocked when the liquid containing suspended solid particles is transported. Hydraulic loss refers to the energy loss caused by the friction loss of the actual fluid

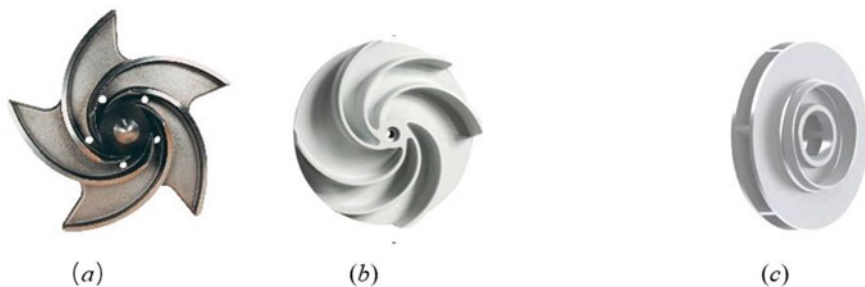


Fig. 2.48 Type of impeller **a** Open, **b** Semi-shielded, **c** Covered

under the action of the limited blades in the pump, including the impact between the liquid and the blade and the shell. The mechanical loss includes the friction between the disk and the liquid of the rotary impeller and the energy loss caused by the mechanical friction of the bearing. The efficiency of the centrifugal pump can reflect the sum of the above three energy losses.

3) Main performance parameters and performance curves of centrifugal pumps

The main performance parameters of the centrifugal pump include pump flow rate, lift head (or pressure head, effective pressure head), power (or shaft power), efficiency, allowable suction vacuum, speed, impeller diameter, weight and so on, as shown in Table 2.10. The lift head H_e , shaft power P_a and efficiency η of the centrifugal pump are all related to the flow rate q_V . The lift head is the static pressure expressed in mH_2O . Because it is difficult to quantitatively calculate the energy loss of centrifugal pumps, the relationship between the above parameters can only be determined by experiments. Before leaving the factory, the manufacturer of the centrifugal pump measures the three performance curves of $H_e - q_V$, $\eta - q_V$, and $P_a - q_V$, which are drawn in the product samples for reference in design.

Figure 2.49 shows the performance curves of a 6 PW centrifugal sewage pump ($n = 1450 \text{ r/min}$). Various types of pumps have their own performance curves and basically the similar shape, and they all have the following common points:

- (1) $H_e - q_V$ curve: This curve is used to express the relationship between the pump lift head (static pressure) and flow rate. The lift head of the centrifugal pump generally decreases with increasing flow rate.
- (2) $P_a - q_V$ curve: This curve is used to express the correlation between the shaft power and the flow rate of the pump. The shaft power of the centrifugal pump generally increases with increasing flow rate, and it is the smallest when the flow rate is zero. Therefore, before starting the centrifugal pump, the outlet valve should be closed to reduce the starting electric current and avoid motor burnout.
- (3) $\eta - q_V$ curve: This curve is used to express the correlation between pump efficiency and flow rate. As seen in Fig. 2.49, when $q_V = 0$ and $\eta = 0$, the efficiency of the pump increases with flow rate, gradually reaches its maximum, and then

Table 2.10 Performance table of PM centrifugal sewage pump (CMENWDRI, 2002)

Model	Flow rate q_v		Lift head H_e (m)	Rotating n (r/min)	Shaft power P_d (kW)	Motor		Efficiency η (%)	Allowable suction vacuum H_s (m)	Impeller diameter d (mm)	Weight (kg)
	m^3/h	L/s				Model	Power (kW)				
$2\frac{1}{2}$ PW	36	10	11.6	1440	2.1	Y112M-4	4	54	7.5	195	65
	60	16.6	9.5		25			62	7.2		
	72	20	8.5		2.72			61.5	7		
$2\frac{1}{2}$ PW	43	12	34	2920	7.8	Y112M ₂ -4	15	51	6	170	65
	90	25	26		11			58	5		
	108	30	24		12.5			56	4.2		
	43	12	48.5	2940	11.5	Y180M-2	22	49	7	195	65
$2\frac{1}{2}$ PW	90	25	43		17			62	5.5		
	108	30	39		19.2			60	4.5		
4PW	72	20	12	960	4	Y160M-6	7.5	59	7	300	125
	100	27.8	11		4.7			64	6.5		
	120	33.2	10.5		5.5			62	5.5		
	108	30	27.5	1460	13.5	Y200L-4	30	60	7.8	300	125
4PW	160	44.4	25.5		18			62	7.5		
	180	50	24.5		19.5			61.5	7		
	25.7	7.15	22.4	2890	2.9	Y112M-2	4	53.3		135	55
2PW	43	11.95	18.3		3.45			61.3			
	51.6	14.43	16.4		3.76			60.8			
6PW	200	56	16	980	13.5	Y225M-6	30	65	7	335	417
	300	83.3	14		17			67	6.8		

(continued)

Table 2.10 (continued)

Model	Flow rate q_v		Lift head H_e (m)	Rotating n (r/min)	Shaft power P_d (kW)	Motor		Efficiency η (%)	Allowable suction vacuum H_s (m)	Impeller diameter d (mm)	Weight (kg)
	m ³ /h	L/s				Model	Power (kW)				
6PW	400	111	12	1450	20	Y250M-4	55	65	6.5	315	417
	250	69	30		34			60	5		
	350	97	27		42			61	4.5		
	450	125	23		47			60	4		
8PW	350	97.2	15.5	730	23	Y280M-8	45	64	7.5	465	750
	500	139	13		29			61	6.5		
	650	185	9.5		33			51			
8PW	400	111	27.5	980	50	Y315S-6	75	60	5.8	465	750
	550	153	25		59.5			63	5.6		
	700	190.4	21		69			58			
4PWB	72	20	18	1450	6	Y160M ₄	10	60	6.5	250	95
	100	27.8	17		7.2			64	6		
	120	33.2	15.8		8.4			61	5		

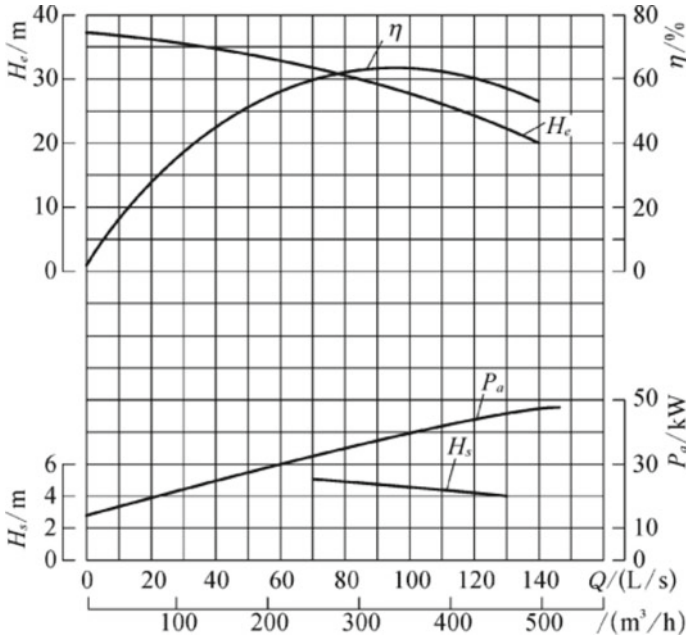
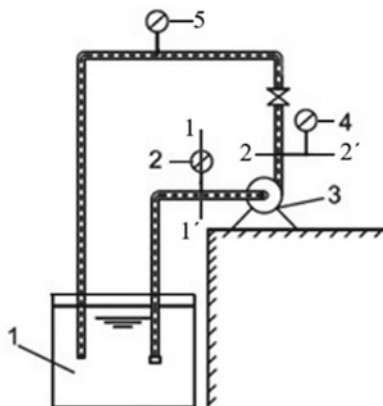


Fig. 2.49 Characteristic curve of a 6 PM centrifugal sewage pump ($n = 1450 \text{ r/min}$) (CMENWDRI, 2002)

decreases with increasing flow rate. From this observation, it is known that the efficiency of the centrifugal pump has a maximum at a certain flow rate, which is usually called the design point. Under this flow rate and lift head, the centrifugal pump works most economically. At this time, the corresponding q_V , H_e , and P_a values are called the optimal operating parameters. The performance parameters marked on the nameplate of the centrifugal pump refer to the state parameters of the highest efficiency of the pump during operation. In actual use, centrifugal pumps are often impossible to operate at the optimal operating point, so only one working range, known as the high efficiency region of pumps, is usually set at the point of the maximum efficiency (as shown in Fig. 2.49, $\eta_{\max} = 62\%$). When choosing a centrifugal pump, the pump should work within this range as much as possible.

Example 2.22 Fig. 2.50 shows an experimental device for measuring the performance curves of a centrifugal pump. It is known that the diameter of the suction pipe $d_1 = 75 \text{ mm}$, the diameter of the discharge pipe $d_2 = 50 \text{ mm}$, the vacuum meter reading $p_1 = 1.89 \times 10^4 \text{ Pa}$ (vacuum degree) at the inlet of the pump, the pressure meter reading $p_2 = 2.10 \times 10^5 \text{ Pa}$ (gauge pressure) at the outlet, and the vertical distance between the two meters is 0.6 m . The pump is driven by a motor, which consumes 6.8 kW of power with 90% efficiency. The centrifugal pump transports clean water at 20°C , and the flow rate is $q_V = 15.0 \times 10^{-3} \text{ m}^3/\text{s}$. Please calculate the lift head H_e , shaft power P_a and efficiency η of the pump under the above conditions.



1. Water channel 2. Vacuum gauge 3. Centrifugal pump
4. Pressure gauge 5. Flow rate meter

Fig. 2.50 Schematic diagram of Example 2.22

Known: $d_1 = 75 \text{ mm}$, $d_2 = 50 \text{ mm}$, $p_1 = 1.89 \times 10^4 \text{ Pa}$ (vacuum degree), $p_2 = 2.10 \times 10^5 \text{ Pa}$ (gauge pressure), $z_{h2} - z_{h1} = 0.6 \text{ m}$, $P_d = 6.8 \text{ kW}$, $\eta' = 90\%$, $T = 20^\circ\text{C}$, $q_V = 15.0 \times 10^{-3} \text{ m}^3/\text{s}$, $\Sigma H_{tf,1-2} = 0$.

Determine the following: H_e , P_a , η

- (1) A Bernoulli equation is set between the cross sections 1–1' and 2–2' of the vacuum gauge and pressure gauge

$$z_{h1} + \frac{p_1}{\rho g} + \frac{u_1^2}{2g} + H_e = z_{h2} + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + \Sigma H_{tf,1-2}$$

$$z_{h2} - z_{h1} = 0.6 \text{ m}$$

$$p_1 = -1.89 \times 10^4 \text{ Pa (gauge pressure)}$$

$$p_2 = 2.10 \times 10^5 \text{ Pa (gauge pressure)}$$

$$u_1 = \frac{4q_V}{\pi d_1^2} = \frac{4 \times 15 \times 10^{-3}}{3.14 \times 0.075^2} = 3.40 \text{ m/s}$$

$$u_2 = \frac{4q_V}{\pi d_2^2} = \frac{4 \times 15 \times 10^{-3}}{3.14 \times 0.05^2} = 7.64 \text{ m/s}$$

If the resistance loss $H_{tf,1-2}$ is neglected, then

$$H_e = 0.6 + \frac{2.10 \times 10^5 + 1.89 \times 10^4}{1000 \times 9.81} + \frac{7.64^2 - 3.40^2}{2 \times 9.81} = 26.32 \text{ mH}_2\text{O}$$

- (2) The power measured by the power meter is the input power of the motor. The motor itself consumes a part of the power. Its efficiency is 0.9, and the output power of the motor is equal to the shaft power) of the pump, and is calculated as follows:

$$P_a = P_d \eta' = 6.8 \times 0.9 = 6.12 \text{ kW}$$

- (3) Pump efficiency

$$\begin{aligned} \eta &= \frac{P_e}{P_a} \times 100\% = \frac{q_V H_e \rho g}{P_a} \times 100\% \\ &= \frac{15 \times 10^{-3} \times 26.32 \times 1000 \times 9.81}{6.12 \times 1000} \times 100\% = 63\% \end{aligned}$$

Answer: Under this flow rate, the pump lift head H_e is 26.32 mH₂O, the shaft power is 6.12 kW, and the pump efficiency is 63%.

In the experiment, if the opening of the outlet valve is changed, the relevant data under different flow rates can be measured, the corresponding values of H_e , P_a and η can be obtained by calculation, and these data can be plotted on coordinate figure. The performance curves of the pump at a fixed rotating of the pump can be obtained.

The performance curves provided by the pump production plant is obtained by experiments with clear water at 20 °C. When the properties of the transported liquid are quite different from those of water, the influence of viscosity and density on the performance curves should be considered.

- (1) The influence of density is shown from the basic equation of the centrifugal pump. The lift head and flow rate of the centrifugal pump are independent of the density of liquid, so the efficiency of the pump does not change with the density of liquid, and the H_e — q_V and η — q_V curves remain unchanged. However, the shaft power of the pump varies with the liquid density. Therefore, when the density of the transported liquid is different from that of clean water, the P_a — q_V curve provided by the pump production plant is no longer applicable, and the shaft power of the pump needs to be recalculated.
- (2) The greater the viscosity of the liquid transported, the more energy loss in the pump, the lower the pump lift head and flow rate, the lower the efficiency, and the higher the needed shaft power, so the performance curves change.

The performance curves of the centrifugal pump are measured at a certain rotating of pump. When the rotating of pump is changed from n_1 to n_2 , the approximate relationship with the flow rate, lift head and shaft power is as follows.

$$\frac{q_{V2}}{q_{V1}} = \frac{n_2}{n_1}, \frac{H_{e2}}{H_{e1}} = \left(\frac{n_2}{n_1}\right)^2, \frac{P_{a2}}{P_{a1}} = \left(\frac{n_2}{n_1}\right)^3 \quad (2.83)$$

Equation (2.83) is called the proportional law of centrifugal pumps. When the rotating of pump change is less than 20%, it can be considered that the efficiency remains unchanged and the calculation error is small.

When the impeller diameter d' changes little and the rotating of pump remains unchanged, the approximate relationship between the impeller diameter and flow rate, lift head and shaft power is as follows:

$$\frac{q_{V2}}{q_{V1}} = \frac{d'_2}{d'_1}, \frac{H_{e2}}{H_{e1}} = \left(\frac{d'_2}{d'_1}\right)^2, \frac{P_{a2}}{P_{a1}} = \left(\frac{d'_2}{d'_1}\right)^3 \quad (2.84)$$

Equation (2.84) is called the cutting law of the centrifugal pump.

4) Operating Point and Flow Rate Regulation of Centrifugal Pump

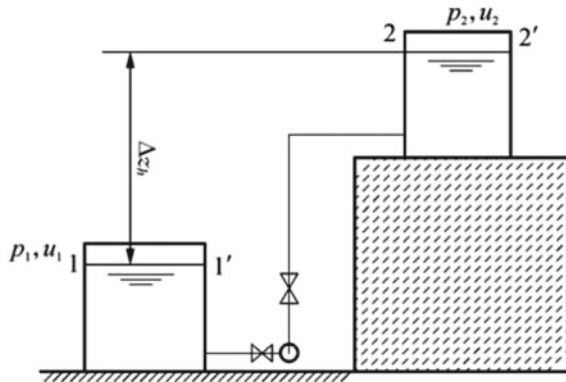
In a specific pipeline system, the actual working lift head and flow rate of the centrifugal pump are related not only to its own performance but also to the pipeline characteristics; that is, when delivering liquid, the pump and pipeline are mutually restricted. Therefore, before discussing the operation of the pump, it is necessary to know the pipeline directly connected to it.

In the system shown in Fig. 2.51, a centrifugal pump is used to transport liquid from low-energy level 1–1' to high-energy level 2–2'. If the energy required per unit mass of liquid is H_{eg} , it can be obtained by the Bernoulli equation.

$$H_{eg} = \Delta z_h + \frac{\Delta p}{\rho g} + \frac{\Delta u^2}{2g} + \Sigma H_{tf,1-2} \quad (2.85)$$

In general, the kinetic energy difference $\Delta u^2/2g$ term can be neglected, and the friction loss is followed as:

Fig. 2.51 Schematic diagram of delivering liquid system



$$\Sigma H_{tf,1-2} = \Sigma \left[\left(\lambda \frac{l}{d} + \zeta \right) \frac{u^2}{2g} \right] \quad (2.86)$$

In which $u = \frac{q_{Vg}}{\frac{\pi}{4}d^2}$.

Equation

q_{Vg} —Flow rate of pipeline system, m³/h.

Thus,

$$\Sigma H_{tf,1-2} = \Sigma \left[\frac{8 \left(\lambda \frac{l}{d} + \zeta \right)}{\pi^2 d^4 g} \right] q_{Vg}^2$$

$$\Sigma H_{tf,1-2} = K q_{Vg}^2 \quad (2.87)$$

in the equation: coefficient $K = \Sigma \frac{8 \left(\lambda \frac{l}{d} + \zeta \right)}{\pi^2 d^4 g}$.

The numerical value is directly related to the pipeline characteristics. When the flow in the pipe enters the resistance square region, the coefficient K is independent of the flow rate in the pipe. Equation (2.87) is substituted for Eq. (2.85) to obtain

$$H_{eg} = \Delta z_h + \frac{\Delta p}{\rho g} + K q_{Vg}^2 \quad (2.88)$$

When the pipeline system and operating conditions are fixed, the values of Δz_h and $\Delta p/\rho g$ are fixed, and the above equation can be written as follows:

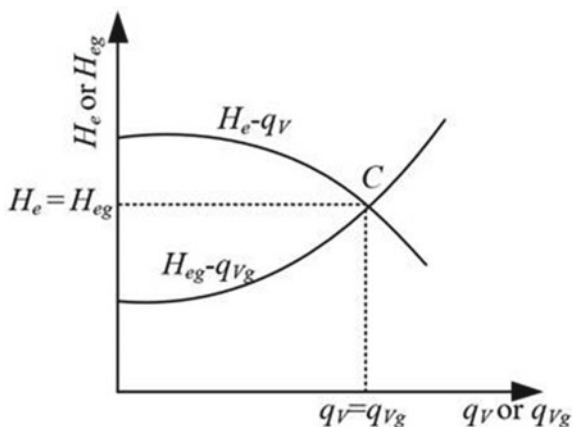
$$H_{eg} = \text{fixed value} + K q_{Vg}^2 \quad (2.89)$$

Equation (2.89) shows that the pressure head H_{eg} required for delivering liquid is positively correlated with the square of flow rate q_{Vg} in a specific pipeline. The pipeline characteristic curve can be obtained by drawing it. As shown in Fig. 2.51, the shape of the pipeline is directly related to the pipeline layout and operating conditions but not to the performance of the pump.

For a specific pipeline, the delivering flow rate q_V of the centrifugal pump is equal to the flow rate q_{Vg} of the pipeline. At this time, the static pressure provided by the centrifugal pump is exactly equal to the static pressure required by the pipeline. Therefore, the actual working condition of the pump is determined by both the pump performance curves and the pipeline characteristic curve.

In Fig. 2.52, the intersection point C of the two curves is called the working point of the pump. When the selected centrifugal pump operates in this particular pipeline system, it can only work at this point. When selecting a pump, the flow rate and lift head corresponding to the working point are required to meet the requirements of the pipeline system, i.e., $q_V = q_{Vg}$, $H_e = H_{eg}$.

Fig. 2.52 Pipeline characteristic curve and pump working point



If the flow rate corresponding to the working point of the pump is greater or less than the actual flow rate, the flow rate should be adjusted to change the position of the working point. The specific methods are as follows.

(1) Regulating the valve

The goal of regulating the valve on the outlet pipeline of the centrifugal pump is to change the characteristic curve of the pipeline. When the valve is closed to create a smaller flow rate, the local resistance of the pipeline increases, and the characteristic curve becomes steeper. As shown in curve 1 in Fig. 2.53, the working point moves from C to C_1 , and the flow rate decreases from q_C to q_{C1} . When the valve is opened to increase flow rate, the resistance of the pipeline decreases, and the pipeline characteristic curve becomes flatter. As shown in curve 2, the working point moves from C to C_2 , and the flow rate increases to q_{C2} .

This method is widely used because it is fast and convenient to use the regulating valve, and the flow rate can be changed continuously. This is suitable for the continuous production of related treatment processes in environmental engineering.

Fig. 2.53 Schematic diagram of flow rate regulation by changing valve opening

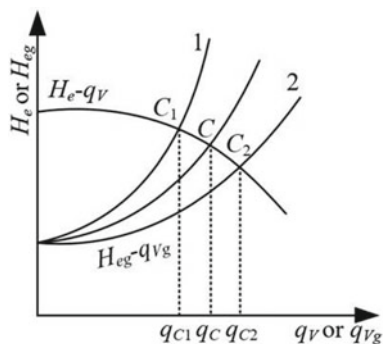
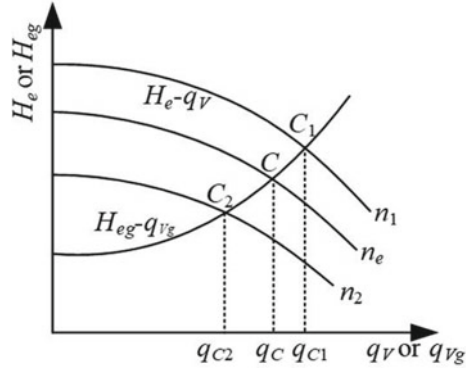


Fig. 2.54 Schematic diagram of adjusting flow rate by changing rotational speed of pump



The disadvantage is that when the valve is closed to create a smaller flow rate, the resistance loss increases and the energy consumption increases, which is not economical.

(2) Changing the rotating of pump

This method essentially changes the performance curves of the pump. As shown in Fig. 2.54, the original pump rotating is n , and the working point is C . If the pump rotating is increased to n_1 , the pump performance curve $H_e - q_v$ moves up, the working point is moved from C to C_1 , and the flow rate is increased from q_c to q_{c1} . If the pump rotating is reduced to n_2 , the working point is moved to C_2 , and the flow rate is reduced to q_{c2} .

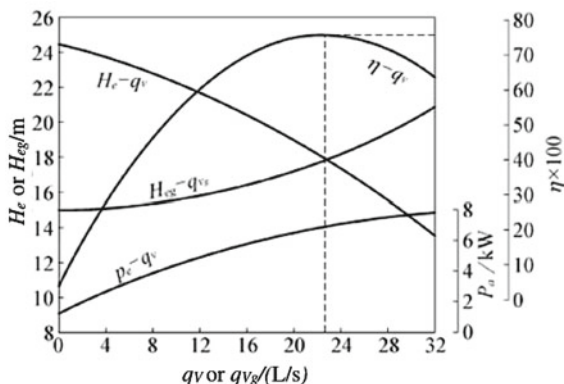
This method can keep the pipeline characteristic curve unchanged. When the flow rate decreases with the rotating, the resistance loss also decreases accordingly, which is more economical and reasonable. However, this method requires additional transmission or expensive variable-speed motors, which are typically used in high-rise buildings.

In addition, reducing the impeller diameter can also change the performance curve of the pump to a certain extent so that the flow rate of the pump can be reduced, but the adjustable range of the impeller diameter is not large, and the improper reduction of the impeller diameter will also reduce the efficiency of the pump, so it is seldom used in an actual operation process.

Example 2.23 Clear water at $20\text{ }^{\circ}\text{C}$ is transported from a storage tank to a water tower. The height difference of the water surfaces is 15 m between the tower and the storage tank. Both the tower and the storage tank are exposed to the atmosphere. A steel pipe of $\Phi 150\text{ mm} \times 5\text{ mm}$ is used for water transportation, and the total length of the steel pipe is 180 m (including all pipes and local resistance loss). The performance curves of the pump at 3000 r/min is shown in Fig. 2.55. Please calculate the flow rate, shaft power and efficiency of the pump during operation. The friction coefficient is 0.02 .

Known: $\Delta z_h = 15\text{ m}$, $d = 150 - 5 \times 2 = 140\text{ mm} = 0.14\text{ m}$, $l + \Sigma l_e = 180\text{ m}$, $\lambda = 0.02$.

Fig. 2.55 Pipeline characteristic curve H_e-q_v and performance curves of the pump



Determine the following: q_v , P_a , η .

Solution: To calculate the flow rate, shaft power and efficiency of the pump when it is running, the working point of the pump must be determined. That is, the pipeline characteristic curve should be plotted on the graph according to the pipeline characteristics.

(1) Pipeline characteristic curve equation

The Bernoulli equation the water surfaces of between the storage tank and the water tower is

$$H_{eg} = \Delta z_h + \frac{\Delta p}{\rho g} + \Sigma H_{tf}$$

In which $\Delta z_h = 15\text{m}$, $\Delta p = 0$.

The flow velocity in the pipe is

$$u = \frac{q_{vg}}{\frac{\pi}{4}d^2 \times 1000} = \frac{q_{vg}}{\frac{\pi}{4} \times 0.14^2 \times 1000} = 0.065q_{vg}$$

$$H_{tf} = \left(\lambda \frac{l + l_e}{d} \right) \frac{u^2}{2g} = 0.02 \times \frac{180}{0.14} \times \frac{(0.065q_{vg})^2}{2 \times 9.81} = 0.00554q_{vg}^2$$

Then, the equation of the pipe characteristic curve is

$$H_{eg} = 15 + 0.00554q_{vg}^2$$

(2) Drawing pipeline characteristic curve

According to the equation of the pipeline characteristic curve, the head value H_{eg} required for different flow rates q_{vg} can be calculated, and the calculation results can be listed as follows:

$q_{Vg} \text{ (L/s)}$	0	4	8	12	16	20	24	28	32
$H_{eg}/(\text{m}\cdot\text{H}_2\text{O})$	15	15.09	15.35	15.80	16.42	17.22	18.19	19.34	20.67

The pipeline characteristic curve $H_{eg}-q_{Vg}$, as shown in Fig. 2.55, is plotted according to the data in the table.

(3) Flow rate, shaft power and efficiency

The intersection point of the pump performance curves and pipeline characteristic curve is the working point of the pump, which can be read from the intersection point.

Pump flow rate $q_V = 22.8 \text{ L/s}$.

Shaft power of pump $P_a = 7 \text{ kW}$.

Pump efficiency $\eta = 75\%$

Answer: The flow rate, shaft power and efficiency of the pump in operation are 22.8 L/s, 7 kW and 75%, respectively.

5) Parallel and serial operation

In the actual design, construction and operation processes, several centrifugal pumps can be used in parallel or in series if it is difficult for a single centrifugal pump to meet the requirements of the transportation task. The operation of parallel and series connections of two pumps with the same performance is discussed below.

(1) Parallel operation

When the flow rate of a pump is low and has difficulty meeting the transportation requirements, two pumps can be connected in parallel to increase the flow rate of the fluid.

The performance curve of a pump is shown in curve S in Fig. 2.56. When two pumps are connected in parallel, the flow rate of two parallel pumps is twice as high as that of a single pump at the same lift head. Therefore, when the abscissa values of the characteristic curve S of a single pump are doubled and the longitudinal coordinates are kept unchanged, the synthetic characteristic curve D of two parallel pumps can be obtained. However, it should be noted that for the same pipeline, the flow rate increases after parallel connection, and the resistance of the pipeline increases, so the flow rate of the pump will not double.

Fig. 2.56 Parallel operation

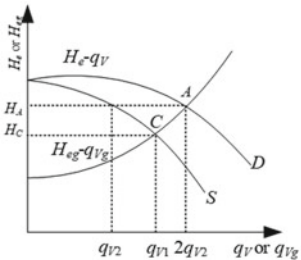
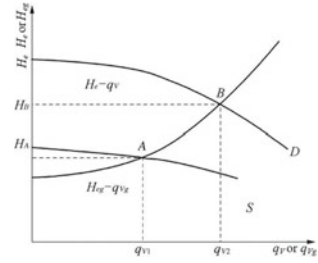


Fig. 2.57 Series operation

(2) Series operation

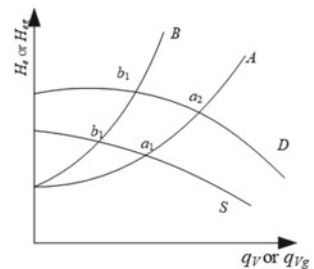
When the lift head of the original pump cannot meet the requirements, it should be considered to use pumps in series.

When two pumps of the same type with the same lift head and flow rate are used in series. Therefore, under the same flow rate, the lift head of the series pump is twice as high as that of a single pump. At this time, the longitudinal coordinates of the characteristic curve S of a single pump can be doubled, and the abscissa coordinates remain unchanged; that is, the composite characteristic curve D of two pumps in series can be obtained Fig. 2.57. The working point of a single pump is A and then moves to B after series connection, as shown in Fig. 2.57. Obviously, the lift head of point B (H_B) is not twice that of point H_A .

(3) Combination mode

If the potential energy difference between the two ends of the pipeline is greater than the maximum lift head provided by a single pump, the series operation must be adopted. In the actual use of the process, a single pump can be used, but the flow rate cannot meet the requirements. At this time, the appropriate combination mode can be selected according to the characteristics of the pipeline to increase the flow rate.

As shown in Fig. 2.58, for low-resistance pipeline A , the flow rate of the parallel combination is greater than that of the series combination, while for high-resistance pipeline B , the flow rate of the series combination is greater than that of the parallel combination. The same is true for lift head. Therefore, for low-resistance pipelines, parallel connections are better than series connections, but for high-resistance pipelines, series connections are preferred.

Fig. 2.58 The choice of combination mode

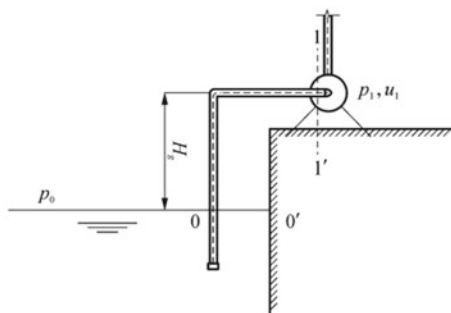


Fig. 2.59 Installation height of centrifugal pump

6) Installation height of centrifugal pump

According to the working principle of a centrifugal pump, a low-pressure zone is formed near the impeller center when the centrifugal pump works. As shown in Fig. 2.59, the pressure at the blade inlet decreases with increasing installation height of the centrifugal pump. When the installation height of the pump reaches a critical position, the pressure near the inlet of the blade may be reduced to the saturated vapor pressure of the transported liquid, resulting in partial vaporization of the liquid and the formation of bubbles. At this time, when the liquid containing bubbles enters the impeller, due to the expansion of the flow passage and the decrease in the velocity, part of the kinetic energy is converted into static pressure energy. The pressure rises, the bubbles condense immediately, and the bubbles disappear, resulting in a local vacuum. The surrounding liquid rushes to the center of the bubbles at high speed, which impacts the impeller and the pump shell. In particular, when bubble coagulation occurs near the blade surface, many liquid particles, such as small high-frequency water hammers, impact the blade. In addition, bubbles may also contain oxygen, which can cause chemical corrosion of metal materials. The long-term operation of the pump in this state will lead to premature damage to the blades, which is called “pump cavitation”. After cavitation, the blades of the pump are shown in Fig. 2.60.

When a centrifugal pump works under the condition of cavitation, the pump body vibrates frequently and produces certain noise. The flow rate, lift head and efficiency of the centrifugal pump decrease obviously and cannot suck and convey

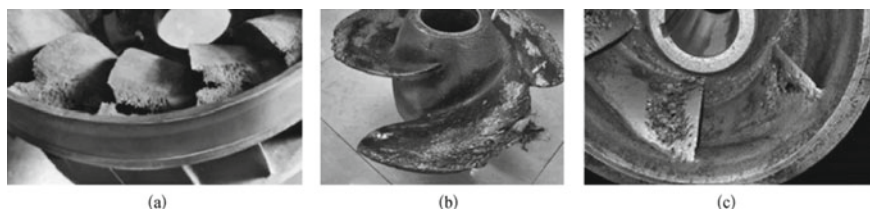


Fig. 2.60 Pump cavitation

liquid normally in serious cases. To avoid cavitation, the installation position of the pump should not be higher than the limit value to ensure that the pressure in the impeller is always higher than the saturated vapor pressure of the liquid. Generally, the performance nameplate of the centrifugal pump is marked with the allowable suction vacuum or cavitation allowance, which is used to express the suction performance of the centrifugal pump.

The allowable installation height of the centrifugal pump, also known as the allowable suction height, refers to the maximum allowable vertical distance between the inlet of the pump and the liquid level of the suction tank, expressed as H_g .

In the specifications of centrifugal pumps in China, two indicators are used to limit the allowable installation height of pumps to avoid cavitation. The two indicators are introduced as follows:

- (1) The allowable vacuum height H_s . The allowable vacuum height H_s is the highest vacuum allowable by the pressure p_1 at the inlet of the pump. Its expression is as follows:

$$H_s = \frac{p_a - p_1}{\rho g} \quad (2.90)$$

where

H_s —Allowable vacuum height. Centrifugal pumps are allowed to suck up the vacuum height. m liquid height

p_a —Atmospheric pressure, Pa

ρ —Density of the transported liquid, kg/m³.

To determine the relationship between the allowable suction vacuum height H_s and the allowable installation height H_g , the Bernoulli equation between the cross section 0–0' shown in Fig. 2.59 and the cross section 1–1' near the pump inlet can be used.

$$H_g = \frac{p_0}{\rho g} - \frac{p_1}{\rho g} - \frac{u_1^2}{2g} - \Sigma H_{tf,0-1} \quad (2.91)$$

where

H_g —The allowable installation height of the pump, m

$\Sigma H_{tf,0-1}$ —Resistance head loss of liquid from cross section 0–0' to 1–1', m

Because the tank is exposed to atmosphere, and p_0 is equal to the atmospheric pressure p_a , the equation above can be written as

$$H_g = \frac{p_a}{\rho g} - \frac{p_1}{\rho g} - \frac{u_1^2}{2g} - \Sigma H_{tf,0-1} \quad (2.92a)$$

Substituting Eq. (2.90) obtains

$$H_g = H_s - \frac{u_1^2}{2g} - \Sigma H_{tf,0-1} \quad (2.92b)$$

Equation (2.91) shows that to increase the allowable installation height of the pump, $u_1^2/2g$ and $\Sigma H_{tf,0-1}$ should be reduced as much as possible. To reduce $u_1^2/2g$, under the same flow rate, a larger diameter suction pipeline should be selected. To reduce $\Sigma H_{tf,0-1}$, the resistance components, such as elbows and cut-off valves, should be minimized, and the suction pipeline should be shortened as much as possible.

Because the actual operating conditions of each pump are different, the layout of their suction pipelines are also different, so the values of $u_1^2/2g$ and $\Sigma H_{tf,0-1}$ are different. The manufacturer of the centrifugal pump can only give the value of H_s , but the value of H_g needs to be calculated and determined according to the specific conditions of the pipeline.

In general, H_s given in the product nameplate of centrifugal pumps for water refers to the values of atmospheric pressure 9.807×10^4 Pa and water temperature 20°C under experimental conditions. If the actual operating conditions of the pumps are different from those of the centrifugal pumps, the H_s values given in the samples should be converted to the H'_s values under the operating conditions by Eq. (2.93).

$$H'_s = \left[H_s + (H_a - 10) - \left(\frac{p_v}{9.81 \times 10^3} - 0.24 \right) \right] \frac{1000}{\rho} \quad (2.93)$$

where

H'_s —Allowable vacuum height for transporting liquid under operating conditions, mH₂O

H_s —Allowable suction vacuum height given in the pump nameplate, mH₂O

H_a —Local atmospheric pressure at the pump installation area, mH₂O. Its value varies with altitude

p_v —Saturated vapor pressure of the transported liquid at the operating temperature, Pa

10—Atmospheric pressure under experimental conditions, mH₂O

0.24—Saturated vapor pressure of water at the experimental temperature (20°C), mH₂O

1000—Density of water at experimental temperature, kg/m³

ρ —Density of liquid at operating temperature, kg/m³.

By replacing H_s with the H'_s substituted Eq. (2.92b), the allowable installation height of the pump can be obtained when the liquid is transported under operating conditions.

Table 2.11 shows atmospheric pressure at different altitudes, of which 1 mH₂O corresponds to 9.807×10^3 Pa.

(2) Critical cavitation allowance ΔH_C .

The critical cavitation allowance ΔH_C is the minimum specified value, which is the sum when the hydrostatic head ($p_1/\rho g$) and the dynamic head of the liquid ($u_1^2/2g$) at the entrance of the centrifugal pump subtract the saturated vapor head of the liquid ($p_v/\rho g$) at the operating temperature.

$$\Delta H_C = \left(\frac{p_1}{\rho g} + \frac{u_1^2}{2g} \right) - \frac{p_v}{\rho g} \quad (2.94)$$

From Eqs. (2.91) and (2.94), it can be concluded that the relationship between the initial cavitation allowance and allowable installation height is as follows:

$$H_g = \frac{p_0}{\rho g} - \frac{p_v}{\rho g} - \Delta H_C - \Sigma H_{f,0-1} \quad (2.95)$$

where p_0 is the pressure above the liquid surface to be transported. If the liquid level tank is open, $p_0 = p_a$.

It should be noted that the ΔH_C value on the nameplate of centrifugal pump products is also regulated by transporting water at 20 °C. When conveying other liquids, due to the difference in density and viscosity of liquids, correction is also needed. Specific methods can be found in the relevant literature.

Centrifugal pumps should be installed at a height of 0.5 ~ 1.0 m less than the allowable installation height for safety in practical engineering applications.

Example 2.24 A centrifugal pump is installed at an altitude of 800 m to transport water at 40 °C. The head (or friction) loss of the suction pipe is 0.8 mH₂O, the dynamic head (or energy) at the inlet of the pump is 0.5 mH₂O, and the allowable vacuum height of the centrifugal pump is $H_s = 7$ m. Is it appropriate to install the pump at a height of 6 m above the water surface?

Known: $H_s = 7$ m, $u_1^2/2g = 0.5$ mH₂O, $\Sigma H_{f,0-1} = 0.8$ mH₂O.

Determine: Whether H_g is greater than 6 m.

Solution: The water temperature and atmospheric pressure used are different from the standard conditions and need to be corrected.

When the water temperature is 40 °C, $p_v = 7377$ Pa, $\rho = 992.2$ kg/m³.

Looking up Table 2.11, the atmospheric pressure at 800 m is $H_a = 9.36$ mH₂O.

$$H'_s = \left[H_s + (H_a - 10) \left(\frac{p_v}{9.81 \times 10^3} - 0.24 \right) \right] \frac{1000}{\rho}$$

Table 2.11 Atmospheric pressure at different altitudes

Altitude/m	0	100	200	300	400	500	600	700	800	1000	1500	2000	2500
Atmospheric pressure/mmHg	10.33	10.20	10.09	9.95	9.85	9.74	9.6	9.5	9.36	9.16	8.64	8.15	7.62

$$= \left[7 + (9.36 - 10) - \left(\frac{7377}{9.81 \times 10^3} - 0.24 \right) \right] \frac{1000}{992.2} = 5.89 \text{ mH}_2\text{O}$$

The allowable installation height of the pump is

$$H_g = H'_s - \frac{u_1^2}{2g} - \Sigma H_{tf,0-1} = 5.89 - 0.5 - 0.8 = 4.59 \text{ m} < 6 \text{ m}$$

Therefore, it is not appropriate to install the pump 6 m above the water surface.

2.7.2 Centrifugal Compressor, Root Blower and Jet Aerator

1) Centrifugal compressor

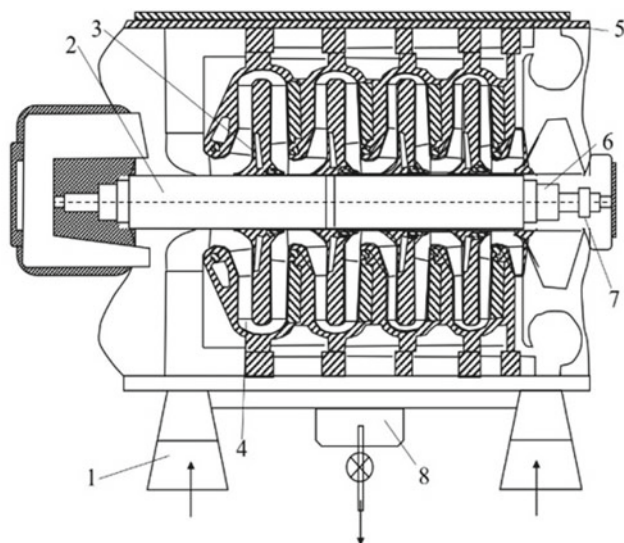
Centrifugal compressors are widely used in various processes of environmental engineering. Their main function is to transport air, various process gases or mixed gases and increase the pressure. The working principle of a centrifugal compressor is the same as that of a centrifugal pump. A high-speed rotating impeller is used to make the fluid obtain kinetic energy, and then the kinetic energy is transformed into static energy in the volute channel. The system is mainly composed of rotors, stators and bearings. The impeller and other parts are sleeved on the main shaft to form a rotor, which is supported on the bearing and rotates at high speed when driven by the power engine. The stator includes a sealing, casing, baffle, intake chamber and worm chamber, and fixed elements such as a diffuser, bend and return current are formed between the baffles, as shown in Fig. 2.61.

A single-stage centrifugal compressor has only one impeller, and a centrifugal compressor with more than two impellers is called a multistage centrifugal compressor. A single-stage centrifugal compressor cannot produce high pressure, so to make the gas outlet pressure reach a certain value, a multistage centrifugal compressor is typically used.

The main performance parameters of a centrifugal compressor include the flow rate, exhaust pressure, rotational speed, power and efficiency. The curve reflecting the relationship between exhaust pressure, power and efficiency and flow rate at the same rotational speed is called the performance curve, as shown in Fig. 2.62. The performance of the centrifugal compressor can be adjusted by changing the rotational speed, inlet throttle, outlet throttle and adjustable inlet guide vane to expand the operating range.

Some advantages of centrifugal compressors are as follows:

- (1) A centrifugal compressor has a large gas volume, single structure barrel, compact structure, small unit size and small floor area.
- (2) These compressors are characterized by reliable operation, fewer friction parts, smooth operation, less spare part consumption, fewer maintenance costs and less workload.

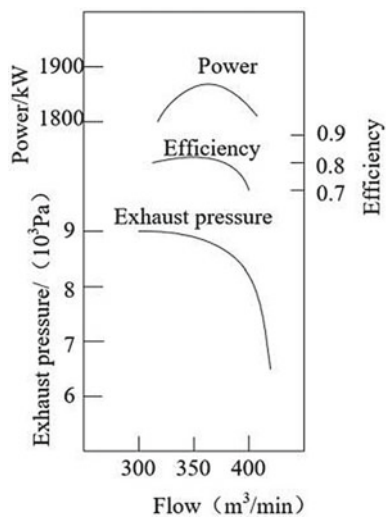


1-Inhalation chamber, 2-Axis, 3-Impeller, 4-Fixed part, 5-Machine shell,

6-Shaft end seal, 7-Bearing, 8-Exhaust cochlear chamber

Fig. 2.61 Structural diagram of centrifugal compressor

Fig. 2.62 Performance curve of centrifugal compressor (CMENWDRI, 2002)



- (3) In environmental engineering applications, centrifugal compressors can achieve an absolute oil-free compression process for chemical media.
- (4) As a rotating machine, a centrifugal compressor is suitable for the direct drive of industrial or gas turbines. For example, in environmental engineering applications, byproduct steam is often used to drive industrial steam turbines, which ensures the comprehensive utilization of heat energy.

Some shortcomings of centrifugal compressors are as follows:

- (1) At present, centrifugal compressors are not suitable for working occasions where the gas volume is too small and the pressure ratio is too high.
- (2) The stable operating range of a centrifugal compressor is narrow, and although its gas regulation is convenient. Moreover, its economy is poor.
- (3) The efficiency of a centrifugal compressor is low.

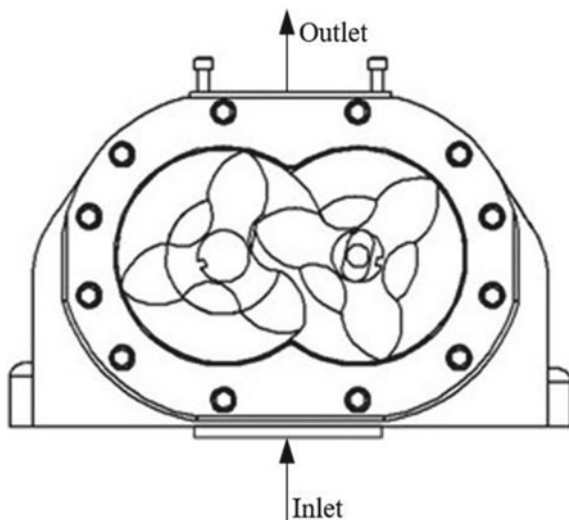
2) Root blower

Root blowers are a kind of volumetric gas compressor, as shown in Fig. 2.63. Their main characteristics are as follows: in the range of the maximum design pressure, the flow rate changes little when the friction loss of the pipeline varies. Therefore, root blowers are more suitable for working occasions where the demand of the gas flow rate is stable and the variation in the friction loss is large. Therefore, root blowers are widely used in wastewater treatment plants to provide oxygen in the form of compressed air for the growth of aerobic microorganisms in activated sludge processes. The following are several common root blowers and their structural features.

(1) R series standard root blower

① Scope of application:

Fig. 2.63 Working principle of root blower



R series standard root blowers are generally used for conveying clean air. The inlet flow rate ranges from 0.45 to 458.9 m³/min, and the outlet boost can reach 9.8 ~ 98 kPa. They are widely used in wastewater treatment, aquaculture, electricity, petroleum, the chemical industry, textiles and other fields.

② Structures and characteristics:

Cycloidal blades and the latest aerodynamic theories are adopted for high efficiency and energy savings. The rotor balancing accuracy is high, vibration is low, noise is low, gas transmission is not polluted by oil, and the working life is long. The transmission modes are divided into direct connections and belt connections.

(2) SSR (Root) blower

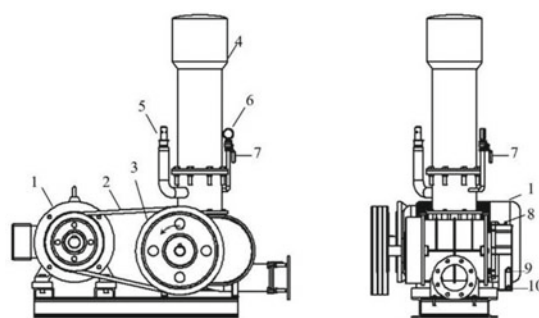
① Scope of application: SSR root blowers are mainly used in wastewater treatment, aquaculture and vacuum packaging industries to transport clean, oil-free air. The inlet air volume ranges from 1.18 to 26.5 m³/min, and the outlet boost can reach 9.8 ~ 58.8 kPa.

② Structures and characteristics: The impeller adopts a three-blade straight line, which further improves the total adiabatic rate and volumetric efficiency. Oil lubrication is not needed inside the chassis, and the output air is clean. SSR blowers have smooth operation, small volume, large flow, low noise, excellent air flow rate and pressure characteristics.

③ Shape: The configuration of an SSR root blower is shown in Fig. 2.64.

(3) L series root blower

Scope of application: The L series root blower is widely used in the cement, chemical, foundry, pneumatic transportation, aquaculture, food processing, wastewater treatment and other industries to transport clean air, gas, sulfur dioxide and other



1—Electric motor, 2—Belt cover, 3—Blower, 4—Imported silencer, 5—Safety valve, 6—Pressure gauge, 7—Pressure gauge switch, 8—Exhaust nozzle, 9—Oil pointer, 10—Drain plug

Fig. 2.64 Schematic diagram of the structure of the SSR root blower

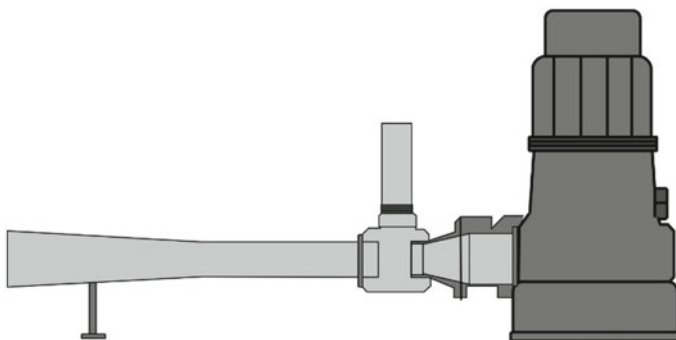


Fig. 2.65 Shape and application schematic diagram of an underwater jet aerator

gases without oil. The model has dense flow rate classification and a wide working coverage. The inlet flow rate ranges between 0.8 and 711 m³/min, and the outlet boost can reach 9.8 to 98 kPa.

3) **Jet aerator**

- (1) A jet aerator consists of a submersible pump and a jet. When the water from a submersible pump flows through the throat of a jet, it generates vacuum and suction, sucks air into the jet through the intake pipe, and sucked air mixes completely with water in the diffusion section pipe. When the mixture of air and water is ejected from the jet, a strong eddy current is formed in the water, which results in a large amount of oxygen in air to dissolve in the water. Scope of application: In an aeration tank for the biochemical treatment of industrial wastewater and municipal wastewater, sewage and sludge, a jet aerator mixes the air and wastewater and provides dissolved oxygen for the growth and reproduction of aerobic bacteria. This jet aerator is suitable for small wastewater treatment plants.
- (2) Structures and characteristics: including two types of slideways and non-slideways. It is characterized by high aeration capacity, low oxygen transfer energy consumption, no need for blower rooms and gas pipelines, and low capital investment.
- (3) Shape: The outline and application schematic diagram of a BER underwater jet aerator are shown in Fig. 2.65.

2.7.3 *Reciprocating Compressor and Reciprocating Pump*

Reciprocating fluid conveying equipment uses the reciprocating motion of a piston to transfer energy to fluid to complete the task of fluid conveyance. A reciprocating compressor is generally used to convey gas, while a reciprocating pump is used

to convey liquid. Although the use and mechanical structure of the two are very different, the basic operating principle is similar.

2.7.3.1 Reciprocating Compressor

Figure 2.66 shows the working process of a reciprocating compressor. When the piston moves to the left end of the cylinder (point A in the figure), the press trip ends. However, because of the mechanical structure, although the piston has reached the left end of the stroke, there is still some volume on the left side of the cylinder, called the clearance volume. The existence of clearance makes the initial stage of the inhalation stroke a high-pressure gas expansion process of the pressure p_2 in the clearance, and the inhalation valve does not open until the pressure drops to the inhalation pressure p_1 (point B in the figure), when the pressure p_1 gas is sucked into the cylinder. During the inhalation process, the pressure p_1 remains unchanged until the piston moves to the right end (point C in the figure), and the inhalation stroke ends. The compression stroke starts, the suction valve closes, and the gas in the cylinder is compressed. When the pressure of gas in the cylinder increases to slightly higher than p_2 (point D in the figure), the exhaust valve opens, the gas is discharged from the cylinder block until the piston moves to the left end of the cylinder, and the exhaust process ends.

The working cycle of the compressor consists of four stages: expansion, inhalation, compression and discharge. The area surrounded by the quadrilateral ABCD in the figure is the work done by the piston to the gas in a working cycle.

2.7.3.2 Reciprocating Pump

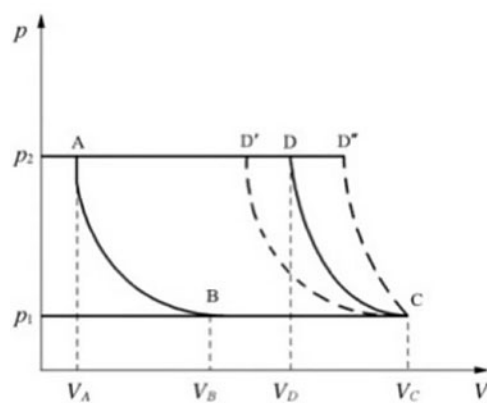
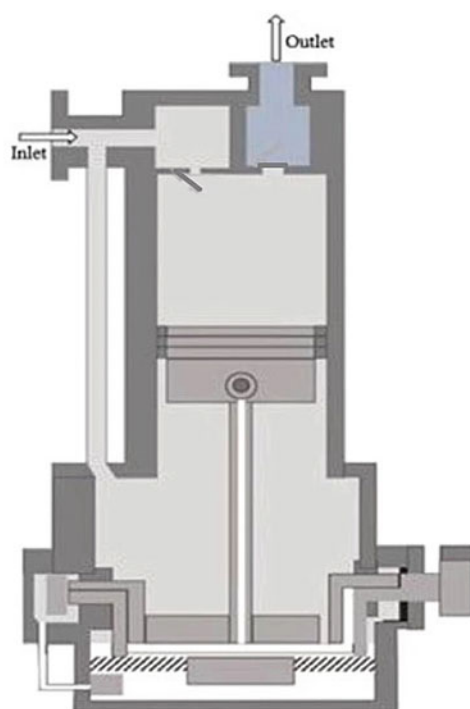
The flow rate of liquid conveyed by the reciprocating pump is related to the displacement of the piston but not to the pipeline condition, but the lift head of the reciprocating pump is only related to the pipeline condition. This characteristic is called a positive displacement characteristic, and the pump with this characteristic is called a positive displacement pump.

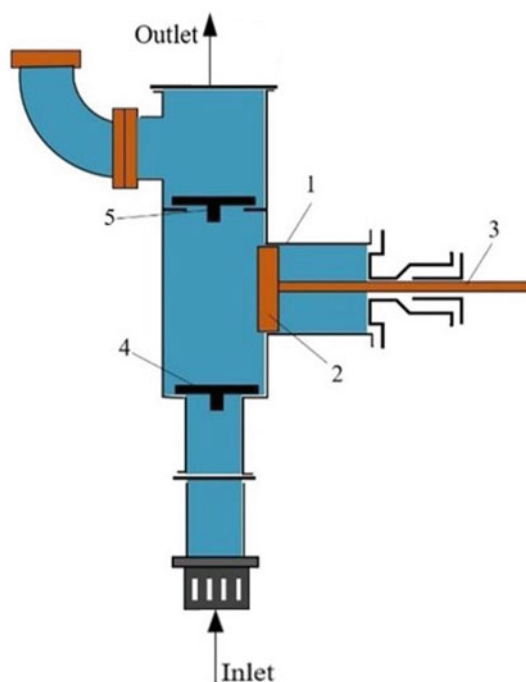
The structure of the reciprocating pump is shown in Fig. 2.67. Its main components include pump cylinder 1, piston 2, piston rod 3, suction valve 4 and discharge valve 5.

The piston of the reciprocating pump is driven by the crank and connecting rod mechanism, which creates a reciprocating motion. During the reciprocating motion of the piston, the volume and pressure of the pump cylinder change periodically. The suction valve and discharge valve open and close alternately to achieve the purpose of conveying fluid.

A piston that reciprocates once with a reciprocating pump that only sucks and discharges liquid once is called a single-action pump. The reciprocating motion of the piston is driven by a crank rotating at equal speed, and its velocity varies in accordance with the sinusoidal curve characteristics, so the discharge volume or flow rate of the piston will undergo the same change in a cycle, as shown in Fig. 2.68a. To

Fig. 2.66 Working process of reciprocating compressor





1 - Pump cylinder, 2 - Piston, 3 - Piston rod, 4 - Suction valve, 5 - Discharge valve

Fig. 2.67 Drawing of reciprocating pump device

overcome the nonuniformity of the flow rate of a single-acting pump, a double-acting pump can be used instead. Its working principle and structure are shown in Fig. 2.69. There are suction valves and discharge valves on both sides of the piston. The piston reciprocates once and sucks and discharges twice so that each stroke of the piston is accompanied by suction and discharge. The flow rate curve of the double-acting pump is shown in Fig. 2.68b.

The theoretical flow rate of the reciprocating pump is determined by the volume swept by the piston, which has nothing to do with the pipeline characteristics. The lift head provided by the reciprocating pump depends only on the pipeline condition. The working point of the reciprocating pump is the intersection of the pipeline characteristic curve and pump characteristic curve, as shown in Fig. 2.70. In practice, the flow rate of the reciprocating pump decreases slightly with increasing lift head, which is caused by volume loss.

For centrifugal pumps, outlet valves can be used to regulate fluid flow rate, but this method cannot be used for reciprocating pumps because reciprocating pumps are positive displacement pumps whose flow rate has nothing to do with pipeline characteristics. The installation of control valves can not only change the flow rate

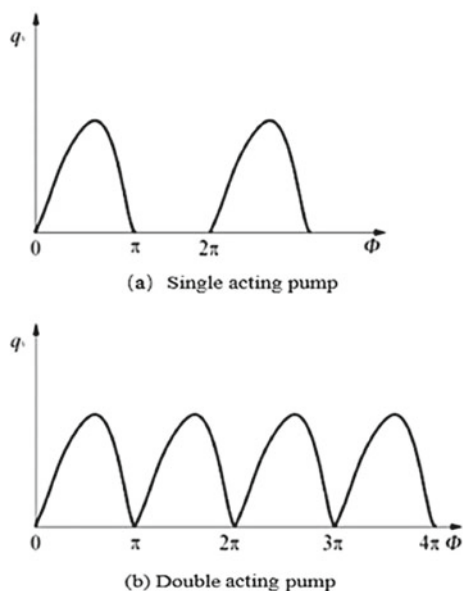
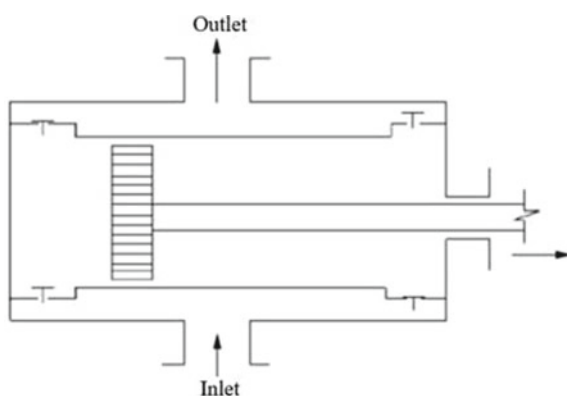


Fig. 2.68 Flow rate curve of reciprocating pump

Fig. 2.69 Double-acting reciprocating pump



but also cause harm. Once the outlet valves are completely closed, the pressure in the cylinder of the pump will rise sharply, resulting in damage to parts or motor burning.

The flow rate regulation method of the reciprocating pump is as follows.

- (1) **Bypass regulation:** Bypass regulation is shown in Fig. 2.71. Because the flow rate of the reciprocating pump is constant, the bypass flow rate can be regulated by the valve so that part of the discharge fluid can be returned to the suction pipeline to regulate the main flow rate. This method is only applicable to regular adjustment with small change range of flow rate.

Fig. 2.70 Working point of reciprocating pump

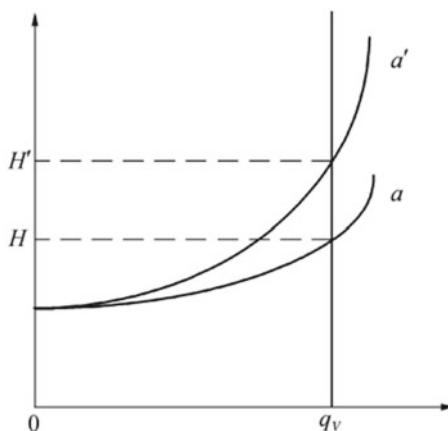
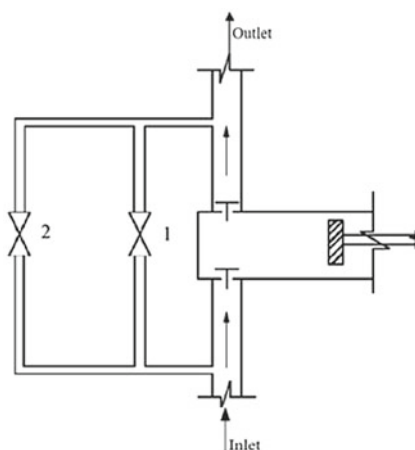


Fig. 2.71 Schematic diagram of regulating flow rate by bypassing for the reciprocating pump



1- Pass valve, 2- Safety valve

- (2) Change the crank speed and piston stroke: The motor is connected with the reciprocating pump through the deceleration device, so changing the transmission ratio of the deceleration device can easily change the crank speed and achieve the purpose of flow regulation. Changing the speed regulation method is the most commonly used and economical method. For conveying flammable and explosive liquids, the reciprocating pump driven by steam can change the intake of steam and the reciprocating frequency of the piston, thus realizing the regulation of the flow rate.

2.7.4 Other Commonly Used Fluid Conveying Equipment

2.7.4.1 Positive Displacement Pump

1) Reciprocating type

(1) Metering pump

A metering pump is a kind of reciprocating pump. The structure of a metering pump is shown in Fig. 2.72. The rotating motion of the motor is changed into reciprocating motion of the plunger through the eccentric wheel. The eccentric distance of the eccentric wheel can be adjusted, and the stroke of the plunger changes accordingly. If the number of reciprocating plungers per unit time remains unchanged, the flow rate of the pump is proportional to the stroke of the plunger; thus, the flow rate can be controlled and regulated more strictly by adjusting the stroke.

The metering pump is suitable for situations requiring the accurate and easy adjustment of infusion volume, such as conveying liquid to the reactor of a chemical plant. Sometimes several metering pumps can be driven by a motor so that each liquid has a stable flow rate, while the proportion of each liquid flow is fixed.

(2) Diaphragm pump

A diaphragm pump is a kind of reciprocating pump. As shown in Fig. 2.73, there is an elastic film in the pump body, which separates the movable column from the conveyed liquid. When conveying a corrosive liquid or suspension, the movable column and cylinder block can be protected. Diaphragms are usually made of corrosion-resistant rubber or elastic metal sheets. The left side of the diaphragm and the liquid contact part are made of corrosion-resistant materials, and the right side of the diaphragm

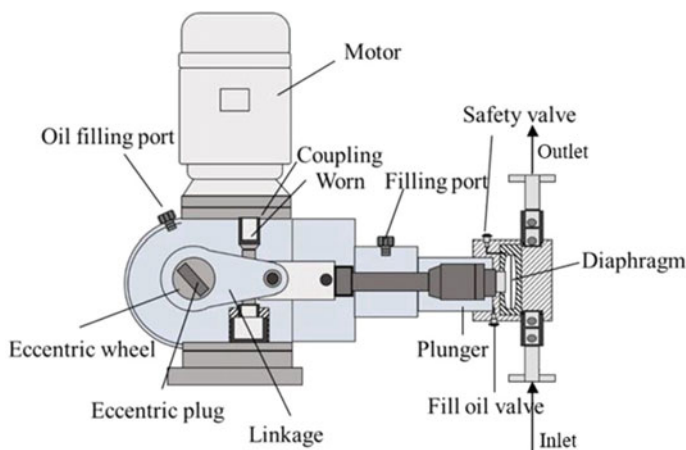
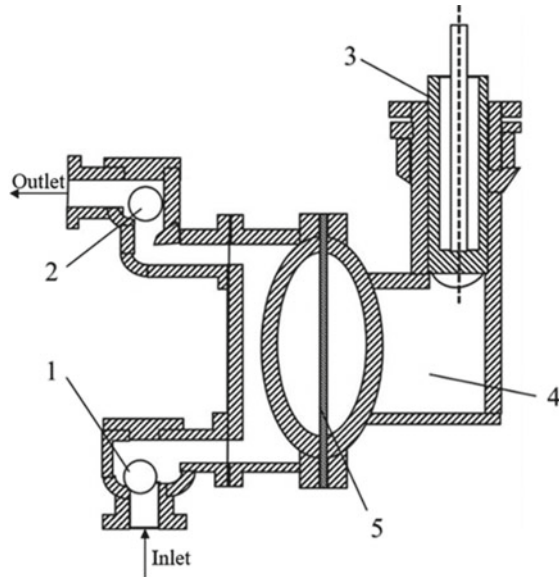


Fig. 2.72 Metering pump

Fig. 2.73 Diaphragm pump



1—The suction valve; 2—Pressing out valve; 3—Living column;
4—Water (or Oil) tank; 5—Diaphragm

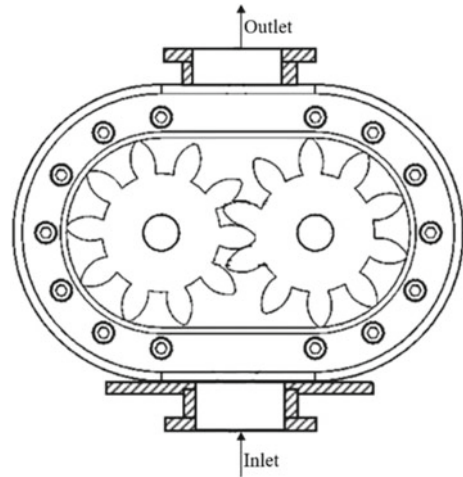
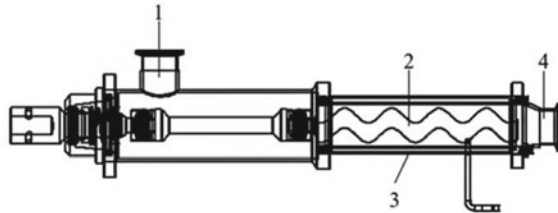
is filled with oil or water. When the movable column moves back and forth, the diaphragm is forced to bend alternately to both sides, and the liquid to be transported is inhaled and discharged.

2) Rotary type

(1) Gear pump

The main components of the gear pump are an elliptical pump housing and two gears, as shown in Fig. 2.74. One of the gears is the driving wheel, driven by the transmission mechanism, and the other is the driven wheel, which engages with the driving wheel and then rotates backward. When the gears rotate, the teeth of the two gears separate from each other to form a low pressure, which sucks the liquid and pushes it along the shell wall to the discharge chamber. In the discharge chamber, the teeth of the two gears are meshed with each other, thus forming high pressure and discharging the liquid. This is done continuously to complete the task of transporting liquid.

The gear pump has a low flow rate and high pressure head. It is very suitable for transporting liquids with high viscosity, such as glycerol and grease.

Fig. 2.74 Gear pump**Fig. 2.75** Screw pump

1- Suction entrance 2- Screw
3- Pump shell 4- Pressure exit

(2) Screw pump

The main components of a screw pump are the pump shell and one or more screw rods. According to the number of screws, it can be divided into a single screw pump, twin screw pump, three screw pumps and five screw pumps.

The structure of the single screw pump is shown in Fig. 2.75. When the screw pump works, screw 2 rotates eccentrically in pump housing 3 with internal threads, pushes the liquid along the axis, sucks from suction port 1, and discharges from pressure outlet 4. A multiscrew pump relies on the volume change of screw meshing to transport liquid.

The working efficiency of the screw pump is higher than that of the gear pump. The noise and vibration of the screw pump are very small, and the flow rate is uniform and stable. It is suitable for the transportation of high-viscosity liquid, such as sludge.

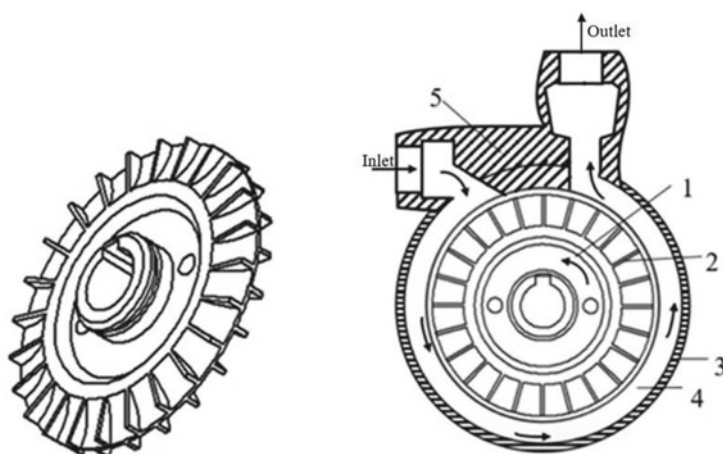
2.7.4.2 Nonpositive Displacement Pump

1) Vortex pump

A vortex pump is a special type of centrifugal pump. The pump shell is round, and the suction and discharge ports are located at the top of the pump shell. As shown in Fig. 2.76, impeller 1 in the pump body is a disc, with radial grooves milling around it, forming blade 2. There is a certain gap between the impeller and pump housing 3 forming flow channel 4. The suction pipe joint is separated from the discharge pipe joint with separator 5.

When the pump body is filled with liquid, the centrifugal force generated by the impeller rotation moves the liquid in the blade groove to the flow channel at a certain velocity. In a flow channel with a wide cross section, the liquid velocity will slow down, and some kinetic energy will be converted into static pressure energy. At the same time, the inner side of the blade groove forms low pressure because the liquid is thrown out. The liquid with higher pressure in the runner can re-enter the blade groove again and is subjected to centrifugal force again, so the pressure continues to increase. In this way, after the liquid is inhaled from the suction port, a higher pressure head can be obtained when it reaches the outlet through repeated whirlpool movement between the blade groove and the channel.

The scroll pump should be filled with liquid before starting. When the flow rate decreases, the head of the scroll pump increases, and the power increases. Therefore, the outlet valve should not be closed before the scroll pump starts, but bypass reflux should be used to regulate the flow rate.



(a) Impeller shape (b) Internal schematic diagram

1-Impeller 2-Blade 3-Pump shell 4-Flow channel 5-Diaphragm

Fig. 2.76 Diagram of vortex pump

The efficiency of a scroll pump generally does not exceed 40%. Its main characteristics are small flow rate, high lift head and simple structure. It is widely used in chemical production and is suitable for liquids with a high pressure head and low viscosity.

2) Axial flow pump

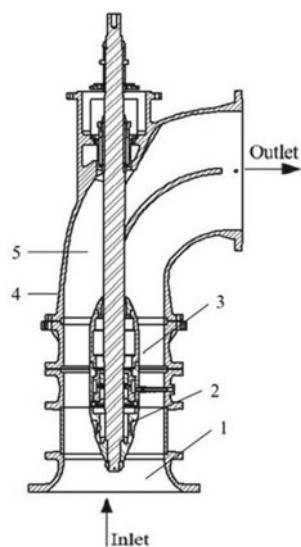
The structure of an axial flow pump is shown in Fig. 2.77. When working, the rotating shaft drives the axle head to rotate, and the axle head is equipped with a blade 2. The liquid enters the pump housing along the arrow direction, passes through the blade, and then flows out through guide vane 3 fixed on the pump housing.

The blade shape of the axial flow pump is different from that of the centrifugal pump. The twist angle of the blade of the axial flow pump increases with the radius. As shown in Fig. 2.77, the angular velocity of the liquid decreases with increasing radius.

By properly choosing the twist angle of the blade, the potential energy $\left(\frac{P}{\rho g} + z\right)$ can be kept basically unchanged along the radius to eliminate the radial flow of liquid. An axial flow pump blade is usually a propeller type, and its purpose is shown here.

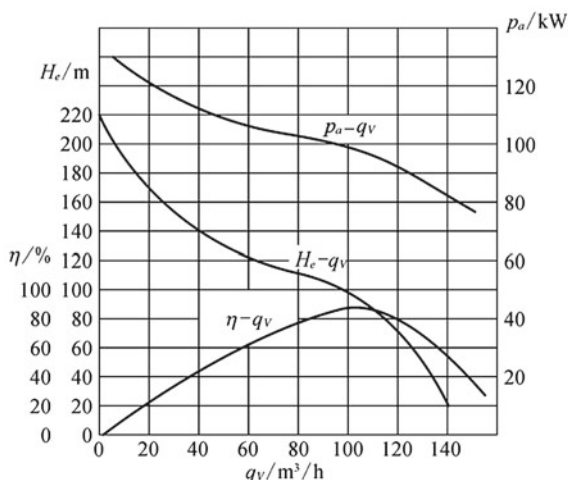
The blade itself rotates at the same angular velocity, while the angular velocity of the liquid varies along the radius, so there must be relative motion between the two in the circumferential direction. That is, the liquid moves around the blade in the opposite direction of rotation at relative speed, which forms a pressure difference on both sides of the impeller, thus producing the pressure head needed to transport the liquid.

Fig. 2.77 Axial flow pump



1-Suction chamber, 2-Blade,
3-Guide vane, 4-Pump body,
5-Outlet bend pipe

Fig. 2.78 Performance curves of axial flow pump (CMENWDRI, 2002)



The impeller of an axial flow pump is usually immersed in the liquid, and the pressure provided is generally low, but the infusion volume is very large. An axial flow pump is suitable for large flow rate and low pressure fluid transportation. They have been widely used in rainwater pumping stations, wastewater reflux and other occasions.

The performance curves of an axial flow pump is shown in Fig. 2.78. The $H-q_v$ curve of the axial flow pump is steep, and the high-efficiency range is very narrow.

Axial flow pumps generally do not set outlet valves but change the flow rate by changing the performance curves of the pump. The more commonly used method is to change the velocity of the shaft or adjust the installation angle of the blade.

Exercises

Part I Multiple choice and fill-in questions

- The continuity equation represents () conservation.
 - mass
 - momentum
 - energy
 - heat
- The Bernoulli equation represents () conservation.
 - mass
 - momentum
 - energy
 - heat
- The reading value of the vacuum gauge is () and that of the pressure gauge is ().
 - absolute pressure
 - absolute pressure – atmospheric pressure
 - absolute pressure + atmospheric pressure
 - atmospheric pressure – absolute pressure

4. If the vacuum gauge reading on a certain equipment is 100 mmHg and the local atmospheric pressure is 760 mmHg, the absolute pressure in the equipment is (), and the gauge pressure is ().
 - A. -100 mmHg
 - B. -760 mmHg
 - C. 860 mmHg
 - D. 660 mmHg
5. Usually, the variation in fluid viscosity with temperature is that ().
 - A. viscosity decreases with increasing temperature
 - B. viscosity increases with increasing temperature
 - C. for liquids, the viscosity decreases with increasing temperature, while for gases, the opposite is true
 - D. for liquids, the viscosity increases with temperature, while for gases, the opposite is true
6. When the diameter of a straight pipe is doubled and its flow rate, length and friction coefficient remain unchanged, the friction loss along the flow path of the fluid in the pipe becomes how many times that of the original ().
 - A. 2
 - B. 4
 - C. 1/16
 - D. 1/32
7. In complete turbulence region (friction loss square region), the friction coefficient λ along a rough pipe is calculated as a value ().
 - A. similar to a smooth pipe
 - B. depending only on Re
 - C. depending only on relative roughness
 - D. having nothing to do with roughness
8. The internal friction loss of fluid flow is proportional to ().
 - A. kinetic energy gradient
 - B. flow rate gradient
 - C. velocity gradient
 - D. pressure gradient
9. When the fluid is complete turbulent in a straight pipe, the friction loss is proportional to the () of flow velocity.
 - A. primary
 - B. square
 - C. cubic
 - D. quintic

10. Which of the following ways is not the way to reduce fluid resistance ().
 - A. reduce unnecessary fittings
 - B. reduce pipe diameter appropriately
 - C. reducing fluid viscosity
 - D. replacing a rough pipe with a smooth pipe
11. The Reynolds number Re reflects the relationship of ().
 - A. viscous force and gravity
 - B. gravity and inertia force
 - C. inertial force and viscous force
 - D. viscous force and hydrodynamic pressure
12. When a fluid is turbulent in a pipe, its average velocity is () times that of the flow in the center of the pipe. When laminar flow occurs, its average velocity is () times that of the flow in the center of the pipe.
 - A. 0.5
 - B. 0.8
 - C. 1
 - D. 2
13. The Bernoulli equation shows that the sum of () per unit mass ideal fluid at any cross section is a constant without external work.
 - A. internal energy, potential energy and kinetic energy
 - B. internal energy, kinetic energy and static pressure energy
 - C. potential energy, kinetic energy and static pressure energy
 - D. internal energy, potential energy and static pressure energy
14. Open channel flow is a fluid () a free surface, belonging to ().
 - A. with, no pressure flow
 - B. with, pressure flow
 - C. without, no pressure flow
 - D. without, pressure flow
15. The relative motion between fluid and solid particles can usually be divided into () cases.
 - A. 1
 - B. 2
 - C. 3
 - D. 4
16. When the free settlement of solid particles in the fluid belongs to the laminar flow region, the size of () has the greatest influence on the settling velocity.
 - A. fluid density
 - B. fluid diameter
 - C. particle density
 - D. particle diameter

17. The following physical parameters which do not belong to the performance parameters of centrifugal pumps are ().
- A. lift head
 - B. efficiency
 - C. shaft power
 - D. effective power
18. The working point of a centrifugal pump is determined () and can be changed by changing () and ().
- A. by both pipeline characteristic curve and centrifugal pump performance curve
 - B. only by centrifugal pump performance curve
 - C. valve
 - D. pump rotating
19. The relationship between the efficiency and flow rate of the centrifugal pump is that ().
- A. efficiency increases with the increase of flow rate
 - B. efficiency first increases and then decreases with increasing flow rate
 - C. efficiency decreases with increasing flow rate
 - D. efficiency decreases and then increases with increasing flow rate
20. If the motion elements of a fluid point particle at any fixed point in the flow field do not change with time, the flow is called _____. For two basic methods of fluid flow investigation, _____ method is often used in describing this flow state.
21. There are two different investigation methods for fluid flow. The _____ method is used to track particles and describe the law of their motion parameters varying with time. The method of studying the distribution field of various motion elements is called _____.
22. The gauge pressure of a device is 100 kPa, and then the absolute pressure of a device is _____ kPa. The vacuum of the other equipment is 400 mmHg, and its absolute pressure is _____. (The local atmospheric pressure is 101.33 kPa).
23. Generally, the viscosity of liquid _____ with increasing temperature, and the gas viscosity _____ with increasing temperature.
24. The resistance can be divided into two types, _____ and _____, when fluid flows in a straight pipeline.
25. The laminar flow of fluid in a circular straight pipe will cause _____ times of original friction loss if the flow velocity is unchanged and the diameter of the pipe is only doubled.
26. When the fluid flow is turbulent region in the pipe, the friction coefficient is related to _____ and _____. If it is a complete turbulence (friction loss square region), then the friction coefficient is only related to _____.
27. (1) When _____, laminar flow must occur, which is called a laminar flow region.
- (2) When _____, it is called a transitional region.

- (3) When _____, turbulence generally occurs, which is called a turbulent region.
28. The flow in the open channel is called the open channel flow, which is generally in the rough turbulent region and can be divided into _____ and _____.
29. Open channel steady flow can be divided into two parts according to whether its streamlines are parallel to each other flow states, _____ and _____.
30. _____ phenomenon will occur when the installation height of centrifugal pump exceeds the allowable installation height.
31. List four types of fluid transport equipment:
_____, _____
_____, _____

Part II Computational questions + answers

- A flue gas can be simplified as a mixture of 15%SO₂ and air, with the gauge pressure of 1750 kPa, the temperature of 127°C and the flow rate of 8400 m³/h (standard state). Before discharge, the gas should be purified. The pretreatment was conducted through a Φ 114 mm \times 4 mm cooling pipe, which was cooled to 27 °C and maintained a constant pressure. The gas is still output from this pipe after pretreatment. Please calculate the average flow velocity and mass flow rate of the flue gas at the inlet and outlet of the pipe. (Steady flow: based on steady mass flow rate, not volume flow rate).
Answer: Inlet gas flow velocity 21.2 m/s, outlet gas flow velocity 15.9 m/s, mass flow rate 3.568 kg/s.
- To understand the performance of a certain type of centrifugal pump, the steady flow pipeline system shown in Fig. 2.79 is adopted. The inlet pipe diameter of the pump is Φ 85 mm \times 4 mm, and the outlet pipe diameter is Φ 75 mm \times 4 mm. The pressure gauge reading is 800 mmHg at the outlet of the pump.

Fig. 2.79 Schematic diagram of question 2

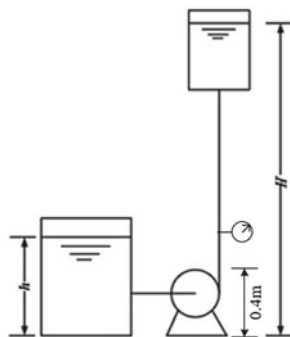
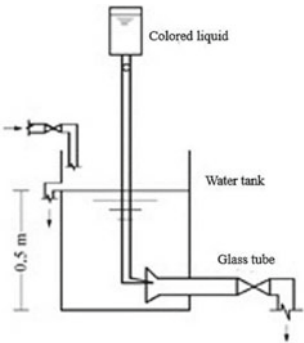


Fig. 2.80 Schematic diagram of question 3

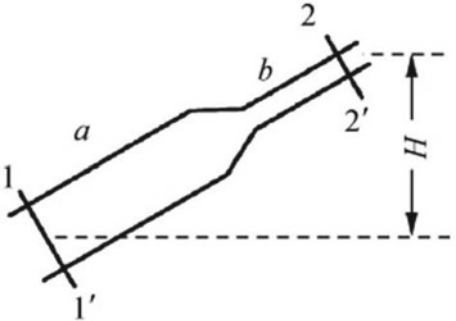


The distance between the pressure gauge and the ground is 0.4 m. All energy losses of the water flowing through the suction pipe and the discharge pipe are calculated according to $\sum h_{f1} = 2u_1^2$ and $\sum h_{f2} = 8u_2^2$, respectively. The liquid level of the low tank is $h = 0.5$ m and that of the high tank is $H = 2.0$ m (negligible pump body height). The output power of the motor is known to be 1.8 kW. Please determine the efficiency of the pump. (The atmospheric pressure in this area is 760 mmHg).

Answer: Pump efficiency is 85.7%.

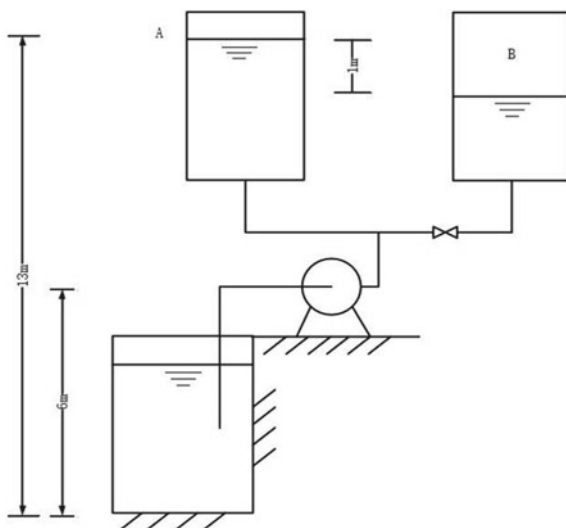
3. Figure 2.80 shows the Reynolds experimental device. The system is in steady flow at 20 °C. The water level of the open water tank is 0.5 m, and the cross section area of the water tank is 0.04 m². The length of the bell-shaped glass pipe in the water tank is 1 m, the absolute roughness is 0.01 mm, the inner diameter is 20 mm, the water level at the outlet of the glass pipe is zero, the gate valve at the end of the pipe is fully opened, and the colored liquid pipe in the bell-shaped glass pipe is very thin. Judge the flow state and velocity of liquid in the glass pipe. Is it feasible to only adjust the water level in the tank separately if the water in the pipe is laminar?
- Answer: 1.634 m/s, turbulent flow in pipe, $h \leq 2.215 \times 10^{-3}$ m, not feasible in practical operation.

Fig. 2.81 Schematic diagram of question 4



4. Figure 2.81 shows a portion of a water supply pipe of a factory. Water with a flow rate of 8×10^4 kg/h flows steadily upward in an inclined pipe. The inner diameter of the pipe is 100 mm and 80 mm from the bottom to the top. In the figure, the 2-2' surface is 6 m higher than the 1-1' surface, and the fluid pressure on the 1-1' surface is 600 mmHg higher than that on the 2-2' surface. Please find the friction loss between the 1-1' and 2-2' surfaces. If the horizontal angle of the water supply pipe is 30 degrees, the absolute roughness of the pipe is 0.046 mm, the viscosity of the water is 1×10^{-3} Pa \cdot s, and the length of the pipeline connection is neglected, how long are the local pipe thick and thin respectively? (Loss coefficient of local resistance $\zeta = 0.06$).
Answer: Friction loss $h_{\text{friction}} = 14.92$ J/kg, $L_a = 8.1$ m, $L_b = 3.9$ m.
5. For partial pressure water supply in an area, water is pumped to a water tank 35 m high with a flow rate of 25 m³/h. The water tank is connected with the atmosphere. The vacuum at the inlet of the pump suction pipe is 41.8 kPa, and the diameter of the suction pipe is 80 mm. The head loss of the suction pipe and pressure pipe is $h_w = 6$ m, and the pump efficiency $\eta = 0.7$. If the motor efficiency $\eta' = 0.9$, how much power motor is needed? If the water pump runs 16 h a day, the daily water demand of the residential area is 1200 m³ and the electricity charge fee is 0.65 yuan/(kW \cdot h), what is the annual electricity charge fee of water supply in the residential area? The atmospheric pressure in this area is 98.8 kPa.
Answer: A 4.82 kW motor is needed with an annual electricity charge fee of 55,460 yuan for water supply.
6. Figure 2.82 shows that an organic solution with a density of 1100 kg/m³ and a viscosity of 0.9×10^{-3} Pa \cdot s is transported to the A and B high tanks by a pump in the laboratory. When the gate valve of the B-end pipeline is closed, the pressure reading of the pump outlet is measured at 4.4×10^5 Pa. At this time,

Fig. 2.82 Schematic diagram of question 6



the fluid flows steadily. The pipeline length from pump outlet to both *A* or *B* is 15 m, and to the tee is 10 m. The pipe is an iron pipe with the inner diameter of 20 mm and the absolute roughness of 15 mm. Atmospheric pressure in this area is 10.33 mH₂O.

Requirements:

- (1) When the *B*-end gate valve closes, calculate the liquid flow rate.
- (2) If the *B*-end gate valve is fully open and the level of tank *A* is 1 m higher than that of tank *B* and the output flow rate of the pump remains unchanged, what is the pressure at the outlet of the pump? What are the flow rates of *A* and *B*, respectively?

Answer: When the *B*-end gate valve closes, the liquid flow rate is 1.49×10^{-3} m³/s. When the *B*-end gate valve is fully opened, the pressure at the pump outlet is 3.32×10^5 Pa. The flow rates of *A* and *B* are 6.85×10^{-4} m³/s and 8.04×10^{-4} m³/s, respectively.

7. The solid particles settled 60 mm in 10 min at an atmospheric pressure at 20 °C in settling test in column, and the true density of solid particles was 1.814×10^3 kg/m³. The physical parameters of wastewater thought to be the same as those of clean water. Please calculate the equivalent diameter of the solid particles. If the removal efficiency of solid particles in one hour is 70%, what is the removal efficiency of solid particles with an equivalent diameter of 10 μm in one hour?

Answer: $d_p = 15$ μm, removal efficiency of solid particles sedimentation at 10 μm is 31.1%.

8. The concrete wastewater pipe of a factory has a bottom slope $i = 0.025$. During the daytime production period, the wastewater filling degree α_f in the pipe is 0.75. At night, the wastewater filling degree becomes 0.3. How much does the wastewater flow rate through the pipe decrease at night?

Answer: The flow rate decreases 79%.

9. Trapezoidal cross section brick channel with rough coefficient $n_r = 0.015$, flow rate $q_V = 1.5$ m³/s, bottom slope $i = 0.008$, and side slope $m_s = 1.25$, the maximum allowable flow velocity is 2.5 m/s. Please design the optimal cross section based on the above data.

Answer: $b_w = 0.7668$ m, $h = 0.451$ m.

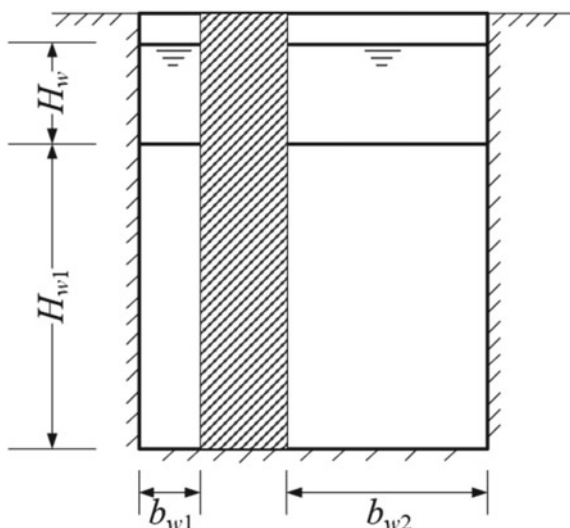
10. For a scraped reinforced concrete trapezoidal cross section channel with bottom width $b_w = 5.1$ m, side slope $m_s = 0.5$, bottom slope $i = 0.003$, what is the flow rate that can be satisfied for optimizing the design? If the navigable velocity $u_{\max} = 5$ m/s is allowed, can the optimal design meet the navigable requirement?

Answer: Optimized design flow rate is 173.0 m³/s, and it cannot meet navigation demand.

11. It is necessary to construct a trapezoidal cross section channel with dry block stone with flow rate $q_V = 1$ m³/s, bottom slope $i = 0.0004$, side slope $m_s = 1.0$, and water depth $h = 0.85$ m, please calculate the required channel width b_w .

Answer: $b_w = 1.96$ m.

Fig. 2.83 Schematic diagram of question 13



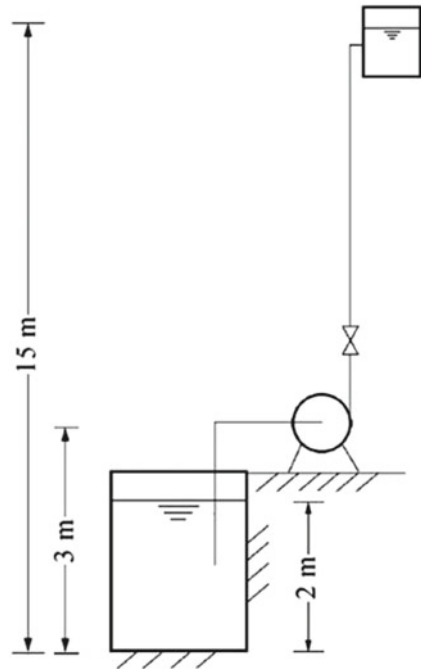
12. A right-angle crest triangular thin wall weir with a head = 0.15 m above the weir is used to calculate the flow rate through the weir. If the discharge is doubled, how much is the water head on the weir?
Answer: $q_V = 0.0122 \text{ m}^3/\text{s}$, $H_w = 0.198 \text{ m}$.
13. As shown in Fig. 2.83, a rectangular thin wall weir without lateral contraction is divided into two parts by a straight partition. The flow rates of each part are $q_{V1} = 20 \text{ L/s}$ and $q_{V2} = 35 \text{ L/s}$, the weir height is $H_{w1} = 0.6 \text{ m}$, and the head is $H_w = 0.2 \text{ m}$. Please calculate the width of each part of weir b_{w1} and b_{w2} . If the partition width is 0.05 m and the discharge flow rate is constant after the partition is removed, what is the head height over the weir?
Answer: Weir width b_{w1} is 0.1166 m, and b_{w2} is 0.204 m. After removing the partition, head on weir H_w is 0.182 m.
14. A centrifugal pump (model: 3B57) is used to transport 20°C water at a flow rate of 45 m³/h. It is known that the vacuum $H_s = 6.7 \text{ m}$ is allowed to be sucked under this flow rate. If the diameter of the suction pipe is 60 mm, the head loss of the suction pipe is estimated to be 0.75 m.
Requirements: (1) If the pump installation height is 5.5 m, can it work normally? Atmospheric pressure in this area is $9.81 \times 10^4 \text{ Pa}$.
(2) What is the allowable installation height if the pump transports water of 50 °C at an altitude of 2000 m? Local atmospheric pressure is $7.9968 \times 10^4 \text{ Pa}$.
Answer: The installation height pump of 5.5 m cannot work properly under the atmospheric pressure and 20 °C working conditions. The allowable installation height is 2.15m.
15. A pump (model: 3B57) is used to transport water at 75°C from an open tank. The required critical cavitation allowance of the pump is 49 kPa. The friction loss of the suction pipe is a 0.5 m water column. Please determine the

installation position of the pump. If the tank is sealed and the maximum allowable vacuum is 45 kPa, how should the pump be installed? The atmospheric pressure in this area is 100 kPa.

Answer: $H_g = 0.301$ m, $H'_g = -4.408$ m.

16. Water at 20°C is transported to a 1.5 bar closed water tower by a centrifugal pump as shown in Fig. 2.84. The lower tank is exposed to the atmosphere, and the water level can be regarded as constant. The water level of the tank is 2 m high. The pipes are seamless steel pipes of $\Phi 106$ mm \times 3 mm. The absolute roughness is 0.05 mm, the length of the suction pipe is 20 m, and the length of the outlet pipe is 100 m. The outlet pipe includes a gate valve, a check valve and three 90-degree elbows. The vertical distance between the inlet and outlet of the pump is 0.5 m. When the gate valve is half open, the vacuum meter reading at the intake of the pump is 43000 Pa, the vertical distance between the inlet and outlet of the pump is 0.5 m. The friction loss of the pump body is neglected. The local coefficient of the gate valve is 0.45 at half open and 0.17 at full open.

Fig. 2.84 Schematic diagram of question 16



Find:

- (1) When the gate valve is half open, please calculate the pipe flow rate and the gauge pressure at the outlet of the pump.
- (2) The gate valve is fully open, which makes the flow rate 1.3 times the original flow rate. The tool table shows that the allowable suction vacuum is 5.6 m under this flow rate, and the local atmospheric pressure is 9.81×10^4 Pa. Can the pump work normally at this time?

Answer: When the gate valve is half open, the flow rate is $94.45 \text{ m}^3/\text{h}$, and the outlet pressure of the pump is 3.47×10^5 Pa. When the gate valve is fully open, the installation height should be 0.0439 m below the water surface. Therefore, and the pump cannot work properly.

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Chapter 3

Heat Transfer



Yan Liu, Liang Li, Guang-Tuan Huang, and Chen-Xi Li

3.1 Overview

3.1.1 *The Basic Mode of Heat Transfer*

Heat transfer, is a common energy transfer process in nature and engineering. According to the second law of thermodynamics, where there is a temperature difference, there must be a spontaneous transfer of heat from high to low temperature, which is called the heat transfer process.

There are three main heat transfer methods: heat conduction, convective heat transfer and radiation heat transfer. Heat transfer is sometimes carried out in one way or in two or three ways simultaneously. For example, a common heat exchange equipment in industry is an interwall heat exchanger. During the process of heat transfer, heat is first transferred from a hot fluid to the inner side of the pipe wall by convection, and then the heat is transferred from the inner side of the pipe wall to the outer side by heat conduction. Finally, heat is transferred from the outer side of the pipe wall to the cold fluid by convection, as shown in Fig. 3.1. The above heat transfer process mainly includes convective heat transfer and heat conduction but is also accompanied by radiation heat transfer. When the temperature is not high, the proportion of radiation heat transfer in the total heat transfer is very small, which can be ignored.

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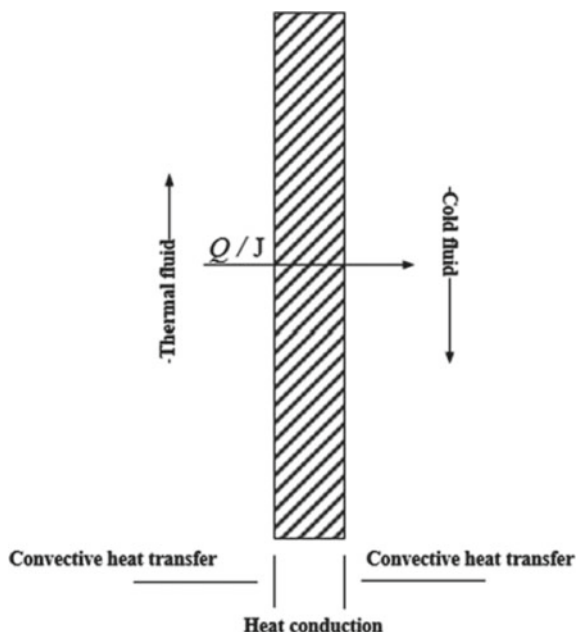
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169

Fig. 3.1 Interwall heat transfer



3.1.2 Heat Transfer Problems in the Field of Environmental Engineering

Research on the heat transfer process is the key to guarantee the normal operation of industrial production and solve the problem of energy efficient utilization. There are various heat transfer problems in the chemical, energy, machinery, electronics, metallurgy, aviation, aerospace and other industrial sectors.

In the field of environmental engineering, the same unit operation process as in industrial processes is adopted, which often requires heating or cooling to maintain the required temperature of the treatment process, which involves many heat transfer steps. The anaerobic treatment of high-concentration organic wastewater and the anaerobic digestion of sludge usually require the heating of wastewater or sludge. It is necessary to heat the wastewater or reactor for the stripping recovery of high concentrations of ammonia, the stripping recovery and treatment of polycyclic aromatic hydrocarbons in coking wastewater, the evaporation treatment of pesticide wastewater, the wet oxidation of high concentrations of organic and toxic wastewater, and supercritical oxidation treatment. Flue gas from waste incineration can heat boiler water to generate steam for power generation. The flue gas cooling of a thermal power plant and condensing to remove organic steam from the waste gas with high temperature require cooling to remove heat. At the same time, to reduce the heat exchange between the system and the external environment, such as reducing the temperature changes of cold and hot fluids in the transportation or reaction process, it is necessary to keep thermal insulation for the pipes or reactors. Therefore, to

ensure the normal operation of various treatment processes in the field of environmental engineering, there are mainly two kinds of intervention processes for heat transfer. One is to strengthen the heat transfer process, such as heat transfer in various heat exchange equipment, by taking measures to improve the heat transfer rate. The second is to weaken the heat transfer process, such as the insulation of reactors and pipes, to reduce the heat loss, that is, to reduce the heat transfer rate.

3.2 Heat Conduction

Heat conduction refers to the heat transfer process that occurs through the vibration, displacement and collision of molecules, atoms and electrons of a material. When there is a temperature gradient inside a material, all gases, liquids or solids conduct heat conduction to some extent.

Gas molecules are in the process of irregular thermal motion, and the constant collisions between them transfer heat from high to low temperature. Molecules or atoms in solids move only slightly around the equilibrium position of the lattice structure, while free electrons move freely between the lattice. For nonconductive solids, heat is transferred mainly by a vibrating lattice of molecules or atoms. For a good electric conductor, the number of free electrons is high, and the heat transferred by the movement of electrons is more than the lattice vibration. Therefore, a good electric conductor is generally a good thermal conductor. The structure of a liquid is between a solid and a gas. Molecules or atoms can move within a certain range and vibrate at the same time. Heat transfer is the common result of molecules or atoms vibrating and colliding with each other.

The main characteristic of the heat conduct process is that there is no macroscopic motion between the parts of the material, the heat transfer can be basically regarded as a molecular transfer phenomenon, and the heat transfer law can be described by the Fourier's law.

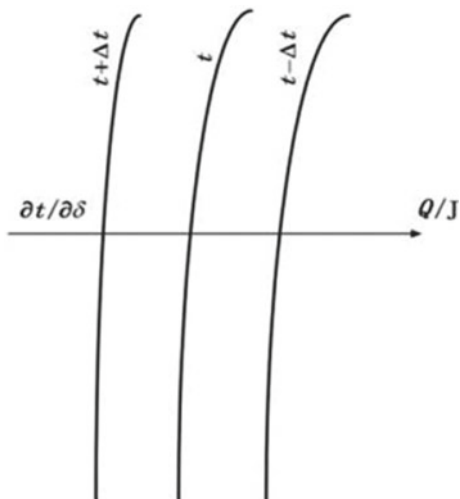
3.2.1 Fourier's Law

Heat conduction depends on the vibration of molecules inside the object and the movement of free electrons. Molecules in the hotter part of an object vibrate and collide with neighboring molecules, passing part of their kinetic energy to the latter, causing heat energy to be transferred from the hotter part to the cooler part. Figure 3.2 shows the diagram of temperature gradient and Fourier's law.

In 1822, Fourier proposed the basic law of heat conduction on the basis of a great number of experiments, and its mathematical expression is

$$\frac{dQ}{d\tau} \propto -A_h \frac{dt}{d\delta} \quad (3.1)$$

Fig. 3.2 Temperature gradient and Fourier's law



which can be written as:

$$\frac{dQ}{d\tau} = -\sigma A_h \frac{dt}{d\delta} \quad (3.2)$$

Under the condition of stable heat conduction, the conducted heat quantity does not change with time; that is, the conducted heat quantity per unit time is a constant value, and the above equation can be written as:

$$\Phi = \frac{Q}{\tau} = -\sigma A_h \frac{dt}{d\delta} \quad (3.3a)$$

where

Q —Heat quantity, J

Φ —Heat transfer quantity per unit time, also known as heat flow rate or heat transfer rate, W or J/s

A_h —Heat conduction area, that is, the cross-sectional area perpendicular to the heat flow direction, m^2

t —Temperature, K

τ —Time, s

σ —Thermal conductivity, $W/(m \cdot K)$ or $(kg \cdot m)/(s^3 \cdot K)$

$\frac{dt}{d\delta}$ —Temperature gradient, K/m.

The Fourier's law shows that heat transfer quantity per unit time is proportional to the heat conduction cross section and temperature gradient perpendicular to the heat flow direction. In the equation, the negative sign indicates that the heat flow direction is opposite to the temperature gradient, that is, the heat flow direction is the direction along which the temperature decreases.

3.2.2 Thermal Conductivity

Equation (3.2) can be rewritten as

$$\sigma = -\frac{\Phi}{A_h \frac{dt}{d\delta}} = -\frac{q_T}{\frac{dt}{d\delta}} \quad (3.3b)$$

$$q_T = \frac{\Phi}{A_h} = \frac{Q}{\tau A_h} \quad (3.3c)$$

where

Q —Heat transfer quantity, J

Φ —Heat transfer quantity per unit time, also known as heat flow rate or heat transfer rate, W or J/s

A_h —Heat conduction area, that is, the cross-sectional area perpendicular to the heat flow direction, m^2

σ —Thermal conductivity, $W/(m \cdot K)$ or $(kg \cdot m)/(s^3 \cdot K)$

$\frac{dt}{d\delta}$ —Temperature gradient, K/m

τ —Time, s

q_T —Heat flux (heat flow density), heat transfer quantity per unit time and area or heat flow rate per unit area, W/m^2 or $J/(s \cdot m^2)$.

The thermal conductivity represents the heat conduction rate of a material per unit area and per unit temperature gradient. Thermal conductivity is the physical property of a material, which represents the heat conduction ability of a material. It is related to the type, temperature and pressure of the material.

3.2.2.1 Thermal Conductivity of Solids

Of all solids, metal is the best conductor of heat. The thermal conductivity of metal generally decreases with increasing temperature and increases with increasing metal purity, so the thermal conductivity of alloys is generally smaller than that of pure metals.

The thermal conductivity of nonmetallic materials is related to the density and temperature of their composition and structure and usually increases with increasing density or temperature.

Usually, a homogeneous solid σ is roughly linear with temperature, which can be expressed as:

$$\sigma = \sigma_0[1 + a_t(t - t_0)] \quad (3.4)$$

where

σ_0 —Thermal conductivity of the solid at temperature t_0 , W/(m · K)

σ —Thermal conductivity of the solid at temperature t , W/(m · K)

t_0 —Temperature, K

t —Temperature, K

a_t —Temperature coefficient, negative for most metallic materials and positive for most nonmetallic materials, K⁻¹.

Although the thermal conductivity is a function of temperature, the numerical value at average temperature is usually taken in engineering calculations. The calculation of heat conduction of most solid materials whose thermal conductivity varies linearly with temperature will not cause much error. Table 3.1 lists the thermal conductivity of some commonly used solid materials.

Table 3.1 Thermal conductivity of common solid materials (Cardarelli, 2018)

Material	Temperature °C	σ W/(m · K)	Material	Temperature °C	σ W/(m · K)
Silver	100	412	Glass	30	1.09
Copper	100	377	Building brick	20	0.69
Aluminum	300	230	Asbestos	200	0.21
Wrought iron	18	61	Asbestos	100	0.19
Nickel	100	57	Asbestos board	50	0.17
Cast iron	53	48	Hard rubber	0	0.15
Steel (1% C)	18	45	Sawdust	20	0.052
Lead	100	33	Interlock	30	0.050
Stainless steel	20	16	Cork	30	0.043
Graphite	0	151	Glass fiber (coarse)		0.041
High alumina brick	430	3.1	Glass fiber (fine)		0.029

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3.2.2.2 Thermal Conductivity of Liquid

The thermal conductivity of liquids is generally lower than that of solids. The thermal conductivity of some liquids is listed in Table 3.2. The thermal conductivity of metallic liquids is larger than that of nonmetallic liquids, and the thermal conductivity of water in nonmetallic liquids is the highest. With the exception of water and glycerol, the thermal conductivity of most liquids decreases slightly with increasing temperature. The thermal conductivity of the liquid mixture is lower than that of the pure liquid, and the value can be estimated as follows:

$$\sigma_m = K_C \sum_{i=1}^n \sigma_i \omega_i \quad (3.5)$$

where

σ_m, σ_i —Thermal conductivity of the liquid mixture and component i , W/(m · K)

ω_i —Mole fraction of component i in the liquid mixture

K_C —Constant; for a general mixture or solution, $K_C = 1.0$, and for an organic aqueous solution, $K_C = 0.9$.

Table 3.2 Thermal conductivity of some liquids (Cardarelli, 2018)

Liquid	Temperature°C	σ W/(m · K)	Liquid	Temperature°C	σ W/(m · K)
50% acetic acid	20	0.35	40% glycerol	20	0.45
Acetone	30	0.17	N-heptane	30	0.14
Aniline	0 ~ 20	0.17	Mercury	28	8.36
Benzene	30	0.16	90% sulfuric acid	30	0.36
30% calcium chloride brine	30	0.55	60% sulfuric acid	30	0.43
80% ethanol	20	0.24	Water	30	0.62
60% glycerol	20	0.38	Water	60	0.66

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3.2.2.3 Thermal Conductivity of Gas

The thermal conductivity of gas is the lowest, which is not conducive to thermal transfer but beneficial to thermal insulation. Industrial application of thermal insulation materials occurs because there is air in the gap, so it can have a thermal insulation effect. Table 3.3 lists the thermal conductivity of some gases.

The thermal conductivity of a gas increases with temperature. In the normal pressure range, its thermal conductivity changes little with pressure, only at high or low pressure (higher than 2×10^5 kPa or less than 3 kPa), and the thermal conductivity increases with pressure.

The thermal conductivity of the gas mixture under normal pressure can be calculated by the following equation:

$$\sigma_m = \frac{\sum_{i=1}^n y_i \sigma_i M_i^{1/3}}{\sum_{i=1}^n y_i M_i^{1/3}} \quad (3.6)$$

where

σ_m, σ_i —Thermal conductivity of the gas mixture and component i , W/(m · K)

y_i —Mole fraction of component i in the gas mixture

M_i —Molar mass of component i in the gas mixture, g/mol.

Table 3.3 Thermal conductivity of some gases (Cardarelli, 2018)

Gas	Temperature °C	σ W/(m · K)	Gas	Temperature °C	σ W/(m · K)
Hydrogen	0	0.17	Water vapor	100	0.024
Carbon dioxide	0	0.015	Nitrogen	0	0.024
Air	0	0.024	Ethylene	0	0.017
Air	100	0.031	Oxygen	0	0.024
Methane	0	0.029	Ethane	0	0.018
Carbon monoxide	0	0.021	Ammonia	0	0.022

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3.2.3 Stable Heat Conduction of Flat Wall

3.2.3.1 Heat Conduction of a Single-Layer Flat Wall

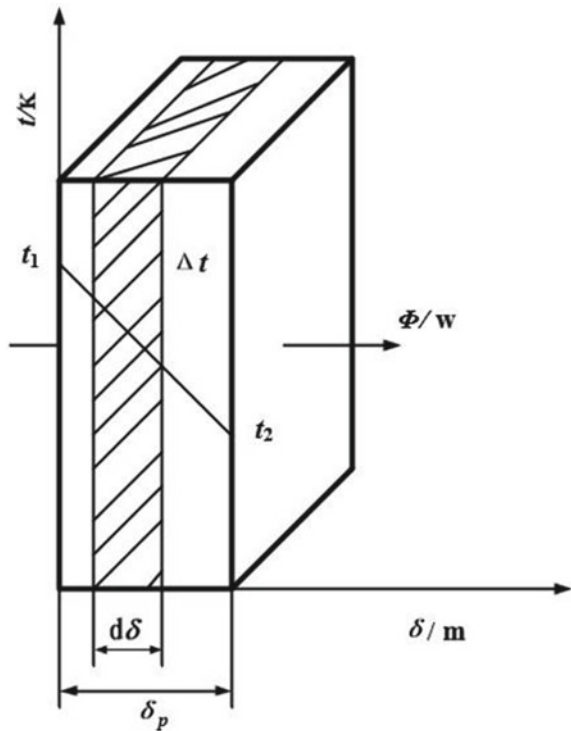
For an infinitely large flat wall, that is, the length and width of the size are much greater than the thickness of the flat wall, the heat dissipation at the edge of the flat wall can be ignored. It is assumed that the temperature inside the flat wall only changes along the δ direction perpendicular to the wall surface, and the temperature field is a one-dimensional temperature field. Assuming that the temperature distribution in the flat wall does not change with time, the heat flow in the flat wall is one-dimensional stable heat conduction (Fig. 3.3).

Assuming that the thermal conductivity of the material does not vary with the temperature (or take the average thermal conductivity), the boundary condition is:

$$\begin{aligned}\delta = 0, t &= t_1 \\ \delta = \delta_p, t &= t_2\end{aligned}$$

Then, the integral of the Fourier's law Eq. (3.3) for stable heat conduction can be obtained as follows:

Fig. 3.3 Heat conduction on the wall of a single-layer plate



$$\begin{aligned}
\Phi &= \frac{Q}{\tau} - \sigma A_h \frac{dt}{d\delta} \\
\frac{Q}{\tau A_h} d\delta &= -\sigma dt \\
\frac{Q}{\tau A_h} \int_0^{\delta_p} d\delta &= -\sigma \int_{t_1}^{t_2} dt \\
\frac{Q}{\tau A_h} \delta_p &= -\sigma (t_2 - t_1) = \sigma (t_1 - t_2)
\end{aligned} \tag{3.7}$$

or

$$\Phi = \frac{Q}{\tau} = \frac{\sigma}{\delta_p} A_h (t_1 - t_2) \tag{3.8a}$$

$$\Phi = \frac{t_1 - t_2}{\frac{\delta_p}{\sigma A_h}} = \frac{\Delta t}{\Omega} \tag{3.8b}$$

where

δ_p —Thickness of flat wall, m

t_1, t_2 —Temperature on both sides of the flat wall, K

Δt —Temperature difference, driving force of heat conduction, K

Ω —Thermal resistance of heat conduction, K/W

Q —Heat transfer quantity, J

Φ —Heat flow rate or heat conduction transfer rate, heat transfer quantity per unit time, W or J/s.

q_T represents the heat transfer quantity per unit time and area, which is called heat flux or heat flow density [J/(s · m²)]. q_T can be expressed as:

$$q_T = \frac{\Phi}{A_h} = \frac{t_1 - t_2}{\delta_p / \sigma} = \frac{t_1 - t_2}{\Omega A_h} \tag{3.9}$$

Example 3.1 A flat heat insulation wall is designed on the outside of the incineration boiler of a waste incineration plant. The heat insulation wall adopts an insulating brick thickness of 600 mm, the temperature inside the brick wall is 500 °C, the temperature outside the wall is 50 °C, and the average thermal conductivity is 0.6 W/(m · K). Please determine:

- (1) Heat flow density
- (2) The temperature t_δ away 300 mm from the inside.

Known: $\delta_p = 600 \text{ mm}$, $t_1 = 500^\circ\text{C} = 773 \text{ K}$, $t_2 = 50^\circ\text{C} = 323 \text{ K}$, $\sigma = 0.6 \text{ W/(mK)}$, $x = 300 \text{ mm}$.

Determine: q_T , t_δ .

Solution: (1) According to the Fourier's law of heat conduction, heat flux q_T is:

$$q_T = \frac{\Phi}{A_h} = \frac{t_1 - t_2}{\delta_p / \sigma} = \frac{773 - 323}{\frac{0.6}{0.6}} = 450 \text{ W/m}^2$$

(2) Suppose that the temperature on the isothermal surface at the distance from the inner x along the direction of wall thickness is t_δ ; then,

$$q_T = \frac{t_1 - t_\delta}{x / \sigma}$$

so

$$t_\delta = t_1 - \frac{q_T}{\sigma} x = 773 - \frac{450}{0.6} \times 0.3 = 548 \text{ K} = 275^\circ\text{C}$$

Answer: The heat flow density (namely heat flux q_T) is 450 W/m^2 and the temperature is 275°C away 300 mm from inside the brick wall.

3.2.3.2 Heat Conduction of a Multilayer Flat Wall

Engineering applications often encounter multiple layers composed of different materials of flat walls, called multilayer flat wall. The wall of a boiler furnace chamber is comprised of firebrick, adiabatic brick and common brick. Take the three-layer flat wall as an example, as shown in Fig. 3.4. The area of the flat wall is A_h , the wall thickness of each layer is δ_{p1} , δ_{p2} and δ_{p3} , and the thermal conductivities are σ_1 , σ_2 and σ_3 . Assume that the contact between layers is good, that is, the two surface temperatures in contact are the same. The surface temperatures are t_1 , t_2 , t_3 and t_4 , $t_1 > t_2 > t_3 > t_4$, and the temperature difference of each layer is Δt_1 , Δt_2 and Δt_3 .

Figure 3.4 shows the schematic diagram of the heat conduction of a multilayer flat wall.

In the process of stable heat conduction, the heat flow through the flat walls of each layer is equal. Therefore,

$$\begin{aligned} \Phi &= \sigma_1 A_h \frac{(t_1 - t_2)}{\delta_{p1}} = \frac{\Delta t_1}{\frac{\delta_{p1}}{\sigma_1 A_h}} = \frac{\Delta t_1}{\Omega_1} \\ &= \sigma_2 A_h \frac{(t_2 - t_3)}{\delta_{p2}} = \frac{\Delta t_2}{\frac{\delta_{p2}}{\sigma_2 A_h}} = \frac{\Delta t_2}{\Omega_2} \end{aligned}$$

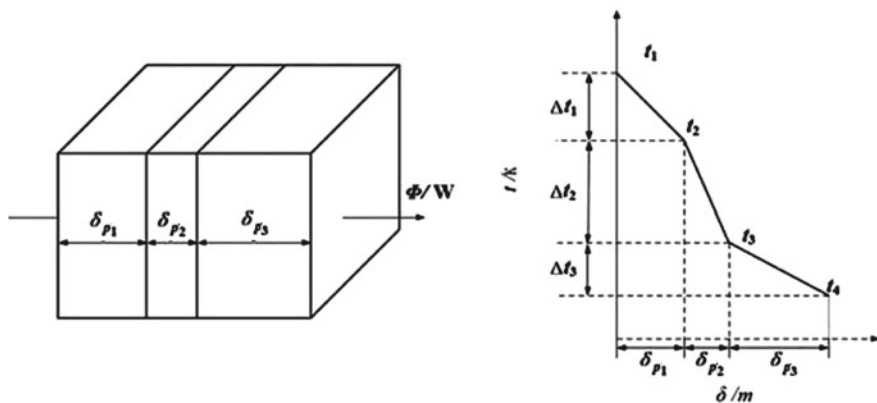


Fig. 3.4 Heat conduction of multilayer flat wall

$$= \sigma_3 A_h \frac{(t_3 - t_4)}{\delta_{p3}} = \frac{\Delta t_3}{\frac{\delta_{p3}}{\sigma_3 A_h}} = \frac{\Delta t_3}{\Omega_3} \quad (3.10a)$$

where

Ω_1 , Ω_2 and Ω_3 —Thermal resistance of conduction in each layer, K/W or (K · s)/J. The following equation can be obtained from the above equation:

$$\Delta t_1 = \Phi \Omega_1, \Delta t_2 = \Phi \Omega_2, \Delta t_3 = \Phi \Omega_3 \quad (3.10b)$$

Add the above three equations to obtain

$$\begin{aligned} \Phi &= \frac{\Delta t_1 + \Delta t_2 + \Delta t_3}{\Omega_1 + \Omega_2 + \Omega_3} \\ &= \frac{t_1 - t_4}{\frac{\delta_{p1}}{\sigma_1 A_h} + \frac{\delta_{p2}}{\sigma_2 A_h} + \frac{\delta_{p3}}{\sigma_3 A_h}} \end{aligned} \quad (3.11)$$

This can be extended to n flat walls:

$$\Phi = \frac{t_1 - t_{n+1}}{\sum_{i=1}^n \Omega_i} = \frac{t_1 - t_{n+1}}{\sum_{i=1}^n \frac{\delta_{pi}}{\sigma_i A_h}} \quad (3.12)$$

As seen from Eqs. (3.11) and (3.12), multilayer flat wall heat conduction is actually a series heat transfer process. The driving force (total temperature difference) of the series heat transfer process is the sum of the temperature differences of each process. The total thermal resistance of the series heat transfer process is the sum of the thermal

resistances of each process. In a stable series heat transfer process, the temperature difference of each process is proportional to the thermal resistance. When the total temperature difference is constant, the heat flow rate depends on the total thermal resistance.

Example 3.2 In waste incineration plant A, in actual operation, the heat insulation wall consists of three layers, namely, the heat preservation layer, the heat insulation layer and the outer layer of the building from the inside to the outside. It is known that the thickness of firebrick used in the heat preservation layer is 300 mm and its thermal conductivity is $1.5 \text{ W}/(\text{m} \cdot \text{K})$. The thickness of heat insulating brick used in the insulation layer is 200 mm, and its thermal conductivity is $0.1 \text{ W}/(\text{m} \cdot \text{K})$. The thickness of building bricks used in the outer layer of the building is 300 mm, and its thermal conductivity is $1 \text{ W}/(\text{m} \cdot \text{K})$. The temperatures are 500 and 50°C , inside and outside the whole heat insulation wall respectively.

Ask: The heat loss of the heat insulation wall per unit area and the wall temperature inside and outside each layer.

Known: $\sigma_1 = 1.5 \text{ W}/(\text{m} \cdot \text{K})$, $\delta_{p1} = 300 \text{ mm}$, $\sigma_2 = 0.1 \text{ W}/(\text{m} \cdot \text{K})$, $\delta_{p2} = 200 \text{ mm}$, $\sigma_3 = 1 \text{ W}/(\text{m} \cdot \text{K})$, $\delta_{p3} = 300 \text{ mm}$, $t_1 = 500^\circ\text{C} = 773 \text{ K}$, and $t_4 = 50^\circ\text{C} = 323 \text{ K}$.

Determine the following: q_T , t_2 , t_3 .

Solution: Set the interface temperature between the firebrick and insulating brick to be t_2 , and set the interface temperature between the insulation brick and building brick as t_3 . Knowing that $t_1 = 773 \text{ K}$ and $t_4 = 323 \text{ K}$. The furnace wall heat loss per unit area is

$$\begin{aligned} \frac{\Phi}{A_h} = q_T &= \frac{t_1 - t_4}{\frac{\delta_{p1}}{\sigma_1} + \frac{\delta_{p2}}{\sigma_2} + \frac{\delta_{p3}}{\sigma_3}} = \frac{773 - 323}{\frac{0.3}{1.5} + \frac{0.2}{0.1} + \frac{0.3}{1}} \\ &= \frac{450}{0.2 + 2 + 0.3} = 180 \text{ W/m}^2 \end{aligned}$$

The temperature at the interface between layers is calculated as follows:

$$\Delta t_1 = t_1 - t_2 = q_T \times \frac{\delta_{p1}}{\sigma_1} = 180 \times \frac{0.3}{1.5} = 36 \text{ K}$$

$$t_2 = t_1 - \Delta t_1 = 773 - 36 = 737 \text{ K} = 464^\circ\text{C}$$

$$\Delta t_2 = t_2 - t_3 = q_T \times \frac{\delta_{p2}}{\sigma_2} = 180 \times \frac{0.2}{0.1} = 360 \text{ K}$$

$$t_3 = t_2 - \Delta t_2 = 737 - 360 = 377 \text{ K} = 104^\circ\text{C}$$

Answer: The heat loss per unit area of the insulation wall is 180 W/m^2 . Interface temperature is 464°C between heat preservation wall and heat insulation wall, and the interface temperature is 104°C between heat insulation wall and building wall.

3.2.4 Heat Conduction on Cylinder Wall

Cylindrical vessels, equipment and pipes are often used in engineering, so the thermal conductivity of long cylindrical walls is very common. This differs from a flat wall in that the heat transfer area of the cylinder wall is not constant but varies with the radius. Figure 3.5 shows a schematic diagram of heat conduction on the wall of a long cylinder. The inner and outer wall radii of the cylinder wall are respectively r_1 , r_2 . The inner and outer wall surface temperatures are t_1 and t_2 , and the cylinder height is l . Heat conduction area $A = 2\pi r l$ at the cylinder wall radius r . If the cylinder wall is very long, the axial heat dissipation can be neglected, and the temperature only changes along the radius direction. If cylindrical coordinates are adopted, the heat transfer problem in Fig. 3.5 is one-dimensional stable heat conduction.

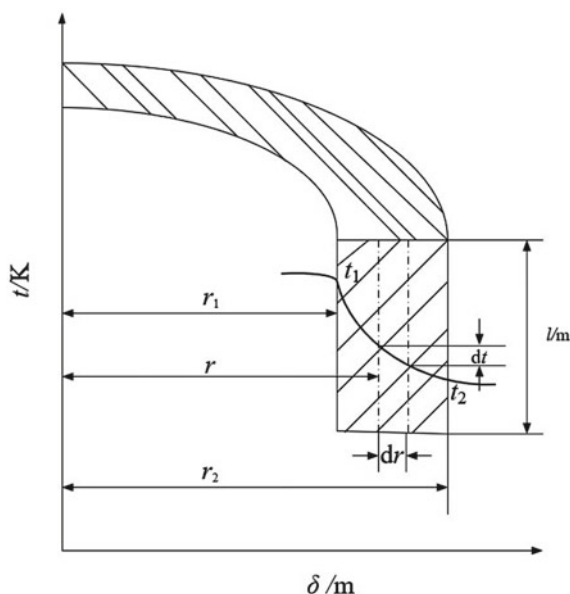
Figure 3.5 Heat conduction on a cylinder wall.

If a thin wall of dr thickness is taken at the inner radius of the cylinder wall r , the differential equation of heat flow of the cylinder wall can be listed as follows according to the Fourier's equation:

$$\Phi = -\sigma A_h \frac{dt}{dr} = -\sigma 2\pi r l \frac{dt}{dr} \quad (3.13)$$

For a single-layer cylinder wall, within a certain temperature range, σ is a constant; then, the integral of the above equation after separating variables can be:

Fig. 3.5 Heat conduction on a cylinder wall



$$\Phi \int_{r_1}^{r_2} \frac{dr}{r} = -\sigma 2\pi l \int_{t_1}^{t_2} dt$$

$$\Phi = 2\pi l \sigma \frac{t_1 - t_2}{\ln(r_2/r_1)} = \frac{2\pi l(t_1 - t_2)}{\frac{1}{\sigma} \ln(r_2/r_1)} = \frac{t_1 - t_2}{\frac{\ln(r_2/r_1)}{2\pi l \sigma}} = \frac{t_1 - t_2}{\Omega} \quad (3.14)$$

where

Φ —Heat flow rate, W, J/s

t_1, t_2 —Temperature on both sides of the cylinder wall, K

r_1, r_2 —Radius of inner and outer sides of cylinder wall, m

l —Length of the wall pipe of the cylinder, m

σ —Thermal conductivity, W/(m · K)

Ω —The thermal resistance of the cylinder wall, $\Omega = \frac{\ln(r_2/r_1)}{2\pi l \sigma}$, K/W.

Consider

$$q'_T = \frac{\Phi}{l} = \frac{2\pi(t_1 - t_2)}{\frac{1}{\sigma} \ln(r_2/r_1)} \quad (3.15)$$

q'_T represents the heat flow rate per unit pipe length, and the unit is W/m.

If the heat conduction equation of the flat wall is used to write the heat conduction equation of the cylinder wall, use $\delta_p = r_2 - r_1, A_h = A_m = 2\pi r l$, so that

$$\Phi = \frac{t_1 - t_2}{\frac{\delta_p}{\sigma A_h}} = \frac{t_1 - t_2}{\frac{1}{\sigma A_m} (r_2 - r_1)}$$

$$= \frac{t_1 - t_2}{\frac{1}{\sigma 2\pi r_m l} (r_2 - r_1)} = \frac{2\pi l(t_1 - t_2)}{\frac{1}{\sigma} \frac{(r_2 - r_1)}{r_m}} \quad (3.16)$$

By comparing Eqs. (3.16) and (3.14), it can be concluded that:

$$r_m = \frac{r_2 - r_1}{\ln(r_2/r_1)} \quad (3.17)$$

where

r_m —Logarithmic mean radius of the cylinder wall, m

A_m —Logarithmic mean of the cylinder wall, $A_m = 2\pi r_m l$, m².

Cylinder walls composed of multilayer materials are also common in engineering applications, such as reactors, steam pipe insulation, and heat exchanger pipe. Taking

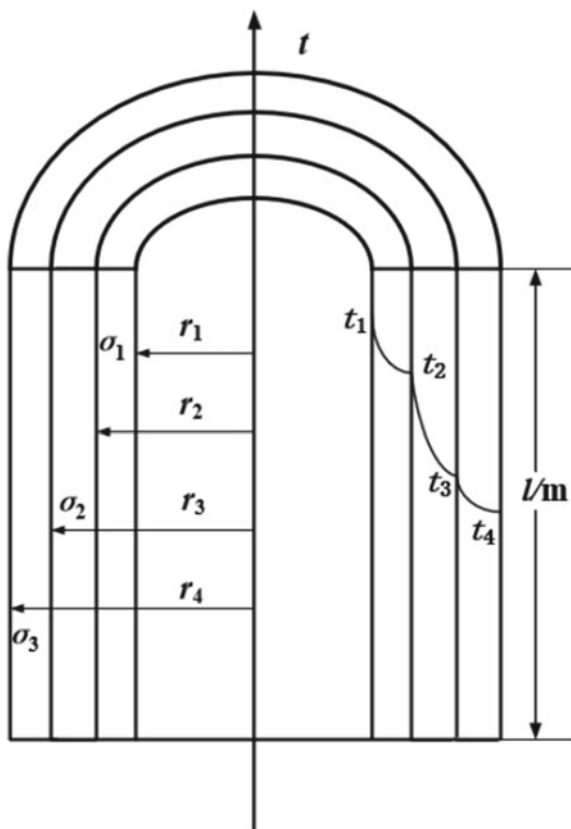
the three-layer material shown in Fig. 3.6 as an example for discussion and assuming that the contact between layers is good, the thermal conductivity of each layer is σ_1 , σ_2 and σ_3 . The thickness is $\delta_{p1} = r_2 - r_1$, $\delta_{p2} = r_3 - r_2$ and $\delta_{p3} = r_4 - r_3$. According to the principle of superposition of series thermal resistance, and similar to the treatment of a multilayer flat wall, the equation of heat flow rate of a three-layer cylinder wall can be obtained as:

$$\Phi = \frac{\Delta t_1 + \Delta t_2 + \Delta t_3}{\Omega_1 + \Omega_2 + \Omega_3} \quad (3.18a)$$

$$= \frac{t_1 - t_4}{\frac{\ln \frac{r_2}{r_1}}{2\pi l \sigma_1} + \frac{\ln \frac{r_3}{r_2}}{2\pi l \sigma_2} + \frac{\ln \frac{r_4}{r_3}}{2\pi l \sigma_3}} \quad (3.18b)$$

or

Fig. 3.6 Heat conduction of a three-layer cylinder wall



$$\Phi = \frac{t_1 - t_4}{\frac{\delta_{p1}}{\sigma_1 A_{m1}} + \frac{\delta_{p2}}{\sigma_2 A_{m2}} + \frac{\delta_{p3}}{\sigma_3 A_{m3}}} \quad (3.18c)$$

When the n -layer cylinder wall is extended, the equation of heat flow is

$$\Phi = \frac{t_1 - t_{n+1}}{\sum_{i=1}^n \frac{\ln \frac{r_{i+1}}{r_i}}{2\pi l \sigma_i}} \quad (3.18d)$$

or

$$\Phi = \frac{t_1 - t_{n+1}}{\sum_{i=1}^n \frac{\delta_i}{\sigma_i A_{mi}}} \quad (3.18e)$$

Equation (3.18c) is exactly the same as the multilayer flat wall heat flow Eq. (3.11) in form. The total driving force of heat conduction on the cylinder wall is also the total temperature difference, and the total thermal resistance is also the sum of the thermal resistance of each layer, except that the heat transfer area used to calculate the thermal resistance of each layer is not equal, which is the average area of each layer.

In addition, since the inner and outer areas of the cylinder walls of each layer are not equal, the heat flow rate through each layer is the same, but the heat flow density or heat flux q_T through each layer are not equal when heat transfer is stable.

Example 3.3 In a waste incineration plant A, the heat generated by waste incinerator is used for thermal power generation. There is a steam conveying pipe of $\Phi 320 \text{ mm} \times 10 \text{ mm}$ with a 20 mm insulation layer outside the pipe in the power generation system. Known steam conveying pipe inner surface temperature is 900°C , and its wall thermal conductivity is $1.5 \text{ W/(m} \cdot \text{K)}$. The outer temperature of the heat insulation layer is 50°C , and its thermal conductivity is $0.05 \text{ W/(m} \cdot \text{K)}$.

Please calculate: Heat loss per meter of pipe length and temperature at the interface between the pipe wall and insulation layer.

Known: $d_1 = 300 \text{ mm} = 0.3 \text{ m}$, $\delta_{p1} = 10 \text{ mm} = 0.01 \text{ m}$, $\delta_{p2} = 20 \text{ mm} = 0.02 \text{ m}$, $\sigma_1 = 1.5 \text{ W/(m} \cdot \text{K)}$, $\sigma_2 = 0.05 \text{ W/(m} \cdot \text{K)}$, $t_1 = 900^\circ\text{C} = 1173 \text{ K}$, $t_3 = 50^\circ\text{C} = 323 \text{ K}$.

Determine: q'_T and t_2 .

Solution: (1) Heat loss:

$$r_1 = \frac{0.3}{2} = 0.15 \text{ m}$$

$$r_2 = r_1 + 0.01 = 0.16 \text{ m}$$

$$r_3 = r_2 + 0.02 = 0.18 \text{ m}$$

If the pipe length is l , then the thermal resistance of each layer is:

$$\Omega_1 = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\sigma_1 l} = \frac{\ln(0.16/0.15)}{2\pi \times 1.5 \times l} = 0.00685/l \text{ K/W}$$

$$\Omega_2 = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\sigma_2 l} = \frac{\ln(0.18/0.16)}{2\pi \times 0.05 \times l} = 0.375/l \text{ K/W}$$

According to Eq. (3.18a), the heat loss per unit time and unit length can be obtained as follows:

$$q'_T = \Phi/l = \frac{\Sigma \Delta t}{(\Sigma \Omega)l} = \frac{t_1 - t_3}{(\Omega_1 + \Omega_2)l} = \frac{1173 - 323}{\left(\frac{0.00685}{l} + \frac{0.375}{l}\right)l} = 2226 \text{ W/m}$$

(2) Calculate the temperature of the contact surface between the pipe wall and the insulation layer

$$q'_T = \frac{\Delta t_1}{\Omega_1 l}$$

$$2226 = \frac{1173 - t_2}{\frac{0.00685}{l} \times l}$$

$$t_2 = 1157.8 \text{ K} = 884.8 \text{ }^\circ\text{C}$$

Answer: The heat loss per meter of pipe length is 2226 W/m, and the interface temperature of the contact surface between the pipe wall and the insulation layer is 884.8 °C.

Example 3.4 In waste incineration plant A, there is a pipe of $\Phi 320 \text{ mm} \times 10 \text{ mm}$ with thermal conductivity of $1.5 \text{ W/(m} \cdot \text{K)}$ for conveying system. There is a 25 mm insulation layer outside the pipe with thermal conductivity of $0.05 \text{ W/(m} \cdot \text{K)}$ and there is also a preservation heat layer with the thermal conductivity $\sigma_3 = 0.15 + 0.00015 t_m \text{ W/(m} \cdot \text{K)}$ outside the insulation. The temperatures inside insulation layer and outside preservation heat layer are $900 \text{ }^\circ\text{C}$ and $30 \text{ }^\circ\text{C}$, respectively. The whole heat loss per meter of pipe with both insulation layer and preservation heat layer is 1500 W/m . Try to find the temperature of interface between insulation layer and preservation heat layer, as well as the thickness of preservation heat layer.

Known: Φ 320 mm \times 10 mm, $\delta_{p1} = 25$ mm, $t_1 = 900^\circ\text{C} = 1173$ K, $t_3 = 30^\circ\text{C} = 303$ K, $q'_T = 1500$ W/m, $\sigma_1 = 1.5$ W/(m \cdot K), $\sigma_2 = 0.05$ W/(m \cdot K), $\sigma_3 = 0.15 + 0.00015t_m$ W/(m \cdot K)

Determine: t_2 and δ_{p2} .

Solution: (1) Find the temperature of interface between insulation layer and preservation heat layer.

As seen from the question:

$$r_0 = \frac{d_0}{2} = \frac{320}{2} = 160 \text{ mm} = 0.16 \text{ m}$$

$$r_1 = r_0 + \delta_{p1} = 0.16 + 0.025 = 0.185 \text{ m}$$

$$t_1 = 1173 \text{ K} \quad \sigma_2 = 0.05 \text{ W/(m} \cdot \text{K)}$$

According to:

$$q'_T = 1500 = \frac{t_1 - t_2}{\frac{\ln(r_1/r_0)}{2\pi\sigma_2}} = \frac{1173 - t_2}{\frac{\ln(0.185/0.16)}{2\pi \times 0.05}}$$

Can be obtained: $t_2 = 481$ K = 208°C .

(2) Determine the thickness of the preservation heat layer.

Because the thermal conductivity of the insulation heat layer changes with temperature, the average temperature of this layer is calculated.

$$t_m = \frac{t_2 + t_3}{2} = \frac{481 + 303}{2} = 392 \text{ K}$$

$$\sigma_2 = 0.15 + 0.00015t_m = 0.15 + 0.00015 \times 392 = 0.2088 \text{ W/(m} \cdot \text{K)}$$

In this case, q'_T remains the same, it can be known from the algorithm of the first question:

$$q'_T = 1500 = \frac{t_2 - t_3}{\frac{\ln(r_2/r_1)}{2\pi\sigma_3}} = \frac{481 - 303}{\frac{\ln(r_2/0.185)}{2\pi \times 0.2088}}$$

$$r_2 = 0.216 \text{ m}$$

$$\delta_{p2} = r_2 - r_1 = 0.216 - 0.185 = 0.031 \text{ m} = 31 \text{ mm}$$

Answer: The temperature of interface between insulation layer and preservation heat layer is 208°C , and the thickness of preservation heat layer is 31 mm.

3.3 Convection Heat Transfer

Convection heat transfer refers to the phenomenon of heat transfer caused by the relative displacement of each part of the fluid. The convection heat transfer in engineering applications mainly refers to the convection heat transfer when the fluid flows over the wall surface with a temperature difference from the fluid. The convection heat transfer law is discussed below.

3.3.1 Mechanism of Convection Heat Transfer

3.3.1.1 Simplified Treatment of the Heat Transfer Boundary Layer

Convection heat transfer occurs in the process of fluid flow. In Chap. 2, it is described that a flow boundary layer is formed when fluid passes over the solid wall surface. A velocity gradient exists in the boundary layer. Even if the fluid reaches turbulence, fluid flows in the sublayer as laminar flow. Most fluids are poor conductors with low thermal conductivity, so they have great thermal resistance at the bottom of laminar flow, forming a considerable temperature gradient. Outside the laminar sublayer, due to the relative displacement of fluid particles caused by eddy motion, heat transfer occurs by convection except for conduction, which reduces the temperature gradient. In a turbulent body, due to eddy motion, heat transfer is dominated by convection, so the thermal resistance is greatly reduced, and the temperature distribution tends to be consistent.

In summary, convective heat transfer is the general term of the heat conduction at the laminar sublayer and the heat transfer outside the sublayer of laminar flow based on the relative displacement and mixing of fluid particles. For the convenience of processing, convection heat transfer is generally considered to be equivalent to the heat conduction across the boundary layer with a thickness δ_t , which is generally composed of two parts. One is the actual thickness δ_b of the laminar sublayer of the heat conduction. The other is thickness δ_f of the virtual heat conduction layer outside the laminar sublayer, which represents the thermal resistance of the fluid particle with relative displacement and mixing, as shown in Fig. 3.7. δ_t (equivalent thickness of heat conduction boundary layer) is neither the thickness of the heat conduction boundary layer nor the thickness of the flow boundary layer. It is the thickness of the virtual layer which concentrates all thermal resistances and temperature differences and transfers the heat via heat conduction.

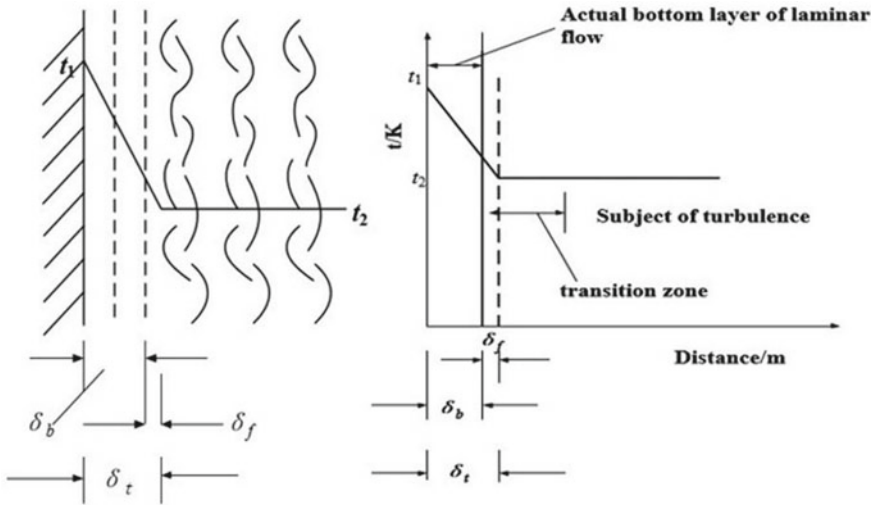


Fig. 3.7 Simplified diagram of liquid heat transfer boundary layer

3.3.1.2 Heat Transfer in Different Flow States

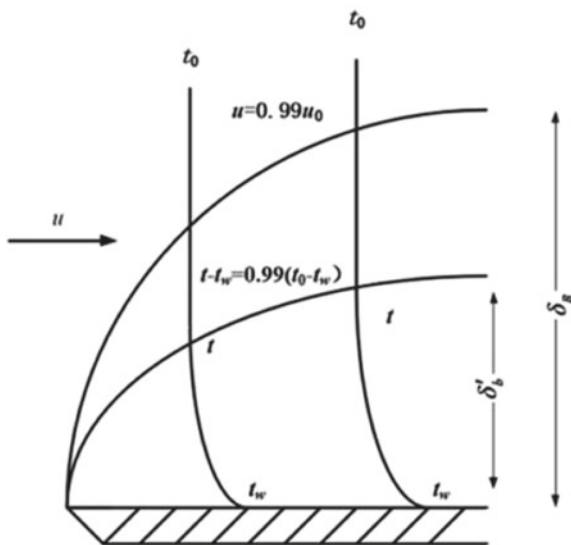
When a fluid flows over a solid wall at a temperature different from its own, a heat transfer process occurs, and a temperature distribution is formed near the wall. The mechanism of heat transfer is not same under different flow states. The following is a preliminary analysis of the convective heat transfer process in which the fluid is forced through the flat wall without phase change.

When cold fluid at a temperature of t_0 flows along a hot solid wall at a temperature of t_w , the wall will transfer heat to the fluid due to the temperature difference between them. When the fluid flows over the wall, due to the action of fluid viscosity, a flow boundary layer is formed near the wall. The different flow conditions in the flow boundary layer determine that the convection heat transfer mechanism between the fluid and the wall is different.

In the case of laminar flow, there is no macroscopic motion of fluid particles between the fluid layers. In the direction perpendicular to the flow direction, heat transfer is carried out through heat conduction. However, because the fluid flow increases the temperature gradient at the wall surface, the heat flow at the wall surface is greater than when at rest; that is, laminar flow enhances the thermal conductivity.

In the turbulent boundary layer, there are three zones, namely, the laminar sublayer, buffer layer and turbulent center. The fluid is in different three flow states. The flow state of the fluid affects the heat transfer and the temperature change near the wall surface, so the temperature distribution curve shown in Fig. 3.8 is formed near the wall surface. In the laminar sublayer near the wall surface, only the flow parallel to the wall surface is available. Heat transfer mainly depends on heat conduction, which conforms to the Fourier's law. The temperature distribution is almost a straight line,

Fig. 3.8 Temperature distribution near solid wall surface



and the slope of the temperature distribution curve is great. In the turbulence center, there is a strong movement of particles in the vertical direction with the flow. Heat transfer mainly depends on thermal convection, and thermal conductivity plays a small role. Therefore, the temperature gradient is small, and the temperature distribution curve tends to be flat. In the buffer layer, the motion of particles perpendicular to the flow direction is weak, and the effect of both convection and heat conduction is roughly equal. Due to the effect of convection heat transfer, the temperature gradient is smaller than that of the laminar sublayer. The random motion of fluid particles in turbulent flows causes the fluid to mix in the y direction of the vertical wall surface, leading to a greatly intensified heat transfer process.

In turbulent heat transfer, the heat transfer process from the main body to the wall surface is a stable series heat transfer process. As mentioned above, in the case of stable heat transfer, the thermal resistance of heat transfer is the sum of the thermal resistance of each layer in series, and the temperature difference is proportional to the thermal resistance. Therefore, the thermal resistance of turbulent heat transfer is concentrated at the laminar sublayer.

In laminar flow, the normal heat transfer along the wall mainly depends on molecular heat transfer, that is, heat conduction. In turbulent flow, the thermal resistance is mainly concentrated in the laminar sublayer near the wall, and the thickness of the laminar sublayer is very thin, so the thermal resistance is much smaller than that of laminar flow, so the turbulent heat transfer rate is much higher than that of laminar flow.

3.3.1.3 Heat Transfer Boundary Layer

Similar to the flow boundary layer, the concept of a heat transfer boundary layer is introduced, and the area near the wall where fluid temperature changes due to heat transfer (that is, the area with a temperature gradient) is called the heat transfer boundary layer. The temperature of the fluid is t_w from the wall to t_0 . The change in temperature is asymptotic, and the temperature is only equal to t_0 at infinity in the normal direction. Therefore, $(t - t_w) = 0.99(t_0 - t_w)$ will be the demarcation of the heat transfer boundary layer, and the distance from the demarcation to the wall surface is called the thickness of the boundary layer (δ'_b). The temperature change outside the boundary layer is very small, so it can be considered that there is no temperature gradient. Therefore, the resistance of the heat transfer process concentrates in the heat transfer boundary layer and mainly depends on the thickness of the heat transfer boundary layer.

The development of the heat transfer boundary layer and the flow boundary layer are usually out of sync. In general, the thicknesses of the two are different, and the thickness relationship depends on the Prandtl number Pr .

$$Pr = \frac{v}{m_t} = \frac{\mu}{\rho} \times \frac{1}{m_t} = \frac{\mu C_p}{\sigma} \quad (3.19)$$

where

Pr —Prandtl number, dimensionless criterion number, represents the characteristic number of physical influence

v —Kinematic viscosity, m^2/s

m_t —Thermal diffusivity, $m_t = \frac{\sigma}{\rho C_p}$, m^2/s

μ —Dynamic viscosity, $\text{Pa} \cdot \text{s}$

ρ —Fluid density, kg/m^3

C_p —Specific heat capacity at constant pressure, or specific heat capacity, the heat absorbed per unit mass of a material by 1K increase in temperature at constant pressure, $\text{J}/(\text{kg} \cdot \text{K})$ or $\text{m}^2/(\text{s}^2 \cdot \text{K})$

σ —Thermal conductivity, $\text{W}/(\text{m} \cdot \text{K})$.

Pr is a dimensionless criterion composed of fluid physical parameters, indicating the ratio of molecular momentum transfer ability to molecular heat transfer ability. v is the kinematic viscosity and is an important physical characteristic that affects velocity distribution and reflects the characteristics of fluid flow. The thermal diffusivity m_t is an important physical property that affects the temperature distribution and reflects the characteristics of heat transfer. The greater the viscosity of the fluid is, the greater the ability of the fluid to transfer momentum and the wider the influence range on the flow rate, that is, the thickening of the flow boundary layer. The larger the thermal diffusivity m_t , the faster the heat transfer is, and the greater the range of temperature

change is, namely, the thickening of the heat transfer boundary layer. Therefore, the dimensionless number composed of the two sets up the relationship between the velocity field and the temperature field, which is an important physical property criterion for studying the convection heat transfer process. For oil, water, gas and liquid metals, the order of magnitude of Pr is $10^2 \sim 10^5$, $1 \sim 10$, $0.7 \sim 1$ and $10^{-3} \sim 10^{-2}$, respectively. The relation between thermal conductivity and thermal diffusivity is as follows: thermal conductivity σ refers to the ability to transfer heat, while thermal diffusivity m_t refers to the ability to characterize the transfer of temperature changes. To put it simply, for example, for a material, the thermal conductivity σ is large, but its specific heat capacity at constant pressure C_p is also great and the temperature change is small after it absorbed heat, then its thermal diffusivity m_t may be small.

The relationship between the flow boundary layer thickness (δ_B) and heat transfer boundary layer thickness (δ'_b) is as follows: when δ_B is greater than or equal to δ'_b , δ_B/δ'_b approximately equals $Pr^{1/3}$. When Pr is equal to 1, δ_B equals δ'_b . Generally, for the flow of a low-viscosity fluid with a small Pr number, $\delta_B < \delta'_b$, because the heat transfer boundary layer is very thick, the thermal resistance distribution throughout the heat transfer process is relatively uniform, and the temperature change in the sublayer with the laminar flow accounts for a small proportion. For a large Pr high-viscosity fluid, $\delta_B > \delta'_b$, the heat transfer boundary layer is very thin, the temperature gradient in the near-wall area is very great, and the temperature change mainly occurs in the laminar sublayer; that is, the thermal resistance is mainly in the laminar sublayer. Understanding the Pr of different fluids is very important for analyzing the heat transfer characteristics between the fluid and the solid wall surface.

3.3.2 Convection Heat Transfer Rate

1) Newton's law of cooling

Although the heat transfer mechanism of a fluid is not same under different circumstances, the convection heat transfer rate can be described by Newton's cooling law; that is, the heat transfer rate through the heat transfer surface is proportional to heat transfer area and the temperature difference between the solid wall surface and the surrounding fluid, and its mathematical expression is:

$$d\Phi = \alpha dA \Delta t \quad (3.20)$$

where

dA —Heat transfer area of the microelement perpendicular to the heat transfer direction, m^2

$d\Phi$ —Convection heat transfer rate through the heat transfer area dA , W or J/s

Δt —Temperature difference between fluid and solid wall surface (When the fluid is cooled, $\Delta t = t - t_w$. When the fluid is heated, $\Delta t = t_w - t$, K)

t_w, t —Temperature of the heat transfer wall surface in contact with the fluid and the fluid, respectively, K

α —Convection heat transfer film coefficient, convection heat transfer coefficient, heat transfer coefficient, heat transfer film coefficient, W/(m² · K).

Newton's law of cooling takes the differential form because during the process of convection heat transfer, the temperature and the convection heat transfer coefficient are affected by the temperature change along the path, so the convection heat transfer coefficient is a local parameter. In practical engineering, the average value is often used for calculation, so Newton's law of cooling can be written as:

$$\Phi = \alpha A \Delta t = \frac{\Delta t}{\frac{1}{\alpha A}} = \frac{\Delta t}{\Omega} \quad (3.21)$$

where

$\Omega = \frac{1}{\alpha A}$ —Heat resistance of convective heat transfer, K/W or (K · s)/J

A —Heat transfer area perpendicular to the heat transfer direction, m²

Φ —Convective heat transfer rate, W or J/s.

2) Convection heat transfer film coefficient

The convection heat transfer film coefficient α is not a properly physical parameter and is related to many factors. Its value depends on the physical properties of the fluid, wall conditions, flow reasons, flow conditions, fluid phase change, and so on and is usually determined by experiments. Generally, for the same fluid, the forced convection heat transfer film coefficient is higher than the natural convection heat transfer film coefficient. The convective heat transfer film coefficient with phase change is higher than that without phase change. Table 3.4 shows the α values in several convection heat transfer cases.

In practical engineering applications, convection heat transfer is often encountered in the flow of fluid in the geometric shape and size—fixed equipment, the fluid heat transfer to the wall or the wall heat transfer to the fluid—so the convection heat transfer film coefficient is related to the following factors:

- (1) Fluid properties: The physical properties of the fluid directly affect convection heat transfer. Main properties are density, specific heat capacity, thermal conductivity, viscosity, etc. In addition, the effect of temperature and pressure on those physical properties need to be considered. In general, the higher the heat capacity at constant pressure of the fluid density is, the higher the specific heat transfer rate between the fluid and the wall surface. The greater the thermal conductivity is, the faster the heat transfer. The greater the viscosity of the fluid, the more unfavorable it is to flow, so it will weaken the heat transfer between the fluid and the wall surface.

Table 3.4 Range of convection heat transfer film coefficients in several conditions (He & Li, 2014)

Heat transfer	α W/(m ² · K)
Natural convection in air	5 ~ 25
Forced convection in gas	20 ~ 100
Natural convection in water	200 ~ 1000
Forced convection in water	1000 ~ 15,000
Steam condensation	5000 ~ 15,000
Organic steam condensation	500 ~ 2000
The water boiling	2500 ~ 25,000

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- (2) Geometric characteristics of the heat transfer surface of the wall: The shape, size and position of the heat transfer surface of the wall will affect convection heat transfer, such as the heat transfer surface of circular pipes, noncircular pipes and other shapes, pipe diameter and pipe length, pipe arrangement, vertical or horizontal placement, etc.
- (3) Flow condition: Heat is transferred between the fluid and the wall surface through heat conduction in laminar flow. In the case of turbulence, the proportion of convection and heat conduction in different regions within the turbulent boundary layer is not same. Generally, turbulent flow promotes the heat transfer process, so the turbulent heat transfer rate is much greater than laminar flow, and the convective heat transfer film coefficient is much higher than that in laminar flow. In addition, the causes of flow (natural convection, forced convection) and convection modes (parallel flow, countercurrent, cross-flow) all affect the convective heat transfer film coefficient.

The fluid velocity of forced convection is higher than that of natural convection, so the heat transfer rate of the former is higher than that of the latter.

If the fluid in the heat transfer produces a phase change (steam condensation, liquid boiling), the influencing factors are more complex. Since the fluid temperature on the phase change side does not change, the heat transfer process always maintains a high temperature gradient, so the heat transfer rate is much greater than that without phase change.

3.3.3 Empirical Equation for the Convection Heat Transfer Film Coefficient

Because there are many factors that influence the convection heat transfer film coefficient, it is difficult to put forward a general equation when considering many factors.

At present, the method commonly used is to adopt the dimensional analysis method to summarize the factors affecting the convection heat transfer film coefficient into several dimensionless groups, which reduces the variable. Then, an experimental method is used to determine the relationship between these parameters under different conditions to identify empirical criteria for calculating the convection heat transfer film coefficient.

3.3.3.1 Dimensionless Number Equation

The research shows that the convection heat transfer film coefficient α is related to the following factors: flow velocity u , heat transfer surface size L (m), fluid viscosity μ (Pa·s), thermal conductivity σ [W/(m·K)], density ρ (kg/m³), specific heat capacity C_p [J/(kg·K)] and $\rho g \beta_g \Delta t$ on the basis of changes in temperature Δt (β_g is the volume expansion coefficient). Therefore, the convection heat transfer film coefficient can be expressed as

$$\alpha = f(u, L, \mu, \sigma, \rho, C_p, \rho g \beta_g \Delta t)$$

Through dimensional analysis, the relationship between physical parameters can be obtained as follows:

$$\frac{\alpha L}{\sigma} = k \left(\frac{Lu\rho}{\mu} \right)^a \left(\frac{\mu C_p}{\sigma} \right)^f \left(\frac{L^3 \rho^2 g \beta_g \Delta t}{\mu^2} \right)^h \quad (3.22a)$$

Equation (3.22) can be written as the relationship expressed by four dimensionless numbers, i.e.,

$$Nu = k Re^a Pr^f Gr^h \quad (3.22b)$$

or

$$Nu = f(Re, Pr, Gr) \quad (3.22c)$$

where

Nu —Nusselt number, dimensionless number, represents the characteristic number of convective heat transfer film coefficient, $Nu = \frac{\alpha L}{\sigma}$

Gr —Grashof number, dimensionless number, represents the characteristic number of natural convection influence, $Gr = L^3 \rho^2 g \beta_g \Delta t$

Pr —Prandtl number, dimensionless number, represents the characteristic number of physical properties, $Pr = \frac{v}{m_t} = \frac{\mu C_p}{\sigma}$

Re —Reynolds number, dimensionless number, represents the characteristic number of influence of fluid flow state, $Re = \frac{Lu\rho}{\mu}$

k, a, f, h —Unknown parameters to be determined experimentally

β_g —Gas expansion coefficient, the inverse of temperature T , K^{-1} . Definition: the ratio of the increased volume to the original volume for every 1 K of temperature increase for a gas.

Assuming the volume of the original gas is V_1 and the volume of the gas is V_2 after the temperature increases 1 °C, then:

$$\beta_g = \frac{V_2 - V_1}{V_1 \Delta T}$$

Derivation the second law of thermodynamics: $pV = nRT$, $p_1 V_1 = nRT_1$, $p_2 V_2 = nRT_2$

$$\text{Thus : } \frac{V_2}{V_1} = \frac{T_2}{T_1}, \text{ subtract 1 from both sides, } \frac{V_2 - V_1}{V_1} = \frac{T_2 - T_1}{T_1}$$

$T_2 - T_1 = \Delta T$, so by plugging in the definition,

$$\beta_g = \frac{V_2 - V_1}{V_1} \cdot \frac{1}{\Delta T} = \frac{T_2 - T_1}{T_1} \cdot \frac{1}{\Delta T} = \frac{\Delta T}{T_1} \cdot \frac{1}{\Delta T} = \frac{1}{T_1}$$

To determine the specific function form under various conditions, it is necessary to conduct relative experiments, sort out the experimental data into the form of dimensionless numbers of Re , Pr and Gr , analyze the relationship between them, determine the unknown parameters k, a, f and h in the function equation, and obtain the empirical correlation equation.

Since the parameters correlation is obtained through experimental data sorting, it must be noted that the heat transfer surface type should be the same, and the dimensionless numbers Re , Pr and Gr should be within the experimental value range and, in principle, cannot be extrapolated. Therefore, the quasi-correlation usually gives the application range of Re , Pr or Gr values.

The dimensionless numbers of Re , Pr and Gr include the parameters of properties, such as μ , σ , ρ and C_p , and the heat transfer surface characteristic size L and flow velocity u . Physical property parameters are related to temperature. Since the temperature distribution along the tangential direction and normal direction of the wall surface is often uneven during the heat transfer process, a representative temperature, namely, the qualitative temperature, is needed to determine the numerical value of physical property parameters during processing experimental data. In principle, the qualitative temperature should be an average value with certain physical significance.

The characteristic size L of the heat transfer surface represents the geometric characteristics of the heat transfer surface, that is, the characteristic size. Usually, a certain geometric size that has a major impact on flow and heat transfer is taken.

Therefore, the characteristic size is often different in systems with different heat transfer types.

The velocity u of the fluid is the characteristic velocity, which is usually taken as the meaningful velocity, such as the average velocity of the cross section fluid when flowing in the pipe.

The dimensionless number corresponds to the qualitative temperature, characteristic size and characteristic velocity, and the equation is changed due to the use of different qualitative temperatures, characteristic sizes and characteristic velocities. Therefore, when using the exact dimensionless number equation, the qualitative temperature, characteristic size and characteristic velocity must be selected strictly in accordance with the provisions of the equation.

Equation (3.22c) represents the general relation of convection heat transfer in the absence of phase change, including both forced convection and natural convection. However, Gr is often negligible in forced convection. For natural convection, Re is not included. In this way, different convection heat transfer situations can be further simplified. Equation (3.22c) can be expressed as:

Forced convection:

$$Nu = f(Re, Pr) \quad (3.23)$$

Natural convection:

$$Nu = f(Gr, Pr) \quad (3.24)$$

The empirical correlations of convection heat transfer film coefficients under several common conditions are given below.

3.3.3.2 Convective Heat Transfer Film Coefficient in the Case of Forced Convection of Fluid

1) The fluid presents laminar flow in the circular straight pipe

Laminar heat transfer in pipes is complex because additional natural convection heat transfer is often affected. The effect of natural convection can only be ignored if the diameter of the pipe and the temperature difference between the fluid and the wall surface are small, i.e., $Gr < 25,000$. In this case, the following dimensionless number equation can be used to calculate the convection heat transfer film coefficient:

$$Nu = 1.86 Re^{1/3} Pr^{1/3} \left(\frac{d_i}{L} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14} \quad (3.25)$$

$$Nu = \frac{\alpha L}{\sigma} = 1.86 Re^{1/3} Pr^{1/3} \left(\frac{d_i}{L} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14} \quad (3.25a)$$

$$\alpha = 1.86 \frac{\sigma}{L} \left(\frac{L u \rho}{\mu} \right)^{1/3} \left(\frac{\mu C_p}{\sigma} \right)^{1/3} \left(\frac{d_i}{L} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14} \quad (3.25b)$$

$$\alpha = 1.86 \frac{\sigma}{d_i} \left(\frac{d_i u \rho}{\mu} \right)^{1/3} \left(\frac{\mu C_p}{\sigma} \right)^{1/3} \left(\frac{d_i}{l} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14} \quad (3.25c)$$

In the above equation, except μ_w , the average fluid temperature was taken for the qualitative temperature, and the inner diameter of the pipe was taken for the characteristic size. Take the viscosity of the fluid at the inner wall temperature for μ_w . The application range is $Re < 2300$, $Pr > 0.6$, $Re \cdot Pr \frac{d_i}{l} > 10$.

When $Gr > 25,000$, α can be calculated according to Eq. (3.25c) and then multiplied by the modified coefficient f :

$$f = 0.8(1 + 0.015 Gr^{1/3}) \quad (3.26)$$

Example 3.5 In the heat exchange system of waste incineration plant A, air with 0.101 MPa pressure flows in a 1 m horizontal pipe with $\Phi 220 \text{ mm} \times 10 \text{ mm}$ at an entrance temperature of 15 °C, outlet temperature of 45 °C, and average air velocity of 0.1 m/s. The pipe inner surface temperature is 150 °C.

Please find the convection heat transfer film coefficient when air flows in the heat exchange pipe.

Known: $d_i = 0.2 \text{ m}$, $d_0 = 0.22 \text{ m}$, $l = 1.0 \text{ m}$, $t_1 = 15 \text{ °C} = 288 \text{ K}$, $t_2 = 45 \text{ °C} = 318 \text{ K}$, $u = 0.1 \text{ m/s}$, $t_w = 150 \text{ °C} = 423 \text{ K}$.

Calculate: convection heat transfer film coefficient α .

Solution: Qualitative temperature is the average temperature of the air in the pipe $t = \frac{1}{2}(288 + 318) = 303 \text{ K} = 30 \text{ °C}$, and the physical parameters of the air at this temperature are as follows:

$C_p = 1.013 \text{ kJ/kg} \cdot \text{K}$, $\sigma = 0.02675 \text{ W/(m} \cdot \text{K)}$.

$\mu = 1.86 \times 10^{-5} \text{ Pa} \cdot \text{s}$, $\rho = 1.165 \text{ kg/m}^3$

$$\beta_g = \frac{1}{T_1} = \frac{1}{303} = 3.33 \times 10^{-3} \text{ K}^{-1}$$

$$Re = \frac{d_i u \rho}{\mu} = \frac{0.2 \times 0.1 \times 1.165}{1.86 \times 10^{-5}} = 1252 < 2300 \text{ (Laminar flow)}$$

$$Pr = \frac{\mu C_p}{\sigma} = \frac{1.86 \times 10^{-5} \times 1.013 \times 10^3}{0.02675} = 0.704 > 0.6$$

$$Gr = \frac{d_i^3 \rho^2 g \beta_g \Delta t}{\mu^2} = \frac{0.2^3 \times 1.165^2 \times 9.81 \times 3.33 \times 10^{-3} \times (150 - 30)}{(1.86 \times 10^{-5})^2}$$

$$= 1.23 \times 10^8 > 25,000$$

$$Re \times Pr \times \frac{d_i}{l} = 1252 \times 0.704 \times \frac{0.2}{1} = 176.3 > 10$$

At 423 K, $\mu_w = 2.41 \times 10^{-5}$ (Pa · s), Re , Pr and $\frac{d_i}{l}$ are within the application range of Eq. (3.25), and thus α_i can be calculated by this equation. Since $Gr > 25,000$, the influence of natural convection should be considered. According to Eq. (3.25), α'_i can be obtained:

$$\begin{aligned}\alpha'_i &= 1.86 \frac{\sigma}{d_i} (Re)^{1/3} (Pr)^{1/3} \left(\frac{d_i}{l} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14} \\ &= 1.86 \times \frac{0.02675}{0.2} \times 1252^{1/3} \times 0.704^{1/3} \times \left(\frac{0.2}{1} \right)^{1/3} \times \left(\frac{1.86 \times 10^{-5}}{2.41 \times 10^{-5}} \right)^{0.14} \\ &= 1.36 \text{ W/(m}^2 \cdot \text{K)}.\end{aligned}$$

$$\begin{aligned}\text{According to the Eq. (3.26) } f &= 0.8(1 + 0.015Gr^{1/3}) \\ &= 0.8[(1 + 0.015 \times (1.23 \times 10^8)^{1/3})] \\ &= 6.76\end{aligned}$$

Therefore, $\alpha = f\alpha'_i = 6.76 \times 1.36 = 9.19 \text{ W/(m}^2 \cdot \text{K)}.$

Answer: The convection heat transfer film coefficient of air flowing in the pipe is $9.19 \text{ W/(m}^2 \cdot \text{K)}.$

2) The fluid presents a strong turbulent flow state in a circular straight pipe

The turbulent flow state is beneficial to heat transfer, so most heat transfer processes in engineering are carried out under turbulent conditions. For fluids with low viscosity (less than 2 times the viscosity of water at room temperature), the following equation is typically used to calculate the convection heat transfer film coefficient:

$$Nu = 0.023 Re^{0.8} Pr^f$$

or

$$\alpha = 0.023 \frac{\sigma}{d_i} \left(\frac{d_i u \rho}{\mu} \right)^{0.8} \left(\frac{\mu C_p}{\sigma} \right)^f \quad (3.27)$$

When the fluid is heated, $f = 0.4$ in the above equation, and $f = 0.3$ when the fluid is cooled. The application conditions and scope of Eq. (3.27) are as follows:

Qualitative temperature: arithmetic mean of fluid inlet and outlet temperature

Characteristic size: inner diameter d_i .

Application: $Re > 10^4$, $0.7 < Pr < 120$, the inner surface of the pipe is smooth, and the ratio of pipe length to pipe diameter $\frac{l}{d_i} > 60$. For $\frac{l}{d_i} < 60$ with respect to

short pipes, it is required to multiply the right side of Eq. (3.27) by the modification coefficient for short pipes φ_L as follows:

$$\varphi_L = 1 + \left(\frac{d_i}{l} \right)^{0.7} \quad (3.28)$$

For high-viscosity fluids, the viscosity is greater than 2 times the viscosity of water at room temperature ($1.0 \times 10^{-3} \text{ Pa} \cdot \text{s}$) and should be calculated by the following modified equation:

$$\alpha = 0.027 \frac{\sigma}{d_i} \left(\frac{d_i u \rho}{\mu} \right)^{0.8} \left(\frac{\mu C_p}{\sigma} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14} \quad (3.29)$$

Except for μ_w , the qualitative temperature and the characteristic size are the same, as shown in Eq. (3.27). For μ_w , take the viscosity of the fluid at the inner wall temperature. Since the wall temperature is unknown, the method of trial and error is generally used to calculate it, which is more troublesome. Therefore, appropriate value of $\left(\frac{\mu}{\mu_w} \right)^{0.14}$ is often adopted in engineering calculation. When the fluid is heated, $\left(\frac{\mu}{\mu_w} \right)^{0.14} \approx 1.05$. When the fluid is cooled, take $\left(\frac{\mu}{\mu_w} \right)^{0.14} \approx 0.95$.

Application range: $Re > 10^4$, $0.7 < Pr < 16$, 700 , $\frac{l}{d_i} > 60$. For $\frac{l}{d_i} < 60$ with respect to short pipes, it is required to multiply the right side of Eq. (3.29) by the modification coefficient for short pipes φ_L calculated according to Eq. (3.28).

Equation (3.27) shows that in the case of turbulence, the convective heat transfer film coefficient is directly proportional to the 0.8 power of the flow velocity and inversely proportional to the 0.2 power of the pipe diameter. Therefore, increasing the flow velocity or using small diameter pipes can enhance heat transfer, especially when the former is more effective. Of course, when taking the above measures, the increased flow resistance in the pipe should be considered.

Example 3.6 There is a horizontal countercurrent tubular heat exchanger with $\Phi 1020 \text{ mm} \times 10 \text{ mm}$ and 3.0 m of length in a waste incinerator A. There are 30 steel pipes with $\Phi 22 \text{ mm} \times 1 \text{ mm}$ inside the exchanger. The cooling water flow velocity in the steel pipe is 1 m/s . The inlet temperature is 20°C , and the outlet temperature is 60°C .

Please determine convection heat transfer film coefficient of the inner wall in the steel pipe on cooling water

Known: $d_i = 0.02 \text{ m}$, $d_o = 0.022 \text{ m}$, $d_w = 1 \text{ m}$, $u_i = 1 \text{ m/s}$, $t_1 = 20^\circ \text{C} = 293 \text{ K}$, $t_2 = 60^\circ \text{C} = 333 \text{ K}$.

Determine: α_i .

Solution: $t_1 = 20^\circ \text{C} = 293 \text{ K}$, $t_2 = 60^\circ \text{C} = 333 \text{ K}$

$$t_{m1} = (t_1 + t_2)/2 = \frac{293 + 333}{2} = 313 \text{ K} = 40^\circ \text{C}$$

The physical property parameters of water at 40 °C can be obtained in Appendix I.

$$C_p = 4174 \text{ J/(kg} \cdot \text{K)}, \rho = 992.2 \text{ kg/m}^3, \mu = 65.60 \times 10^{-5} \text{ Pa} \cdot \text{s}$$

$$\sigma = 63.38 \times 10^{-2} \text{ W/(m} \cdot \text{K)}, Pr = \frac{\mu C_p}{\sigma} = \frac{65.60 \times 10^{-5} \times 4174}{63.38 \times 10^{-2}} = 4.32$$

and $d_i = 22 - 1 \times 2 = 20 \text{ mm} = 0.02 \text{ m}$.

$$\text{Thus: } Re = \frac{d_i u \rho}{\mu} = \frac{0.02 \times 1 \times 992.2}{65.6 \times 10^{-5}} = 30250 > 10^4, \text{ belong to turbulence,}$$

$$\frac{l}{d_i} = \frac{3}{0.02} = 150 > 60$$

The above data and calculation show that, $Re > 10^4$, $120 > Pr > 0.7$, $\frac{l}{d_i} > 60$ and the water is heated, $n = 0.4$. Therefore, the following equation can be used:

$$\begin{aligned} \alpha_i &= 0.023 \frac{\sigma}{d_i} (Re)^{0.8} (Pr)^{0.4} \\ &= 0.023 \times \frac{0.6338}{0.02} \times 30250^{0.8} \times 4.32^{0.4} \\ &= 5028 \text{ W/(m}^2 \cdot \text{K)} \end{aligned}$$

Answer: The convection heat transfer film coefficient of the inner wall of the steel pipe on the cooling water is 5028 W/(m² · K).

(3) The fluid presents a transition flow state in the circular straight pipe

For a transition flow with $Re = 2300 \sim 10,000$, the convection heat transfer film coefficient can be first calculated using the empirical equation of turbulent flow and then calculated α times with a correction coefficient φ (< 1):

$$\varphi = 1 - \frac{6 \times 10^5}{Re^{1.8}} \quad (3.30)$$

3.3.3.3 Convective Heat Transfer Film Coefficient in Natural Convection of Fluid

The convection heat transfer film coefficient of natural convection is only related to the Gr and Pr which reflect natural convection. The correlation equation is as follows:

$$Nu = C(Gr \cdot Pr)^n \quad (3.31)$$

Table 3.5 C and n values and characteristic sizes of natural convection in large space (He & Li, 2014)

Shape and position of heat transfer surface	$Gr \cdot Pr$	C	n	Characteristic size L
A vertical flat or round pipe	$10^4 \sim 10^9$	0.59	1/4	Height H
	$10^9 \sim 10^{13}$	0.10	1/3	
Horizontal circular pipe	$10^4 \sim 10^9$	0.53	1/4	Outside diameter d
	$10^9 \sim 10^{11}$	0.13	1/3	
The horizontal plate faces up hot or down cold	$2 \times 10^4 \sim 8 \times 10^6$	0.54	1/4	Take the average of both sides of the rectangle The disk is $0.9d$ Long strips take short edges
	$8 \times 10^6 \sim 10^{11}$	0.15	1/3	
The horizontal plate faces down hot or up cold	$10^5 \sim 10^{11}$	0.58	1/5	

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$$\alpha = C \frac{\sigma}{L} (Gr \cdot Pr)^n \tag{3.31a}$$

For natural convection in a large space, such as the convection heat transfer between the surface of a pipe or heat exchanger and the surrounding atmosphere, the C , n values and characteristic dimensions (L) are listed in Table 3.5. The arithmetic mean of the wall temperature and atmospheric temperature is taken as the qualitative temperature (He & Li, 2014).

Example 3.7 In the cooling system of waste incinerator A, there is a vertical $\Phi 33 \text{ mm} \times 1.5 \text{ mm}$ steam pipe with a length of 8 m. The outside pipe wall temperature is 95°C , and the ambient temperature is 25°C .

Calculate:

- (1) Heat loss rate per unit pipe length (ignoring heat radiation).
- (2) If the steam pipe is placed horizontally, calculate the heat loss rate per unit pipe length at this time.

Known: $d = 30 \text{ mm} = 0.03 \text{ m}$, $d_0 = 33 \text{ mm} = 0.033 \text{ m}$, $t_w = 95^\circ\text{C} = 368 \text{ K}$, $t_0 = 25^\circ\text{C} = 298 \text{ K}$, $H = 8 \text{ m}$.

Determine: (1) q'_T , (2) q'_T , when placed horizontally.

Solution: (1) This is natural convection heat transfer in a large space. The qualitative temperature $t_m = \frac{1}{2}(t_w + t_0) = \frac{1}{2}(368 + 298) = 333 \text{ K} = 60^\circ\text{C}$, and the air physical parameters at 60°C are as follows:

$$C_p = 1.017 \text{ kJ}/(\text{kg} \cdot \text{K}), \sigma = 0.02896 \text{ W}/(\text{m} \cdot \text{K}).$$

$$\mu = 2.01 \times 10^{-5} \text{ Pa} \cdot \text{s}, \rho = 1.060 \text{ kg}/\text{m}^3$$

$$\beta_g = \frac{1}{t_m} = \frac{1}{333} = 3.00 \times 10^{-3} \text{ K}^{-1}$$

Because the pipeline is placed vertically, its characteristic size $L = H = 8 \text{ m}$

$$\begin{aligned}
 Gr &= \frac{L^3 \rho^2 g \beta_g \Delta t}{\mu^2} = \frac{H^3 \rho^2 g \beta_g \Delta t}{\mu^2} \\
 &= \frac{8^3 \times 1.060^2 \times 9.81 \times 3.00 \times 10^{-3} \times (368 - 298)}{(2.01 \times 10^{-5})^2} = 2.93 \times 10^{12} \\
 Pr &= \frac{\mu C_p}{\sigma} = \frac{2.01 \times 10^{-5} \times 1.017 \times 10^3}{0.02896} = 0.7059 \\
 Gr \cdot Pr &= 2.93 \times 10^{12} \times 0.7059 = 2.07 \times 10^{12}
 \end{aligned}$$

Looking up Table 3.5 gives $C = 0.1$ and $n = 1/3$. Thus,

$$\alpha = C \frac{\sigma}{L} (Gr \cdot Pr)^n = 0.1 \times \frac{0.02896}{8} \times (2.07 \times 10^{12})^{1/3} = 4.61 \text{ W}/(\text{m}^2 \cdot \text{K})$$

Heat loss rate per unit pipe length:

$$\begin{aligned}
 q'_T &= \frac{\Phi}{H} = \frac{\alpha A \Delta t}{H} = \alpha \pi d_0 \Delta t = 4.61 \times \pi \times 0.033 \times (368 - 298) \\
 &= 33.46 \text{ W/m} = 33.46 \text{ J}/(\text{s} \cdot \text{m})
 \end{aligned}$$

(2) When the pipeline is placed horizontally, the characteristic size $L = d_0$

$$\begin{aligned}
 Gr &= \frac{L^3 \rho^2 g \beta_g \Delta t}{\mu^2} = \frac{d_0^3 \rho^2 g \beta_g \Delta t}{\mu^2} \\
 &= \frac{0.033^3 \times 1.060^2 \times 9.81 \times 3.00 \times 10^{-3} \times (368 - 298)}{(2.01 \times 10^{-5})^2} = 2.06 \times 10^5 \\
 Gr \cdot Pr &= 2.0 \times 10^5 \times 0.7059 = 1.45 \times 10^5
 \end{aligned}$$

Looking in Table 3.5, C is equal to 0.53, and n is equal to $1/4$

$$\begin{aligned}
 \alpha &= C \frac{\sigma}{L} (Gr \cdot Pr)^n = C \frac{\sigma}{d_0} (Gr \cdot Pr)^{1/4} = 0.53 \times \frac{0.02896}{0.033} \times (1.45 \times 10^5)^{1/4} \\
 &= 9.08 \text{ W}/(\text{m}^2 \cdot \text{K})
 \end{aligned}$$

At this point, the heat loss rate per unit pipe length is:

$$\begin{aligned}
 q'_T &= \frac{\Phi}{H} = \frac{\alpha A \Delta t}{H} = \alpha \pi d_0 \Delta t = 9.08 \times \pi \times 0.033 \times (368 - 298) \\
 &= 65.87 \text{ W/m} = 65.87 \text{ J}/(\text{s} \cdot \text{m})
 \end{aligned}$$

Answer: The heat loss rate per unit pipe length is $33.46 \text{ J/(s} \cdot \text{m)}$ when it is placed vertically. The heat loss rate per unit pipe length is $65.87 \text{ J/(s} \cdot \text{m)}$, when it is placed horizontally.

3.3.4 Critical Diameter of Insulation Layer

Equipment and pipes operated at or below ambient temperature are often insulated by wrapping insulation materials on the outside. Generally, the heat loss decreases with increasing insulation thickness. However, for small-diameter pipes, the opposite may occur; that is, heat loss increases with insulation thickness. As shown in Fig. 3.9, the internal surface temperature of the insulation layer is set to t_i , and the ambient temperature is t_f . The inner and outer radius of the insulation layer are r_i and r_0 , respectively, and the convection heat transfer film coefficient of the insulation surface facing the environment is α . When heat transfer is stable, according to the principle of superposition of thermal resistance, heat loss rate in the pipeline can be written as:

$$\Phi_l = \frac{t_i - t_f}{\frac{\ln(r_0/r_i)}{2\pi\sigma l} + \frac{1}{2\pi r_0 \alpha l}} = \frac{t_i - t_f}{\Omega_1 + \Omega_2} \quad (3.32)$$

where

Ω_1 —Thermal resistance of conductivity in insulation layer, $\Omega_1 = \frac{\ln(r_0/r_i)}{2\pi\sigma l}$, K/W

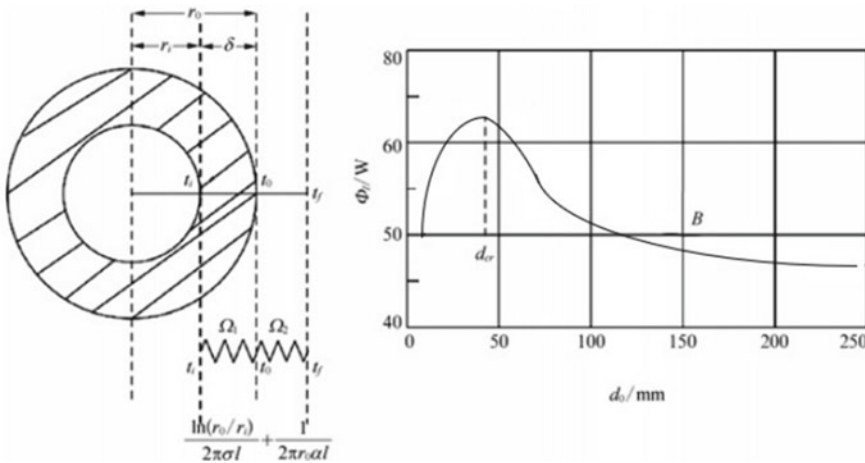


Fig. 3.9 Critical insulation diameter

Ω_2 —Thermal resistance of the convection heat transfer from insulation layer surface to the environment, $\Omega_2 = \frac{1}{2\pi r_0 \alpha l}$, K/W

t_i —Internal surface temperature of insulation layer, K

t_f —Ambient temperature, K

t_0 —External surface temperature of insulation layer, K

σ —Heat conductivity of the insulation material, W/(m · K)

r_i —Radius of inner surface of the insulation layer, m

r_0 —Radius of outer surface of the insulation layer, m

l —Length of pipe, m

Φ_l —Heat flow rate, W or J/s

α —Convection heat transfer film coefficient from insulation layer surface to the environment, W/(m² · K).

As seen from Eq. (3.32), when the insulation layer thickness increases (i.e., r_0 increases), although the thermal resistance of solid conduction increases, the convective thermal resistance of convection heat transfer decreases with increasing surface area. Therefore, whether the heat loss increases or decreases with increasing radius of the outer surface of the insulation layer r_0 depends on whether the sum of the two thermal resistances increases or decreases. Eq. (3.32) is derived on r_0 , and the derivative is equal to zero.

$$\frac{d\Phi_l}{dr_0} = \frac{-2\pi l(t_i - t_f) \left(\frac{1}{\sigma r_0} - \frac{1}{\alpha r_0^2} \right)}{\left(\frac{1}{\sigma} \ln(r_0/r_i) + \frac{1}{\alpha r_0} \right)^2} = 0$$

Therefore, the insulation layer diameter can be obtained when the heat loss rate Φ_l reaches its maximum:

$$d_0 = 2r_0 = \frac{2\sigma}{\alpha} = d_{cr} \quad (3.33)$$

d_{cr} is called the critical diameter of the insulation layer, and $0.5(d_{cr} - d_i)$ is the critical thickness of the insulation layer. If the outer diameter of the insulation layer is less than the critical diameter, that is, $d_0 < d_{cr}$, $d\Phi_l/dr_0$ is a positive value, and heat loss in the pipe will increase with the thickness of the insulation layer.

Example 3.8 In a waste incineration plant, there is a standpipe ($\Phi 45 \text{ mm} \times 2.5 \text{ mm}$) for steam transportation. To reduce the heat loss rate in the process of pipeline transportation, a layer of insulation material is adopted on the outer wall of the pipe. The temperature outside the pipe wall is 300 °C, the thermal conductivity of insulating material is 0.25 W/(m · K), and the convection heat transfer film coefficient of the

outer wall of the insulation layer on air is $10 \text{ W}/(\text{m} \cdot \text{K})$. The influence of the convection heat transfer film coefficient derived by the thickness of insulation layer can be neglected. The ambient air temperature is 25°C .

Calculate:

- (1) Heat loss rate per meter of pipe and the temperature of the outer wall of the insulation layer when the insulation layer thickness is 5, 10 and 15 mm, respectively.
- (2) The thickness of the insulation layer, heat loss rate per meter of pipe length and the temperature of the outer wall of the insulation layer in this case, when the heat loss rate of the insulation layer is at its maximum.
- (3) The thickness of the insulation layer at least if the role of insulation is played.

Known: $\Phi 45 \text{ mm} \times 2.5 \text{ mm}$, $t_i = 300^\circ\text{C} = 573 \text{ K}$, $t_f = 25^\circ\text{C} = 298 \text{ K}$, $\sigma = 0.25 \text{ W}/(\text{m}^2 \cdot \text{K})$, $\alpha = 10 \text{ W}/(\text{m}^2 \cdot \text{K})$

Determine the following:

- (1) $\frac{\Phi_l}{l}$ and t_0 when δ is 5, 10 and 15 mm, respectively
- (2) $\frac{\Phi_l}{l}$ and t_0 , and the thickness of the insulation layer, when the heat loss of the insulation layer is the maximum
- (3) The thickness of the insulation layer be at least if the role of insulation is played.

Solution: $r_i = 45 \div 2 = 22.5 \text{ mm} = 0.0225 \text{ m}$

- (1) According to Eq. (3.32), the heat loss rate per meter of pipe length is

$$\frac{\Phi_l}{l} = \frac{t_i - t_f}{l(\Omega_1 + \Omega_2)} = \frac{t_i - t_f}{\Omega'_1 + \Omega'_2}$$

$$t_i - t_f = 573 - 298 = 275 \text{ K}$$

Let Ω'_1 be the thermal conduction resistance of the insulation layer per meter pipe length and Ω'_2 be the convective heat transfer resistance of the insulation layer facing the environment per meter pipe length.

$$\Omega'_1 = \frac{\ln(r_0/r_i)}{2\pi\sigma} = \frac{\ln(r_0/0.0225)}{2\pi \times 0.25}$$

$$\Omega'_2 = \frac{1}{2\pi r_0 \alpha} = \frac{1}{2\pi r_0 \times 10}$$

Let the temperature of the outer surface of the insulation layer be t_0 . When heat transfer is stable, the heat transfer rate of each layer is equal, i.e.,

$$\frac{\Phi_l}{l} = \frac{t_i - t_f}{\Omega'_1 + \Omega'_2} = \frac{t_0 - t_f}{\Omega'_2}$$

Therefore, when the insulation layer thickness is 5 mm and $r_0 = 0.0275$ m, then

$$\Omega'_1 = \frac{\ln\left(\frac{0.0275}{0.0225}\right)}{2\pi \times 0.25} = 0.128 \text{ (K} \cdot \text{m)/W}$$

$$\Omega'_2 = \frac{1}{2\pi \times 0.0275 \times 10} = 0.579 \text{ (K} \cdot \text{m)/W}$$

$$\frac{\Phi_l}{l} = \frac{275}{0.128 + 0.579} = 388.96 \text{ W/m}$$

$$t_0 = t_f + \frac{\Phi_l}{l} \times \Omega'_2 = 298 + 388.96 \times 0.579 = 523.21 \text{ K} = 250.21^\circ\text{C}$$

Similarly, when the insulation thickness is 10 mm and $r_0 = 0.0325$ m, there is

$$\Omega'_1 = \frac{\ln\left(\frac{0.0325}{0.0225}\right)}{2\pi \times 0.25} = 0.234 \text{ (K} \cdot \text{m)/W}$$

$$\Omega'_2 = \frac{1}{2\pi \times 0.0325 \times 10} = 0.490 \text{ (K} \cdot \text{m)/W}$$

$$\frac{\Phi_l}{l} = \frac{275}{0.234 + 0.490} = 379.83 \text{ W/m}$$

$$t_0 = t_f + \frac{\Phi_l}{l} \times \Omega'_2 = 298 + 379.83 \times 0.490 = 484.12 \text{ K} = 211.12^\circ\text{C}$$

When the thickness of the insulation layer is 15 mm and $r_0 = 0.0375$ m, there is

$$\Omega'_1 = \frac{\ln\left(\frac{0.0375}{0.0225}\right)}{2\pi \times 0.25} = 0.325 \text{ (K} \cdot \text{m)/W}$$

$$\Omega'_2 = \frac{1}{2\pi \times 0.0375 \times 10} = 0.424 \text{ (K} \cdot \text{m)/W}$$

$$\frac{\Phi_l}{l} = \frac{275}{0.325 + 0.424} = 367.16 \text{ W/m}$$

$$t_0 = t_f + \frac{\Phi_l}{l} \times \Omega'_2 = 298 + 367.16 \times 0.424 = 453.67 \text{ K} = 180.67^\circ\text{C}$$

When the insulation layer is 5 ~ 15 mm, the heat loss rate decreases with increasing thickness.

- (2) When the heat loss rate reaches the maximum, the insulation layer diameter is the critical diameter, i.e.,

$$d_0 = d_{cr} = \frac{2\sigma}{\alpha} = \frac{2 \times 0.25}{10} = 0.05 \text{ m}$$

The critical thickness of the insulation layer is

$$\frac{d_{cr} - d_i}{2} = \frac{0.05 - 0.045}{2} = 0.0025 \text{ m}$$

At this point, $r_0 = 0.025 \text{ m}$; then,

$$\Omega'_1 = \frac{\ln\left(\frac{0.025}{0.0225}\right)}{2\pi \times 0.25} = 0.067 \text{ (K} \cdot \text{m)/W}$$

$$\Omega'_2 = \frac{1}{2\pi \times 0.025 \times 10} = 0.637 \text{ (K} \cdot \text{m)/W}$$

$$\frac{\Phi_l}{l} = \frac{275}{0.067 + 0.637} = 390.6 \text{ W/m}$$

$$t_0 = t_f + \frac{\Phi_l}{l} \times \Omega'_2 = 298 + 390.6 \times 0.637 = 546.66 \text{ K} = 273.66^\circ\text{C}$$

- (3) Heat loss rate should be less than that of bare pipe after adding an insulation layer to play an insulation role. Therefore, the outer diameter of the insulation layer should be satisfied:

$$\frac{\Phi_l}{l} = \frac{t_i - t_f}{\Omega'_1 + \Omega'_2} = \frac{t_i - t_f}{\frac{1}{2\pi r_i \alpha}}$$

namely,

$$\frac{\ln(r_0/r_i)}{2\pi\sigma} + \frac{1}{2\pi r_0 \alpha} = \frac{1}{2\pi r_i \alpha}$$

$$\frac{\ln(r_0/0.0225)}{2\pi \times 0.25} + \frac{1}{2\pi r_0 \times 10} = \frac{1}{2\pi \times 0.0225 \times 10}$$

The solution is $r_0 = 0.0279 \text{ m}$, and then the thickness of the insulation layer should be greater than

$$0.0279 - 0.0225 = 0.0054 \text{ m}$$

The insulation layer thickness of the steel pipe with an outer diameter of 45 mm shall be greater than 5.4 mm to play an insulation role.

Answer: When the insulation layer thicknesses are 5 mm, 10 mm and 15 mm, the heat loss rate per meter of pipe is 388.96 W/m, 379.83 W/m, and 367.16 W/m, and the surface temperatures are 250.21 °C, 211.12 °C and 180.67 °C, respectively. When the insulation layer thickness is 0.0025 m, heat loss reaches its maximum, the heat loss per meter of pipe is 390.6 W/m, and the surface temperature is 273.66 °C. To play the role of insulation, the thickness of the insulation layer should be at least 0.0054 m.

3.4 Calculation of Heat Transfer Process

There are two main types of calculations of the heat transfer process in engineering: one is design calculation, that is, according to the requirements of the production process, determining the heat transfer area of the heat exchanger and other relevant dimensions of the heat exchanger to design or choose the heat exchanger. The other is to check the calculation, that is, determining whether a heat exchanger meets the requirements of the production process or predicting the influence of changes in some parameters in the production process on the heat transfer capacity of the heat exchanger. In this section, the total heat transfer rate equation of the whole heat transfer process is derived based on the heat balance calculation of the heat transfer process, combined with the heat transfer of the interwall, on the basis of the heat conduction rate equation and the convection heat transfer rate equation, and it is used to solve the actual heat transfer problem in the production process.

3.4.1 Heat Balance

The heat balance of the interwall heat exchanger is calculated. Assuming that the heat exchanger is well insulated and the heat loss is ignored, the heat released by the hot fluid per unit time is equal to the heat absorbed by the cold fluid.

For the micro element segment of the heat exchanger, the heat balance equation is:

$$d\Phi_e = -q_{mh}d\mathcal{H}_h = q_{mc}d\mathcal{H}_c \quad (3.34)$$

For the whole heat exchanger, the heat balance equation is:

$$\Phi_e = -q_{mh}(\mathcal{H}_{h1} - \mathcal{H}_{h2}) = q_{mc}(\mathcal{H}_{c1} - \mathcal{H}_{c2}) \quad (3.35)$$

If there is no phase change in the fluid in the heat exchanger, the specific heat capacity at the average temperature is adopted. The above two equations can be

expressed as:

$$d\Phi_e = -q_{mh}C_{ph}dT = q_{mc}C_{pc}dt \quad (3.34a)$$

$$\Phi_e = q_{mh}C_{ph}(T_1 - T_2) = q_{mc}C_{pc}(t_2 - t_1) \quad (3.35a)$$

If the heat in the heat exchanger comes from the condensation of saturated steam, the balance equation is:

$$\Phi_e = q'_{mh}r_e = q_{mc}C_{pc}(t_2 - t_1) \quad (3.36)$$

If the steam condenses and cools before it flows out of the heat exchanger, the heat balance equation is:

$$\Phi_e = q'_{mh}[r_e + C_{ph}(t_s - T_2)] = q_{mc}C_{pc}(t_2 - t_1) \quad (3.37)$$

The meaning of the symbols in Eqs. (3.34) ~ (3.47) are as follows:

Φ_e —The heat load of a heat exchanger, namely, the heat transfer quantity per unit time, the heat transfer rate, $d\Phi_e$ is its derivative, W or J/s

\mathcal{H} —Enthalpy of a fluid; $d\mathcal{H}$ is its differential denivative, J/kg

q_m —Mass flow rate of a fluid, kg/s

q'_m —Mass flow rate of a condensation steam, kg/s

C_p —Specific heat capacity at the average temperature of a fluid, J/(kg·K)

r_e —Condensation heat of saturated steam, J/kg

T and t —Temperatures of hot and cold fluids, respectively, K

t_s —Saturation temperature of a condensate, K.

Subscripts h and c represent hot and cold fluids, respectively, and subscripts 1 and 2 represent the inlet and outlet of the heat exchanger, respectively.

3.4.2 Total Heat Transfer Rate Equation

Interwall heat exchangers are widely used in environmental engineering. In such heat exchangers, the heat transfer process from hot to cold fluids through an interwall is shown in Fig. 3.10.

The process which hot fluid transfers heat to cold fluid through an interwall is divided into following three steps: (1) Heat is transferred from the hot fluid to the surface of the interwall. (2) Heat from the hot side of the interwall is transferred to its cold side. (3) Heat is transferred from the surface of its cold side of the interwall to the cold fluid.

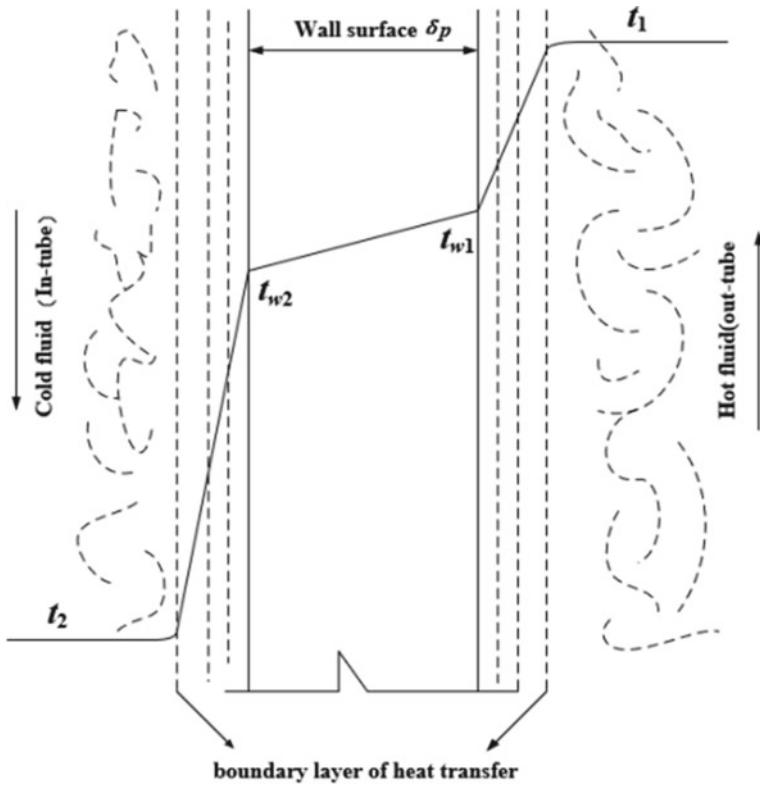


Fig. 3.10 Heat transfer process from hot to cold fluids through an interwall

The second step of interwall heat transfer is heat conduction, and both the first step and the third step belong to heat transfer between the fluid and solid wall, which mainly relies on convection heat transfer.

According to Newton's law of cooling, the convection heat transfer rate from hot fluid to the surface of an interwall is:

$$\Phi_1 = \alpha_1 A_1 (t_1 - t_{w1})$$

$$t_1 - t_{w1} = \frac{\Phi_1}{\alpha_1 A_1} \quad (3.38)$$

According to the fundamental law of solid heat conduction of Fourier's law, the heat conduction rate of the interwall is:

$$\Phi_2 = \frac{A_m (t_{w1} - t_{w2})}{\delta_p / \sigma}$$

or

$$t_{w1} - t_{w2} = \frac{\Phi_2}{\sigma A_m / \delta_p} \quad (3.39)$$

According to Newton's law of cooling, the convection heat transfer rate from the surface of the interwall to the cold fluid is:

$$\Phi_3 = \alpha_2 A_2 (t_{w2} - t_2)$$

or

$$t_{w2} - t_2 = \frac{\Phi_3}{\alpha_2 A_2} \quad (3.40)$$

where

t_1 —Main body temperature of the hot fluid, K

t_{w1} —Temperature of the interwall surface in the hot fluid side, K

t_{w2} —Temperature of the interwall surface in the cold fluid side, K

t_2 —Main body temperature of the cold fluid, K

A_1 —Heat transfer area on the hot fluid side of the interwall, m^2

A_m —Average heat transfer area of the interwall, m^2

A_2 —Heat transfer area on the cold fluid side of the interwall, m^2

α_1 —Convection heat transfer film coefficient of hot fluid, $\text{W}/(\text{m}^2 \cdot \text{K})$

δ_p —Interwall thickness, m

σ —Thermal conductivity of the interwall, $\text{W}/(\text{m} \cdot \text{K})$

α_2 —Convection heat transfer film coefficient of cold fluid, $\text{W}/(\text{m}^2 \cdot \text{K})$.

Add Eqs. (3.38), (3.39) and (3.40) above:

$$\Phi_1 \left(\frac{1}{\alpha_1 A_1} \right) + \Phi_2 \left(\frac{\delta_p}{\sigma A_m} \right) + \Phi_3 \left(\frac{1}{\alpha_2 A_2} \right) = (t_1 - t_{w1}) + (t_{w1} - t_{w2}) + (t_{w2} - t_2)$$

Under the condition of stable heat transfer, the overall heat transfer rate of the heat exchanger Φ is equal to the heat transfer rate of each layer, i.e. $\Phi = \Phi_1 = \Phi_2 = \Phi_3$, so

$$\Phi = \frac{(t_1 - t_{w1}) + (t_{w1} - t_{w2}) + (t_{w2} - t_2)}{\frac{1}{\alpha_1 A_1} + \frac{\delta_p}{\sigma A_m} + \frac{1}{\alpha_2 A_2}}$$

$$\begin{aligned}
&= \frac{t_1 - t_2}{\frac{1}{\alpha_1 A_1} + \frac{\delta_p}{\sigma A_m} + \frac{1}{\alpha_2 A_2}} \\
&= \frac{\Delta t}{\Omega} = \frac{\text{driving force of heat transfer}}{\text{thermal resistance}} \quad (3.41)
\end{aligned}$$

make

$$\Omega = \frac{1}{KA} = \frac{1}{\alpha_1 A_1} + \frac{\delta_p}{\sigma A_m} + \frac{1}{\alpha_2 A_2} \quad (3.42)$$

Then, Eq. (3.41) becomes

$$\Phi = KA(t_1 - t_2) = KA\Delta t \quad (3.43)$$

Equation (3.43) is called the overall heat transfer rate equation or the heat transfer rate equation, where K is the overall heat transfer coefficient, abbreviated as the heat transfer coefficient [$\text{W}/(\text{m}^2 \cdot \text{K})$]. A is the heat transfer area, and $(t_1 - t_2)$ is the overall heat transfer temperature difference.

3.4.3 Overall Heat Transfer Coefficient

The overall heat transfer coefficient is an important parameter reflecting the heat transfer capacity of a heat transfer equipment and the basis for the heat transfer calculation of a heat transfer equipment. Whether evaluating the performance of a heat exchanger or the design of a heat exchanger, the calculation of the K value is the most basic requirement. The value of K depends on the physical properties of the fluid, the operating conditions of the heat transfer process and the type of a heat exchanger.

The overall heat transfer coefficient K corresponds to the heat transfer area selected. In general, the outside surface area of the interwall is selected to be the reference value for the K calculation. K satisfies the following equation:

$$\Omega = \frac{1}{KA_1} = \frac{1}{\alpha_1 A_1} + \frac{\delta_p}{\sigma A_m} + \frac{1}{\alpha_2 A_2}$$

thus

$$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{\delta_p A_1}{\sigma A_m} + \frac{A_1}{\alpha_2 A_2} \quad (3.44)$$

The physical significance of K is the heat quantity transferred per the interwall area and per unit of time when the temperature difference between the fluid bodies

on both sides of the interwall is $1/K$. Its unit is $W/(m^2 \cdot K)$, the same unit of convection heat transfer film coefficient α .

For the pipe wall, Eq. (3.44) can be written as

$$\frac{1}{K_0} = \frac{1}{\alpha} \frac{d_0}{d_i} + \frac{\delta_p}{\sigma} \frac{d_0}{d_m} + \frac{1}{\alpha_0} \quad (3.45)$$

where

d_i, d_o, d_m —Inner, outer and average diameter of the pipe wall, respectively, m

K_0 —Overall heat transfer coefficient calculated according to the outer surface (or d_o), $W/(m^2 \cdot K)$.

For the flat wall or the thin wall pipe, $A_1 = A_m = A_2$. The following equation can be obtained from Eq. (3.44):

$$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{\delta_p}{\sigma} + \frac{1}{\alpha_2} \quad (3.46)$$

where

α_1, α_2 —Convective heat transfer film coefficients of the outer and inner sides of the flat wall, respectively, $W/(m^2 \cdot K)$.

After the heat exchanger runs for a period of time, there will be fouling on the heat transfer surface, and there will be additional heating resistance to heat transfer, which is called as a fouling thermal resistance. Although the thickness of the fouling is relatively thin, its thermal conductivity is much smaller than that of the pipe wall material, and the fouling thermal resistance is often much greater than that of the interwall. Therefore, the influence of fouling thermal resistance should be considered in heat transfer calculations. There are many factors influencing the formation of the fouling, including the properties of the fluid material, characteristics of heat transfer wall materials, operating conditions, equipment structure, cleaning cycle time and so on.

For a wall, with the hot fluid in the outer wall and cold fluid in the inner wall, the fouling thermal resistance per unit heat transfer area on the outer and inner surfaces is Ω_{s1} , and Ω_{s2} , respectively. According to the superposition principle of series thermal resistance, Eq. (3.44) can be expressed as

$$\frac{1}{K} = \frac{1}{\alpha_1} + \Omega_{s1} + \frac{\delta_p A_1}{\sigma A_m} + \Omega_{s2} \frac{A_1}{A_2} + \frac{A_1}{\alpha_2 A_2} \quad (3.47)$$

Equation (3.47) shows that the overall thermal resistance between fluids on both sides of the wall is equal to the sum of the fouling thermal resistance and convection heat transfer resistance of fluids on both sides of the wall, as well as heat conduction resistance of the wall.

For a flat wall or thin wall pipe, there is

$$\frac{1}{K} = \frac{1}{\alpha_1} + \Omega_{s1} + \frac{\delta_p}{\sigma} + \Omega_{s2} + \frac{1}{\alpha_2} \quad (3.48)$$

When the thermal resistance of the interwall and fouling can be ignored, Eq. (3.48) can be simplified as

$$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \quad (3.49)$$

If $\alpha_2 \gg \alpha_1$ and $\frac{1}{K} \approx \frac{1}{\alpha_1}$, the overall thermal resistance is controlled by the convective heat transfer outside the interwall. To improve the K value, the key is to improve the convective heat transfer film coefficient outside the interwall. If $\alpha_2 \ll \alpha_1$ and $\frac{1}{K} \approx \frac{1}{\alpha_2}$, the overall thermal resistance is controlled by the convection heat transfer inside the interwall. To improve the K value, the key is to improve the convection heat transfer film coefficient inside the interwall. Similarly, if the fouling thermal resistance is great, it is controlled by the fouling thermal resistance. At this time, to increase the K value, it is necessary to remove fouling or slow down the fouling formation rate.

The general range of the heat transfer coefficient and fouling thermal resistance of industrial heat exchangers are listed in Tables 3.6 and 3.7.

3.4.4 The Mean Temperature Difference of Heat Transfer

In the heat transfer process of a heat exchanger, if the fluid temperature on both sides of the wall remains constant, it is called constant temperature heat transfer. For example, one side of the heat transfer wall is heated by saturated steam (temperature t_1). On the other side is the boiling liquid temperature (t_2), where the temperature of both fluids remains constant. At this time, the heat transfer temperature difference is constant, which can be simply expressed as

$$\Delta t = t_1 - t_2$$

Generally, the temperature of the cold and hot fluid in the heat exchanger changes along the flow direction, and the heat transfer temperature difference also varies along the path, which is called variable temperature heat transfer. Since the heat transfer temperature difference is variable, the heat transfer rate Eq. (3.43) should be written as

$$\Phi = K A \Delta t_m \quad (3.50)$$

Table 3.6 Overall heat transfer coefficient K of the tubular heat exchanger (He & Li, 2014)

Cold fluid	Thermal fluid	K W/(m ² ·K)
Water	Water	850 ~ 1700
Water	Gas	17 ~ 280
Water	Organic solvent	280 ~ 850
Water	Light oil	340 ~ 910
Water	Heavy oil	60 ~ 280
Organic solvent	Organic solvent	115 ~ 340
Water	Condensation of water vapor	1420 ~ 4250
Gas	Condensation of water vapor	30 ~ 300
Water	Condensation of low boiling Hydrocarbons	455 ~ 1140
Water's boiling	Condensation of water vapor	2000 ~ 4250
The boiling of light oil	Condensation of water vapor	455 ~ 1020

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Table 3.7 Fouling thermal resistance Ω_S of common fluids (Hu et al., 2015)

Fluid	Ω_S (m ² ·K)/kW	Fluid	Ω_S (m ² ·K)/kW
Water (1 m/s, $t > 50$ °C)		Liquid	
Distilled water	0.09	Treated brine	0.264
The sea	0.09	Organic matter	0.176
Clean water	0.21	Fuel oil	1.056
Untreated cooling water	0.58	Tar	1.76
Treated cooling water	0.26	Water vapor	
Treated boiler water	0.26	High quality (no oil)	0.052
Well water, hard water	0.58	Poor quality (no oil)	0.09
Gas		Discharge from the regenerator	0.176
Air	0.26 ~ 0.53		
Vapor	0.14		

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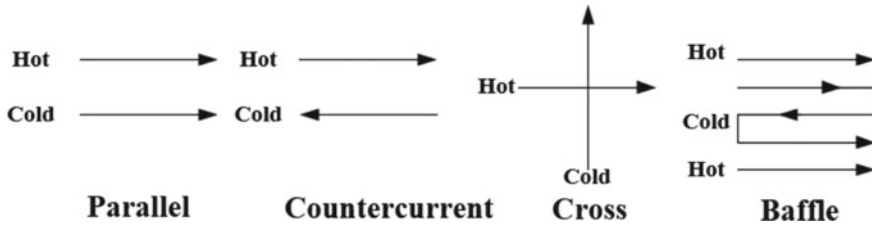


Fig. 3.11 Schematic diagram of the flow form of the cold and hot fluids

where

Δt_m —Mean temperature difference, K.

According to the different flow directions between cold and hot fluids, they can be divided into different flow forms, as shown in Fig. 3.11. When they are parallel and flow in the same direction, it is called parallel flow. When they are parallel but flow in opposite directions, this is called countercurrent flow. Vertical cross flow is called cross flow. When a fluid flows only in one direction, while another fluid is repeatedly deflected, this is called a baffle flow. A combination of several flow forms is called a complex flow.

1) Mean temperature difference of parallel or countercurrent flow

Parallel and countercurrent are the most common forms of flow in industry. Figure 3.12 shows the variety in the temperature difference along the heat transfer wall surface in parallel and countercurrent flow. T is the temperature of the hot fluid, and t is the temperature of the cold fluid. Δt_1 , Δt_2 represents the temperature difference between the two ends of the heat exchanger. Take the logarithmic mean of the inlet and outlet temperature difference, i.e.,

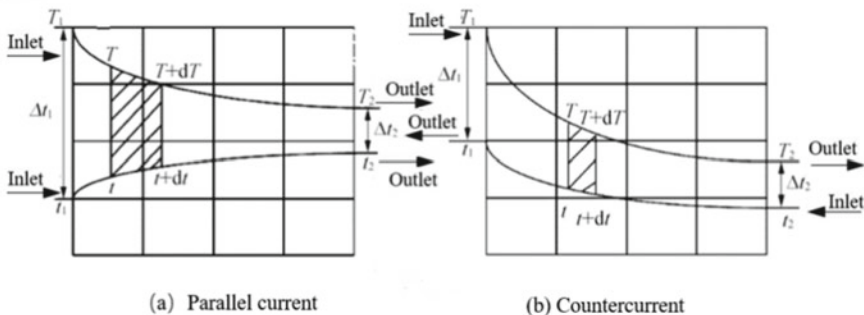


Fig. 3.12 Temperature variation of fluid on both sides of the surface of a pipe in a heat exchanger

$$\begin{aligned}\Delta t_m &= \frac{\Delta t_1 - \Delta t_2}{\ln \frac{\Delta t_1}{\Delta t_2}} \\ &= [(T_1 - t_1) - (T_2 - t_2)] / \ln[(T_1 - t_1)/(T_2 - t_2)]\end{aligned}\quad (3.51)$$

If $\Delta t_1/\Delta t_2$ is less than or equal to 2, Δt_m can be replaced by the arithmetic mean temperature difference:

$$\Delta t_m = \frac{\Delta t_1 + \Delta t_2}{2} \quad (3.52)$$

In the casing heat exchanger shown in Fig. 3.12b, the cold and hot fluids flow in a countercurrent, and the fluid temperature varies along the path. The inlet and outlet temperatures of the hot fluids are T_1 and T_2 , and the inlet and outlet temperatures of the cold fluid are t_1 and t_2 , respectively. Hypotheses: (1) For a heat exchanger in stable operation, the mass flow rates of hot and cold fluid, q_{mh} and q_{mc} , along the heat transfer surface are constant. (2) The specific heat capacity of hot and cold fluid, C_{ph} and C_{pc} , and the overall heat transfer coefficient K remain constant along the heat transfer surface. (3) The heat exchanger has no heat loss.

For the heat transfer situation of the microelement heat transfer area of dA , when the temperature of the hot fluid flow in the cell surface is T and the temperature of the cold fluid is t , the temperature difference between the two is $\Delta t = T - t$.

According to the heat transfer rate Eq. (3.43), the heat transfer through the microelement surface dA is

$$d\Phi = K(T - t)dA = K\Delta t dA \quad (3.53a)$$

The temperature of the hot fluid decreases by dT after it releases heat $d\Phi$ in the microelement surface dA . The cold fluid receives heat $d\Phi$, and the temperature increases dt . According to the flow direction of the hot fluid, the temperature of both fluids decreases after dA . The heat balance of hot and cold fluids can be obtained

$$d\Phi = -q_{mh}C_{ph}dT, dT = -\frac{d\Phi}{q_{mh}C_{ph}} \quad (3.53b)$$

$$d\Phi = -q_{mc}C_{pc}dt, dt = -\frac{d\Phi}{q_{mc}C_{pc}} \quad (3.53c)$$

If Eq. (3.53c) is subtracted from (3.53b), the following equation will be obtained:

$$dT - dt = d(T - t) = d(\Delta t) = \left(\frac{1}{q_{mc}C_{pc}} - \frac{1}{q_{mh}C_{ph}} \right) d\Phi \quad (3.53d)$$

Substitute Eq. (3.53a) into Eq. (3.53d) to obtain

$$d(\Delta t) = \left(\frac{1}{q_{mc}C_{pc}} - \frac{1}{q_{mh}C_{ph}} \right) K \Delta t dA \quad (3.53e)$$

or

$$\frac{d(\Delta t)}{\Delta t} = \left(\frac{1}{q_{mc}C_{pc}} - \frac{1}{q_{mh}C_{ph}} \right) K dA \quad (3.53f)$$

Substitute the boundary conditions: when $A = 0$, $\Delta t = \Delta t_1 = T_1 - t_1$. When $A = A$, $\Delta t = \Delta t_2 = T_2 - t_2$, the integral (3.53f) can be obtained:

$$\begin{aligned} \int_{\Delta t_1}^{\Delta t_2} \frac{d(T - t)}{T - t} &= \int_0^A \left(\frac{1}{q_{mc}C_{pc}} - \frac{1}{q_{mh}C_{ph}} \right) K dA \\ \ln \frac{\Delta t_2}{\Delta t_1} &= \left(\frac{1}{q_{mc}C_{pc}} - \frac{1}{q_{mh}C_{ph}} \right) K A \end{aligned} \quad (3.53g)$$

The heat flow rate through heat transfer surface A is Φ

$$\Phi = q_{mc}C_{pc}(t_1 - t_2) = q_{mh}C_{ph}(T_1 - T_2)$$

It can be obtained from the above equation that

$$\begin{aligned} \frac{1}{q_{mc}C_{pc}} &= \frac{t_1 - t_2}{\Phi} \quad \frac{1}{q_{mh}C_{ph}} = \frac{T_1 - T_2}{\Phi} \\ \frac{1}{q_{mc}C_{pc}} - \frac{1}{q_{mh}C_{ph}} &= \frac{1}{\Phi} [(t_1 - t_2) - (T_1 - T_2)] \\ &= \frac{1}{\Phi} [(T_2 - t_2) - (T_1 - t_1)] = \frac{1}{\Phi} (\Delta t_2 - \Delta t_1) \end{aligned} \quad (3.53h)$$

Substitute Eq. (3.53h) into Eq. (3.53g) to obtain:

$$\ln \frac{\Delta t_2}{\Delta t_1} = K \frac{1}{\Phi} (\Delta t_2 - \Delta t_1) A$$

or

$$\Phi = K A \frac{\Delta t_2 - \Delta t_1}{\ln \frac{\Delta t_2}{\Delta t_1}} = K A \frac{\Delta t_1 - \Delta t_2}{\ln \frac{\Delta t_1}{\Delta t_2}} \quad (3.53i)$$

By comparing Eqs. (3.53i) to (3.50), (3.51) can be obtained.

The above is derived as Δt_m . This is called the logarithmic mean temperature difference. If the two fluids in the heat exchanger are parallel flows, the same result can be derived as that in Eq. (3.51). Therefore, this equation is a general equation for calculating the logarithmic mean temperature difference under the condition of countercurrent or parallel flows.

To apply Eq. (3.51) conveniently, the higher value of the temperature difference between the two ends of the heat exchanger Δt is written as Δt_1 , and the smaller value is written as Δt_2 .

2) Mean temperature difference of baffle or cross flow

The mean temperature difference between baffle and cross flow can be calculated first according to the countercurrent flow and then multiplied by the correction coefficient ϕ . Figure 3.13 shows the correction coefficient curve of a heat exchanger with one-way shell and two-way pipe.

$$\Delta t_m = \phi \frac{\Delta t_1 - \Delta t_2}{\ln \frac{\Delta t_1}{\Delta t_2}} \quad (3.54)$$

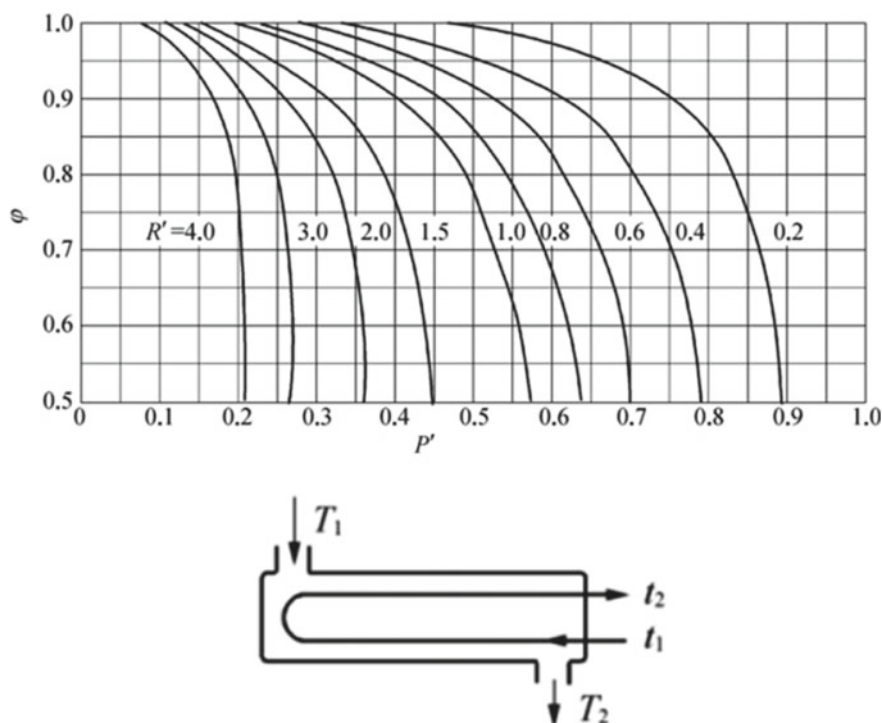


Fig. 3.13 Correction coefficient curve of a heat exchanger with one-way shell and two-way pipe (Lin, 1999)

The correction coefficient under various flow conditions can be determined by referring to Fig. 3.13 for the R' and P' parameters.

$$R' = \frac{\text{temperature decrease of hot fluid}}{\text{temperature increase of cool fluid}} = \frac{T_1 - T_2}{t_2 - t_1}$$

$$P' = \frac{\text{temperature increase of cool fluid}}{\text{Initial temperature difference between two fluids}} = \frac{t_2 - t_1}{T_1 - t_1}$$

3.4.5 Examples of Heat Transfer Calculation

3.4.5.1 Calculation of the Mean Temperature Difference

Example 3.9 A cooling system of an incinerator with a $\Phi 320 \text{ mm} \times 10 \text{ mm}$ pipe heat exchanger will be used to decrease the temperature of hot water. In the design, the heat exchanger needs to cool 5000 t of water per hour from 100 °C to 60 °C. Under running conditions, the temperatures of import and export of cooling water are 20 °C and 40 °C, respectively. The overall heat transfer coefficient of the heat exchanger is known to be 3000 W/(m² · K).

Calculate:

- (1) The flow rate of cooling water.
- (2) When the flow state of cooling water and hot water is parallel and in a countercurrent, respectively, the pipe length of the heat exchanger.

Known: $\Phi 320 \text{ mm} \times 10 \text{ mm}$, $q_{mh} = 5000 \text{ t/h} = 13.89 \text{ kg/s}$, $T_1 = 100 \text{ °C} = 373 \text{ K}$, $T_2 = 60 \text{ °C} = 333 \text{ K}$, $t_1 = 20 \text{ °C} = 293 \text{ K}$, $t_2 = 40 \text{ °C} = 313 \text{ K}$, $K = 3000 \text{ W}/(\text{m}^2 \cdot \text{K})$.

Determine: q_{mc} and l of the parallel flow and the countercurrent

Solution: (1)

$$T_m = \frac{T_1 + T_2}{2} = \frac{373 + 333}{2} = 353 \text{ K} = 80 \text{ °C}$$

$$t_m = \frac{t_1 + t_2}{2} = \frac{293 + 313}{2} = 303 \text{ K} = 30 \text{ °C}$$

According to the Appendix A:

When $t_m = 30 \text{ °C}$, $C_{pc} = 4174 \text{ J}/(\text{kg} \cdot \text{K})$

When $T_m = 80 \text{ °C}$, $C_{ph} = 4195 \text{ J}/(\text{kg} \cdot \text{K})$.

Heat balance: $q_{mc} C_{pc} (t_2 - t_1) = -q_{mh} C_{ph} (T_2 - T_1) = \Phi$

$$q_{mc} \times 4174 \times (313 - 293) = -13.89 \times 4195 \times (333 - 373)$$

Then: $q_{mc} = 27.92 \text{ kg/s}$.

(2) Length of pipe required for parallel flow and countercurrent flow of two fluids

Countercurrent flow:

$$\begin{aligned} 100\text{ }^{\circ}\text{C} &\longrightarrow 60\text{ }^{\circ}\text{C} \\ 20\text{ }^{\circ}\text{C} &\longleftarrow 40\text{ }^{\circ}\text{C} \end{aligned}$$

Or

$$\begin{aligned} 373\text{ K} &\longrightarrow 333\text{ K} \\ 313\text{ K} &\longleftarrow 293\text{ K} \end{aligned}$$

$$\Delta t_1 = T_1 - t_1 = 373 - 313 = 60\text{ K}$$

$$\Delta t_2 = T_2 - t_2 = 333 - 293 = 40\text{ K}$$

$$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\ln \frac{\Delta t_1}{\Delta t_2}} = \frac{60 - 40}{\ln \frac{60}{40}} = 49.33\text{ K}$$

$$l = \frac{\Phi}{K \pi d_m \Delta t_m} = \frac{13.89 \times 4195 \times (373 - 333)}{3000 \times 3.14 \times 0.31 \times 49.33} = 16.18\text{ m}$$

Parallel flow:

$$\begin{aligned} 100\text{ }^{\circ}\text{C} &\longrightarrow 60\text{ }^{\circ}\text{C} \\ 20\text{ }^{\circ}\text{C} &\longrightarrow 40\text{ }^{\circ}\text{C} \end{aligned}$$

Or

$$\begin{aligned} 373\text{ K} &\longrightarrow 333\text{ K} \\ 293\text{ K} &\longrightarrow 313\text{ K} \end{aligned}$$

$$\Delta t_1 = T_1 - t_1 = 373 - 293 = 80\text{ K}$$

$$\Delta t_2 = T_2 - t_2 = 333 - 313 = 20\text{ K}$$

$$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\ln \frac{\Delta t_1}{\Delta t_2}} = \frac{80 - 20}{\ln \frac{80}{20}} = 43.28\text{ K}$$

$$l = \frac{\Phi}{K \pi d_m \Delta t_m} = \frac{13.89 \times 4195 \times (373 - 333)}{3000 \times 3.14 \times 0.31 \times 43.28} = 18.44\text{ m}$$

Answer: The flow rate of cooling water is 27.92 kg/s. The lengths of the pipe required for the countercurrent and parallel countercurrent of the two fluids are 16.18 m and 18.44 m, respectively.

3.4.5.2 Calculation of Heat Transfer Area

1) Overall heat transfer coefficient K is constant

As mentioned above, the logarithmic mean temperature difference is derived when the specific heat capacity, mass flow rate and overall heat transfer coefficient of the cold and hot fluid are assumed to be constant along the whole heat transfer surface of the heat exchanger. In general, when the physical properties of fluids do not

vary greatly with temperature, the changes in the overall heat transfer coefficient are even smaller. In general, when the overall heat transfer coefficient does not change much, the average overall heat transfer coefficient at the inlet and outlet of the heat exchanger is treated as a constant in engineering calculations. At this time, the heat transfer area of the heat exchanger is directly calculated using Eq. (3.50):

$$A = \frac{\Phi}{K \Delta t_m}$$

where $\Phi = q_{mc} C_{pc} (t_2 - t_1)$ or $\Phi = q_{mh} C_{ph} (T_1 - T_2)$.

C_{pc} and C_{ph} are determined by the average temperature of the heat inlet and outlet fluid.

2) Overall heat transfer coefficient K is a variable

If the temperature of the fluid changes greatly in the heat transfer process, the physical properties vary significantly with the temperature, and the overall heat transfer coefficient changes greatly. The calculation of the heat transfer area using the above method will result in a considerable error. Thus, the method of subsection calculation is adopted. The physical properties and overall heat transfer coefficient K are calculated as constants in each cross section. Every Δt_{mj} with corresponding Φ_j and heat transfer area A_j are obtained. Heat transfer area A is obtained by adding together every obtained A_j .

Example 3.10 In the incineration system of waste incinerator A, water is heated in steel pipes of $\Phi 45 \text{ mm} \times 2.5 \text{ mm}$ in a boiler. The water inlet temperature is 30°C , the outlet temperature is 95°C , and the thermal conductivity of the pipe wall is $50 \text{ W}/(\text{m} \cdot \text{K})$. The environmental temperature outside the pipe is 120°C . The convection heat transfer film coefficient outside the pipe is $5000 \text{ W}/(\text{m}^2 \cdot \text{K})$, and the convection heat transfer film coefficient of water in the pipe is $1000 \text{ W}/(\text{m}^2 \cdot \text{K})$.

Calculate: Heat transfer rate per meter of the pipe.

Known: $d_0 = 0.045 \text{ m}$, $\delta_t = 2.5 \text{ mm}$, $\sigma = 50 \text{ W}/(\text{m} \cdot \text{K})$, $t_1 = 30^\circ\text{C} = 303 \text{ K}$, $t_2 = 95^\circ\text{C} = 368 \text{ K}$,

$T_1 = T_2 = 120^\circ\text{C} = 393 \text{ K}$, $\alpha_1 = 1000 \text{ W}/(\text{m}^2 \cdot \text{K})$, $\alpha_0 = 5000 \text{ W}/(\text{m}^2 \cdot \text{K})$.

Determine: Φ/l .

The outer diameter $d_0 = 0.045 \text{ m}$, inner diameter $d_i = 0.045 - 2 \times 0.0025 = 0.04 \text{ m}$

$$\text{According to } \frac{1}{K_0} = \frac{1}{\alpha_i} \frac{d_0}{d_i} + \frac{\delta_t}{\sigma} \frac{d_0}{d_m} + \frac{1}{\alpha_0}$$

$$K_0 = \frac{1}{\frac{1}{\alpha_i} \frac{d_0}{d_i} + \frac{\delta_t}{\sigma} \frac{d_0}{d_m} + \frac{1}{\alpha_0}}$$

$$\begin{aligned}
&= \frac{1}{\frac{1}{1000} \times \frac{0.045}{0.04} + \frac{0.0025}{50} \times \frac{0.045}{0.0425} + \frac{1}{5000}} \\
&= 726 \text{ W}/(\text{m}^2 \cdot \text{K})
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\Delta t_m &= \frac{(\Delta t_1 - \Delta t_2)}{\ln(\frac{\Delta t_1}{\Delta t_2})} = [(T_1 - t_1) - (T_2 - t_2)]/\ln[(T_1 - t_1)/(T_2 - t_2)] \\
&= \frac{(393 - 303) - (393 - 368)}{\ln \frac{393 - 303}{393 - 368}} = 50.78 \text{ K}
\end{aligned}$$

The heat transfer rate per meter of pipe length is:

$$\begin{aligned}
\Phi/l &= K_0 \pi d_0 t_m \\
&= 726 \times \pi \times 0.045 \times 50.78 \\
&= 5209 \text{ W/m}
\end{aligned}$$

Answer: The heat transfer rate per meter of pipe length is 5209 W/m.

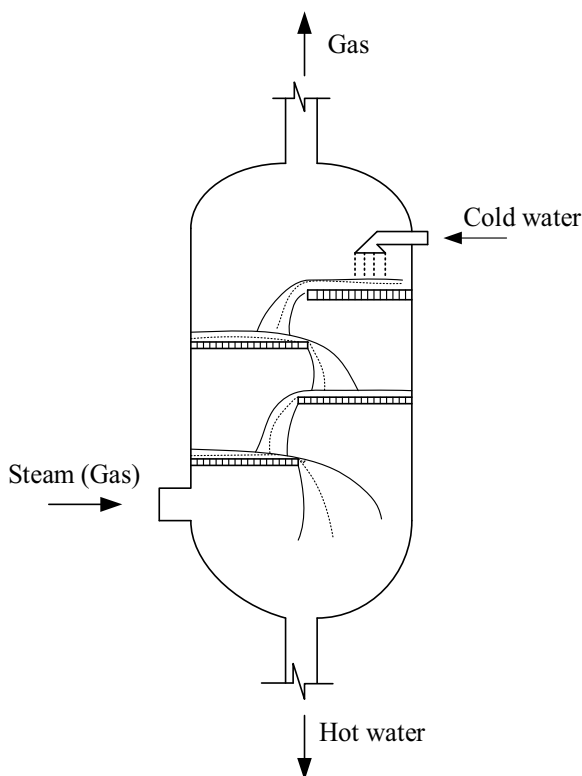
3.5 Heat Exchangers

Heat exchangers are widely applied in power, chemical, petroleum, environmental protection and other industrial sectors and play an important role in production. Because the different properties and scales of production, related materials, heat transfer requirements, etc. are different, there are many types of heat exchangers. On the basis of their use, heat exchangers can be categorized into heaters, coolers, condensers, evaporators and reboilers. According to the heat transfer mode, they can be divided into direct contact, heat storage and interwall types, among which the interwall type heat exchanger is the most popular, especially the tubular heat exchanger. In actual production, the choice of heat exchanger can be made according to the process requirements.

3.5.1 Direct Contact Heat Exchanger

The working characteristic of a direct contact heat exchanger is that cold and hot fluids exchange heat directly in the heat exchanger, so it is also called a mixed heat exchanger. The structure of the heat exchanger is simple, and it is convenient and effective to allow two fluids to mix with each other. Figure 3.14 shows a plate-type sprinkling heat exchanger. This heat transfer method is often used for gas cooling or the condensation of water vapor.

Fig. 3.14 Sprinkler heat exchanger



3.5.2 Regenerative Heat Exchanger

Regenerative heat exchangers are also known as heat accumulators. The main component is a heat storage chamber with a large specific heat capacity. The heat storage chamber is filled with firebrick and other fillers. Regenerative heat exchangers usually have two regenerative chambers used alternately. This kind of heat exchanger is simple in structure and resistant to high temperature. The main disadvantage is the large volume of equipment, and the two kinds of fluid in the heat storage room alternately will have a certain degree of mixing. Its structure is shown in Fig. 3.15.

3.5.3 Interwall Heat Exchanger

Interwall heat transfer is the most commonly used heat transfer equipment in industrial production. The characteristic of an interwall heat exchanger is that cold and hot fluids are separated by materials with certain thermal conductivities (metal walls, graphite, plastic, etc.) so that the two fluids do not need to be mixed for heat

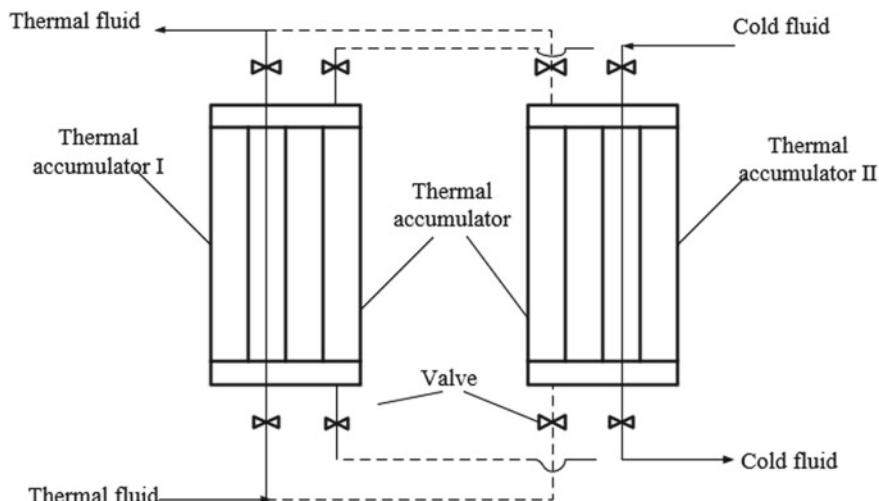


Fig. 3.15 Regenerative heat exchanger

exchange and transfer. Commonly used interwall heat exchangers include tubular heat exchangers, jacket heat exchangers, coil heat exchangers, plate heat exchangers and heat pipe heat exchangers, among which tubular heat exchangers are the most common.

3.5.3.1 Jacket Heat Exchanger

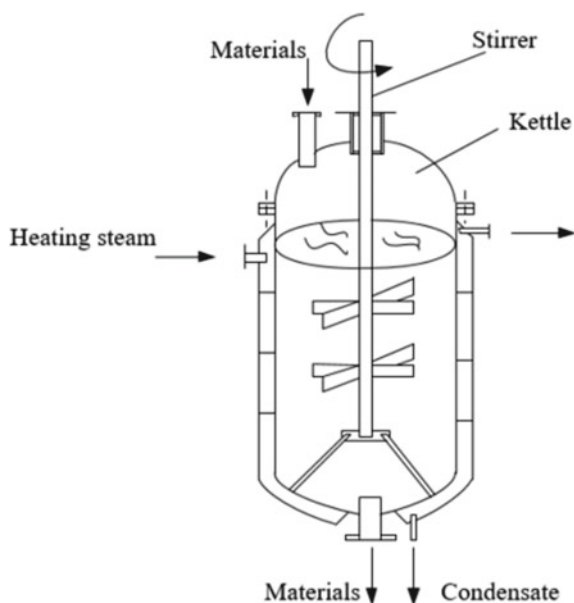
A jacketed heat exchanger is made by installing jackets on the outer wall of the container, and the space formed between the jacket and the wall of the device can accommodate the heating or cooling medium. This kind of heat exchanger is a simple plate heat exchanger, as shown in Fig. 3.16, and is mainly used for heating or cooling during the reaction process. When steam is used for heating, steam enters the jacket from the upper pipe, and condensate flows out from the lower pipe. When used as a cooler, the cooling medium, such as cooling water, enters from the lower pipe of the jacket and flows out from the upper pipe.

The heating surface of the jacketed heat exchanger is limited by the wall surface of the container, and the heat transfer coefficient is not high. To improve the heat transfer coefficient, a stirring device can be installed in the heat exchanger.

3.5.3.2 Coil Heat Exchanger

The structure of a coil-type heat exchanger is also relatively simple. Coil-type heat exchangers are generally composed of straight pipes connected by elbow pipes or

Fig. 3.16 Jacket heat exchanger



curved pipes coiled into spiral shapes, which can be divided into immersion coil types and spray coil types.

1) Immersion coil type

In a submerged coil heat exchanger, the coil is made into a shape suitable for the size of the container and placed in the container. Heat transfer is completed when hot and cold fluids flow in and out of the pipe, respectively. Figure 3.17 shows four types of common submerged coil heat exchangers, which have the advantages of a relatively simple structure, corrosion resistance and high pressure. However, as the volume of the container is much larger than that of the coil, the heat transfer coefficient of the fluid outside the pipe is smaller. If the coil design can be improved, the outer space of the pipe should be reduced, or if the mixing device can be installed, the heat transfer effect can be improved.

2) Spray coil type

A spray coil heat exchanger is mainly used to cool hot fluid in the pipe. The coil pipe is fixed on a steel frame in rows, and the hot fluid flows in the pipe. The cooling water is uniformly poured by the sprinkler device above the pipe, as shown in Fig. 3.18. The heat transfer efficiency of this heat exchanger is higher than that of the immersion type, and it is convenient for maintenance and cleaning. Therefore, the requirements on the quality of cooling water can be appropriately reduced, but cooling water is required to be evenly sprayed.

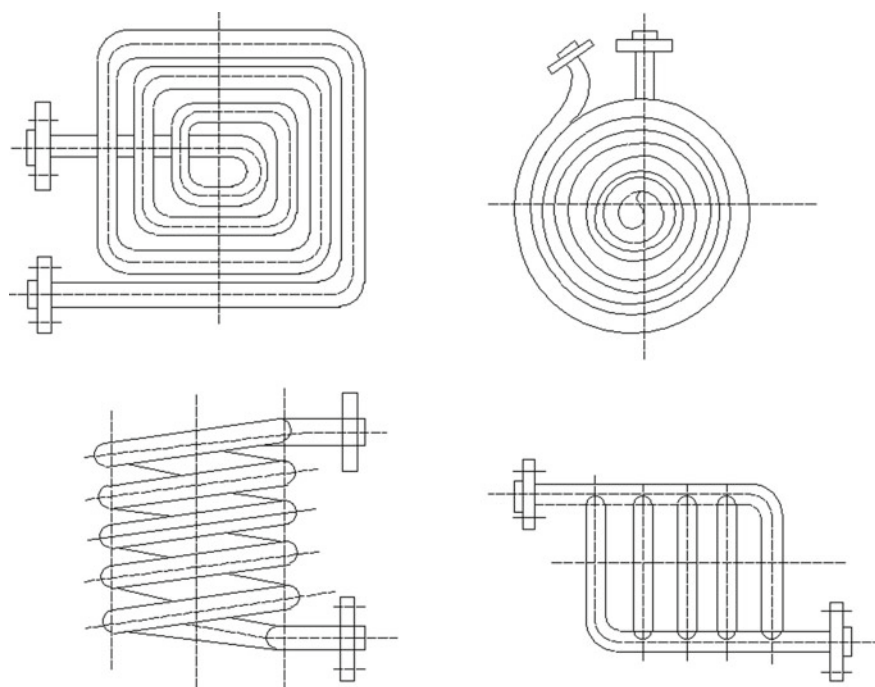


Fig. 3.17 Immersion coil heat exchanger

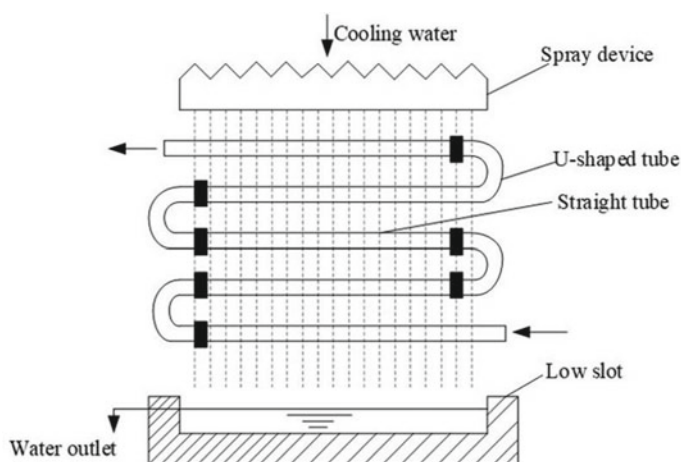


Fig. 3.18 Spray coil heat exchanger

3.5.3.3 Casing Heat Exchanger

The casing heat exchanger contains straight tubular concentric bushings of different sizes, and all pipe segments are connected in series by U-bends, as shown in Fig. 3.19. Each section of the casing is called a trip, and the effective length of each trip is 4 ~ 6 m. The pipe must not be too long, otherwise the middle of the pipe will bend down.

In a casing heat exchanger, one fluid flows inside the pipe and the other flows in the annular gap. The proper selection of the diameter of the two pipes can make both fluids reach a higher flow rate to improve the heat transfer coefficient. The two fluids can be designed for reverse flow so that the mean temperature difference can be increased and the heat transfer efficiency can be further improved. The casing heat exchanger has a simple structure, high pressure resistance and flexible heat transfer area, which can be achieved by increasing or decreasing the number of the sections

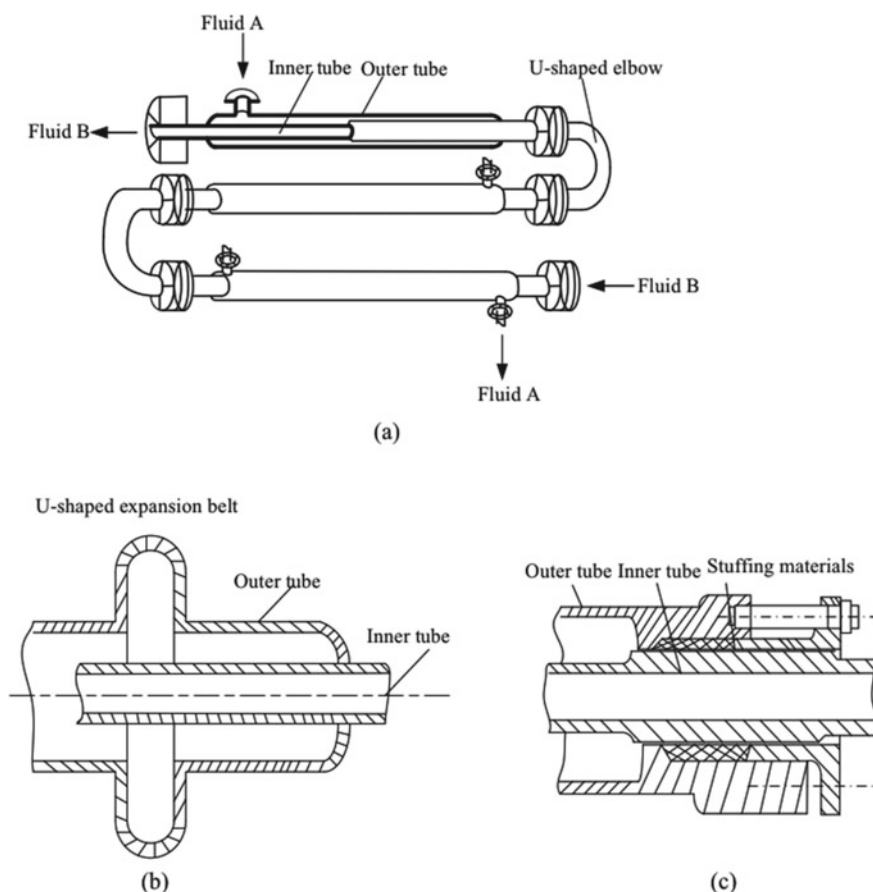


Fig. 3.19 Casing heat exchanger

as required, and the application is flexible and convenient. Its main shortcomings are that there are many pipe union joints, which are prone to leaks, and a high metal consumption amount per unit heat transfer area. Therefore, this approach is suitable for occasions where the required flow rate is low, the heat transfer area is not large, and the pressure is high.

3.5.3.4 Tubular Heat Exchanger

Tubular heat exchangers are the most widely used heat exchangers in industrial production. The main advantages are a large heat transfer surface per unit volume, compact and strong structure, and good heat transfer effect, and they can be made of a variety of materials. Therefore, the applicability of tubular heat exchangers is very high. In high temperature, high pressure and other workplace environments needing large equipment, tubular heat exchangers are adopted.

A tubular heat exchanger is mainly composed of a shell, tube bundle, tube plate and sealing head (also known as the top cover). The tube bundle is installed in the shell, and its two ends are fixed on the tube plate. The sealing head is connected with the flanges at both ends of the shell by screws to facilitate overhaul and disassembly. Its structure is shown in Fig. 3.20.

When heat exchange is carried out, a fluid enters from the inlet tube of the top cover and flows out from the outlet pipe of the top cover at the other end through parallel tube bundles. This distance is called the tube path length. Another kind of fluid enters from the shell nozzle, flows through the gap of the tube bundle in the shell, and flows out by the shell's other nozzle. This distance is called the shell path length. The total surface area of the tube bundle is the heat transfer area, which can be calculated as follows:

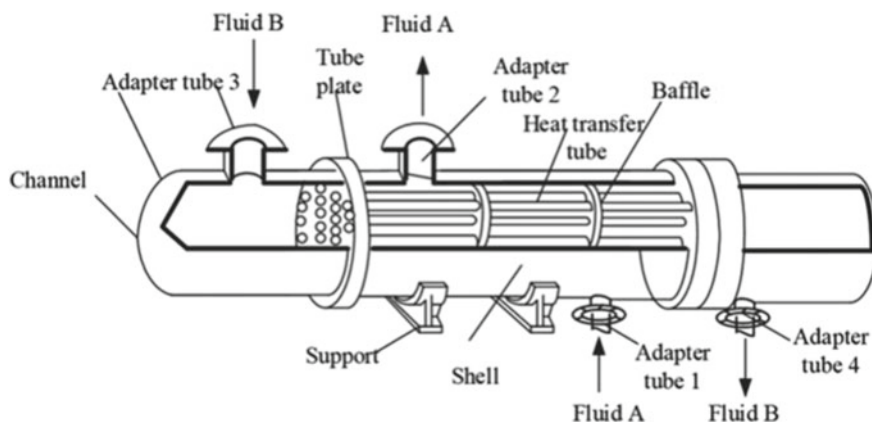


Fig. 3.20 Tubular heat exchanger

$$A_0 = n\pi d_0 l \quad (3.55)$$

where

d_0 —Outer diameter of tube bundle, m

l —Effective length of tube bundle, m

n —Number of tubes in the bundle.

A fluid through the tube is called a single tube, and a single shell through the shell is called a single shell process. Figure 3.20 shows a single-shell heat exchanger.

When the required heat transfer area is large and the number of required tubes is high, the flow velocity of the fluid in the tube may be slow, and the heat transfer coefficient of the fluid will be smaller. At this time, to improve the flow velocity in the tube and thus improve the heat transfer coefficient of the fluid, a partition can be added in the top cover to divide all the tubes into several groups evenly. The fluid only flows through one group of tubes at a time and then enters another group of tubes. After flowing through each group of tubes in turn, it flows out from the outlet. This kind of heat exchanger is called a multipass exchanger, and each group of tubes through which the fluid flows is called one tube length. In general, 2, 4 and 6 tube lengths are the most common.

A multipass exchanger can increase the heat transfer coefficient of fluid by increasing the flow velocity of the fluid. For the same heat transfer task, the heat transfer area required by the heat exchanger can be reduced. However, the increase in the number of passes and the flow velocity increase the resistance along the flow and the local resistance at the same time, which will eventually result in an increase of energy consumption and lead to an increase of operating costs. Therefore, an optimal flow velocity should be selected to minimize the sum of equipment and operating costs.

To improve the fluid velocity in the shell, a certain number of baffles perpendicular to the tube bundle can be installed in the shell. This can not only improve the fluid velocity but also change the direction of fluid flow, forcing the shell side fluid to cross flow through the pipe bundle many times, which is beneficial to increase the heat transfer coefficient. In addition, baffles can also reduce the dead zone of flow, preventing dirt deposition. However, adding baffles will cause an increase in the flow resistance on the shell side, thus increasing the power consumption.

The selection and design calculation of a tubular heat exchanger can refer to the following steps:

(1) Estimate the heat transfer area and try to select an exchanger

The heat transfer area is obtained according to Eq. (3.50) after the heat transfer rate and the mean temperature difference of fluids are calculated, and the overall heat transfer coefficient is estimated. Then, try to select an appropriate type and specification of heat exchanger according to the obtained heat transfer area.

(2) Calculate tube and shell pressure loss

One method is that after selecting the tube and shell side fluids and preliminarily determining the main size of the heat exchanger, the tube and shell side velocity and pressure loss can be calculated. The other method is that the flow velocity of the tube is selected, and then the number of tubes passes and baffle spacing are determined, and then calculating whether the pressure loss is reasonable. The number of tubes passes and baffle spacing are adjustable parameters, and the shell diameter can be repeatedly selected for calculation until the result is reasonable.

(3) Checking calculate the overall heat transfer coefficient

The overall heat transfer coefficient is obtained after the heat transfer coefficients of the tube and shell are calculated, respectively, and the fouling heat resistance is determined. The value of calculated overall heat transfer coefficient is compared to the that of the estimated overall heat transfer coefficient in (1). If the difference is great, the overall heat transfer coefficient should be re-estimated until the difference is small.

(4) Calculate the heat transfer area

The heat transfer area is obtained by the Eq. (3.50) according to the calculated overall heat transfer coefficient in (3) and the mean temperature difference of the fluids. In general, the chosen or designed heat transfer area should be greater than the calculated area by approximately 10 ~ 15%.

3.5.3.5 Plate Heat Exchanger

Plate heat exchangers are divided into two types: flat plate heat exchangers and spiral plate heat exchangers.

1) Flat plate heat exchanger

The flat plate heat exchanger is composed of a long square metal plate arranged in parallel, clamped and assembled on a bracket. The edges of two adjacent metal plates are lined with gaskets, and a sealed fluid channel can be formed between the plates after compression. The thickness of the gaskets can be changed to adjust the size of the channel. There are four circular holes at each of the four corners of each plate, in which a pair of circular holes communicate with an interplate flow passage, and gaskets are placed around the other pair of circular holes to prevent fluid entering the interplate passage. The positions of the two pairs of circular holes are staggered on the adjacent plates, forming the passage of hot and cold fluids. The two fluids flow interlaced on both sides of the sheet metal, and heat is transferred through the sheet metal. The thickness of the plate is 0.5 ~ 3 mm, and the plate is usually pressed into a concave and convex corrugated shape to increase the heat transfer area and increase the stiffness of the plate to prevent the deformation of the plate under pressure. At the same time, the fluid distribution is uniform, enhancing its turbulence, which is

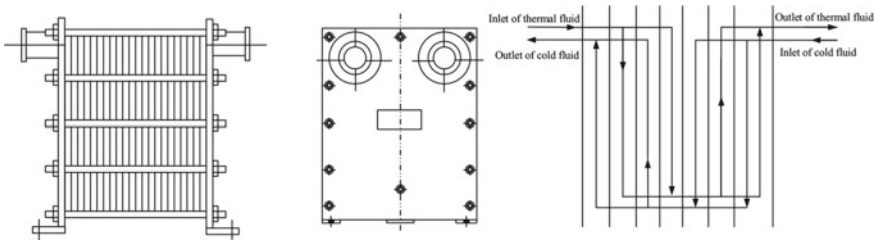


Fig. 3.21 Schematic diagram of flat plate heat exchanger

beneficial to heat transfer. Figure 3.21 shows the flow direction of cold and hot fluid in the plate heat exchanger. Figure 3.22 shows a herringbone corrugated plate.

2) Spiral plate heat exchanger

A spiral plate heat exchanger is made up of two parallel thin metal plates separated by a certain distance, as shown in Fig. 3.23. It forms two concentric spiral channels. A baffle plate is set in the center of the heat exchanger to separate the spiral channel. A fixed distance column is welded between the two plates to maintain the channel distance. A cover plate is welded on both sides of the spiral plate. Cold and hot fluids flow in opposite directions in the two channels, and heat exchange is carried out through thin plates.

The flat plate heat exchanger and spiral plate heat exchanger are characterized by a compact structure, low material consumption and a high heat transfer coefficient. This kind of heat exchanger is generally not able to withstand high temperature and pressure. However, in terms of low pressure, low temperature or strong corrosion and the use of valuable materials, they show great superiority. They have been widely used in light industrial, food, and chemical applications.

3.5.3.6 Hot Pipe Heat Exchanger

As shown in Fig. 3.24, a hot pipe heat exchanger is made by adding a certain amount of ammonia, water or alcohol and other working liquid in a closed metal pipe in which condensable gas is exhausted. The wall of the hot pipe must be able to withstand certain pressure. The inner wall of the pipe is adhered to a layer of an absorbent core network with a capillary structure made of porous materials, such as multilayer stainless steel wire mesh. The working liquid in the pipe is a heat carrier. The two ends of the pipe are the hot end (evaporation end) and the cold end (condensation end), respectively. The middle part of the pipe is insulated to separate the cold and hot ends. In the hot end of pipe, the working liquid is evaporated into steam after heating by the outside heat source. Then the hot stream passes the central part of the pipe of the steam chamber, and is cooled down and condensed in the cool end of pipe with relative low temperature and pressure. At the same time, the heat is transferred to the outside from the cool end of the pipe. The condensed working liquid is sucked back to the hot end by action of surface tension from the capillary structure. The

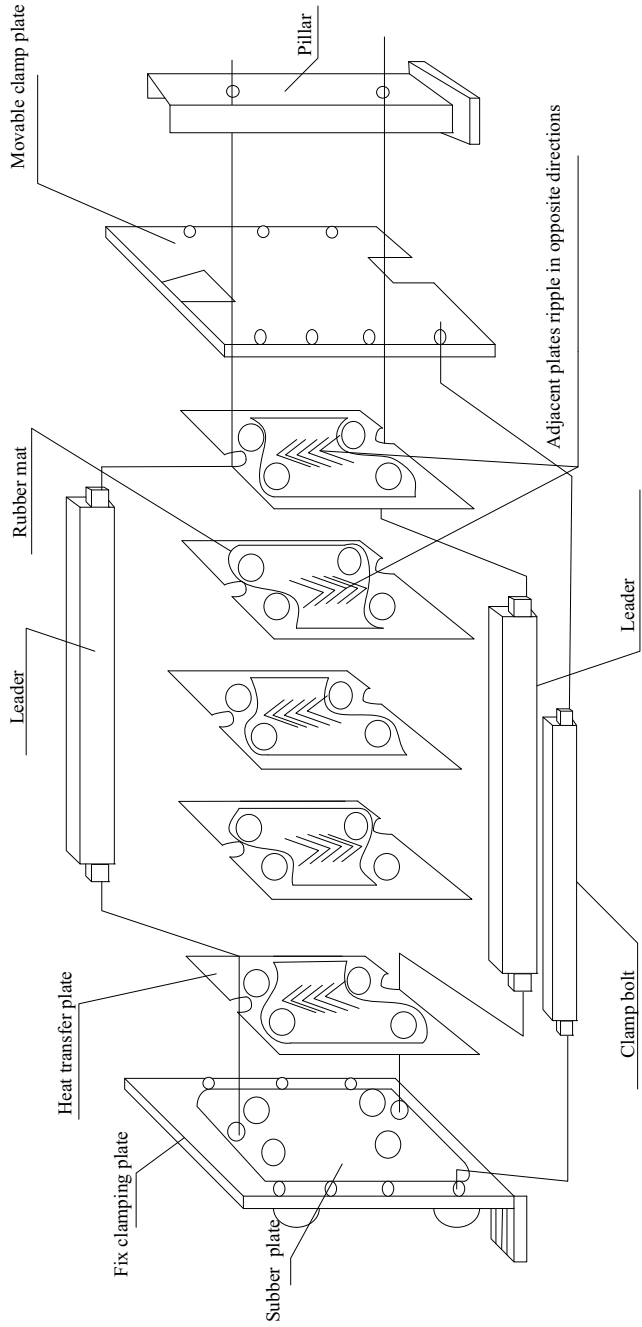


Fig. 3.22 Herringbone corrugated plate structure

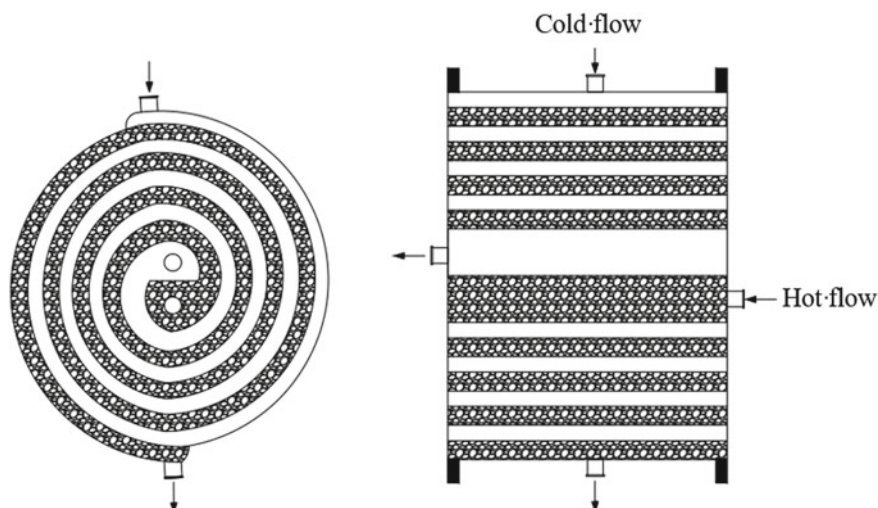
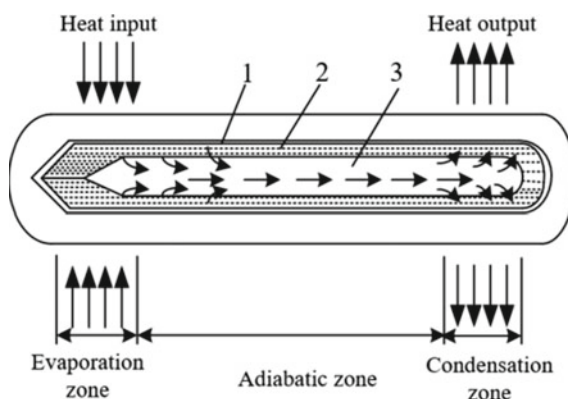


Fig. 3.23 Spiral plate heat exchanger

Fig. 3.24 Heat pipe heat exchanger 1-Shell, 2-Endothermic core, 3-Steam



working liquid circulated repeatedly between hot and cool ends, which achieves the purpose of the heat transfer through evaporation and condensation.

In the heat transfer process of a hot pipe heat exchanger, heat transfer in the hot pipe mainly occurs through boiling vaporization, steam flow and steam condensation. Because the convection heat transfer intensity of boiling and condensation is very great, the surfaces of both ends of the pipe are much larger than the cross section of the pipe, and the resistance loss of steam flow is small, so the temperature difference between the two ends of the hot pipe is very small but can also transfer much heat flow rate. Compared with the thermal conductivity of a metal wall, the thermal conductivity of a hot pipe of the same area can reach $10^3 \sim 10^4$ times the metal thermal conductivity. Therefore, hot pipe heat exchanger has wide application prospects in

low-temperature differential heat transfer and some occasions with high isothermal requirements.

The hot pipe heat exchanger has the advantages of a simple structure, long service life, reliable performance and wide application range. At first, it was mainly favored in aerospace and electronic industry applications. In recent years, it has been applied in more fields, especially in the utilization of waste heat in industrial production, and has achieved good results.

3.6 Measures to Strengthen the Heat Transfer Process of Heat Exchangers

The heat transfer rate increases with area A , the mean temperature difference Δt_m , and the overall heat transfer coefficient K according to the heat transfer rate equation of $\Phi = KA\Delta t_m$. In the design, operation or improvement of heat exchangers, measures to strengthen heat transfer are also considered from these three aspects.

1) Increase the heat transfer area

For an interwall heat exchanger, increasing the heat transfer area means increasing the amount of metal materials and increasing the cost of equipment, so simply increasing the size of the heat exchanger to achieve heat transfer enhancement is not economical. Generally, in terms of structure, one can increase the heat transfer area to achieve the effective method of strengthening heat transfer by improving the heat transfer area per unit volume, such as the use of small diameter pipes, spiral pipes, corrugated pipes instead of smooth pipes, and finned heat exchangers.

2) Increase the mean temperature difference

The mean temperature difference depends mainly on the temperature of the two fluids. The temperature of the material is determined by the production process and cannot be changed at will. The temperature of the heating medium or the cooling medium varies greatly with the medium. For example, the most commonly used heating medium in industry is saturated steam. Increased temperature of the steam is obtained by increasing the pressure of the steam. When there is no phase change between the two fluids in the heat exchanger, the countercurrent flow or near the countercurrent flow should be used in the structure as much as possible to obtain a greater mean temperature difference. For example, a spiral plate heat exchanger can make two fluids flow completely in a countercurrent type.

3) Increase overall heat transfer coefficient

Increasing the overall heat transfer coefficient is the main way to strengthen the heat transfer process. The heat transfer process in a heat exchanger is a steady series process, and the overall heat transfer coefficient depends on the sum of the thermal resistance of the fluid on both sides, the heat conduction resistance of the pipe wall

and the fouling thermal resistances. Reducing one of the three kinds of thermal resistances can improve the overall heat transfer coefficient, but the main thermal resistances with the largest value has the most significant effect. In a heat exchanger, the metal pipe wall is relatively thin, and the thermal conductivity is high; generally, it will not become the main thermal resistance, so the main consideration is some strengthening measures which reduce the other two kinds of thermal resistances.

(1) Increase the flow velocity of the fluid

Increasing the flow velocity can reduce the thickness of the laminar sublayer in the heat transfer boundary layer. For example, increasing the number of pipe passes and baffles in the shell of a tubular heat exchanger can increase the flow velocity of pipe and shell, respectively. However, with increasing flow velocity, the resistance of the fluid flow increases quickly and even is faster than the corresponding overall heat transfer coefficient, so it must be considered comprehensively.

(2) Change the flow conditions

By designing a special heat transfer wall surface, the flow direction of the fluid is constantly changed during the flow process to promote the formation of turbulence or increase the degree of turbulence to improve the overall heat transfer coefficient, such as by using corrugated or rough heat transfer surfaces, adopting special-shaped pipes or adding twisted iron, spiral rings or metal coils to the pipe, using a tubular or spiral plate heat exchanger, installing baffle plates in a tubular heat exchanger, etc.

(3) Adopt a short pipe heat exchanger

By taking advantage of the strong heat transfer characteristics of the inlet section of the heat exchanger, the adoption of short pipes not only improves the disturbance of the fluid but also increases the convection heat transfer intensity due to the short flow passage and the thin thickness of the boundary layer.

(4) Prevent dirt deposition

Fouling thermal resistance is a variable factor in a heat exchanger when put into use. Fouling thermal resistance is a small factor at first and not the main concern. Over time, fouling gradually increases and may become a major factor affecting heat transfer. Therefore, the formation and development of dirt should be slowed by increasing the flow velocity and other means, and dirt should be removed in time. For a heat exchange system with circulating cooling water, fouling can be prevented by adding some water treatment chemicals.

Exercises

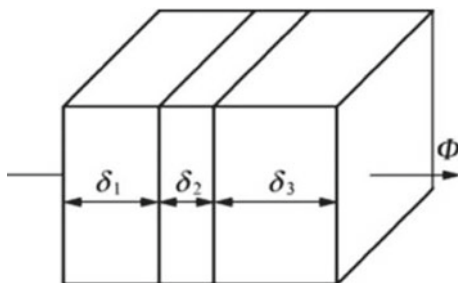
Part I Multiple choice and fill in the blanks

1. In the heat conduction of a stable multilayer flat wall, the () the thermal resistance of a certain layer is, the greater the temperature difference of that layer.

- A. larger
 - B. smaller
 - C. constant
 - D. cannot be determined
2. () refers to the heat energy transferred from the hot fluid to the cold fluid by the unit heat transfer area in unit time when the temperature difference between the cold fluid and the hot fluid on both sides of the interwall is 1 K.
- A. Thermal conductivity
 - B. Heat transfer film coefficient
 - C. Overall heat transfer coefficient
 - D. Resistance coefficient
3. Stable heat transfer refers to the temperature of each point in the heat transfer system in which ().
- A. changes with time and position
 - B. only changes with time, not with position
 - C. only changes with position, not with time
 - D. does not change with time, nor does it change with position
4. In the equation for calculating the convection heat transfer coefficient, the Prandtl number (Pr) is the parameter representing ().
- A. convection heat transfer
 - B. flow state
 - C. physical properties
 - D. influence of natural convection
5. In the case of stable heat transfer (including heat conduction and convective heat transfer), the heat flow rate of heat conduction is () heat flow rate of the convective heat conduction.
- A. greater than
 - B. equal to
 - C. less than
 - D. dependent on the ratio of the two processes
6. Generally, heat loss increases with the thickness of the insulation layer (). However, for small diameter pipes, the opposite may occur, that is, with the increase in insulation thickness, heat loss will ().
- A. decrease
 - B. increase
 - C. remain constant
 - D. cannot be determined
7. For tubular heat exchangers, the efficiency of parallel flow heat transfer is () than that of countercurrent flow heat transfer.

- A. higher
 - B. the same
 - C. lower
 - D. dependent on the temperature
8. Water is used to cool a material in a countercurrent casing heat exchanger, and the temperature of the hot fluid T_{import} , T_{export} and flow rate q_{mh} is constant. The flow rate of cooling water q_{mc} increases to ensure the completion of production tasks under the conditions of increased influent temperature T_{import} of cooling water. Therefore, the overall heat transfer coefficient K () heat transfer rate Φ (), and temperature difference Δt ()
- A. increases
 - B. decreases
 - C. keeps constant
 - D. cannot be determined
9. In a tubular heat exchanger, the heat transfer process between cold and hot fluids is () under the condition that the temperature is not too high.
- A. heat conduction as the main mode
 - B. thermal radiation as the main mode
 - C. thermal convection as the main mode
 - D. both heat conduction and convection as the main mode
10. The basic modes of heat transfer include _____, _____, and _____.
11. For heat conduction, the thermal conductivity of various materials is usually measured by experiments. The ranking of the thermal conductivity of metal, nonmetal solid, liquid and gas is _____.
Generally, convection heat transfer is simplified to heat conduction of a heat transfer boundary layer of thickness of _____, which includes _____ and _____.
12. In the process of heat conduction, liquid and gas are usually compared, and the higher the heat conductivity is _____.
13. Three different thickness (δ_p) materials constitute three layers of flat walls, and every layer has good contact, $\delta_{p1} > \delta_{p2} > \delta_{p3}$ and $\sigma_1 < \sigma_2 < \sigma_3$ are known. In the process of stable heat transfer, the thermal resistance of every layer Ω_1 _____ Ω_2 _____ Ω_3 and the thermal conductivity of every layer Φ_1 _____ Φ_2 _____ Φ_3 .
14. Convective heat transfer resistance between fluid and solid wall is mainly concentrated in _____.
15. In the heat transfer process, according to the different directions of flow between hot and cold fluids, it can be divided into different flow forms: _____, _____, _____, _____. When the temperature of hot and cold fluids entering and leaving the heat exchanger has been determined, the flow form with the highest mean temperature difference is _____.

Fig. 3.25 Schematic diagram of question 1



16. There are three basic modes of heat transfer. List three common heat exchange equipments: _____, _____, _____.
17. In the heat exchange process, the three general ways to strengthen are _____, _____ and _____.

Part II Computational questions + answers

1. As shown in Fig. 3.25, for a furnace, its flat walls consisted from inside to outside are: firebrick: $\sigma_1 = 1.047 \text{ W/(m} \cdot \text{K)}$, ordinary brick: $\sigma_2 = 0.835 \text{ W/(m} \cdot \text{K)}$ and insulation brick: $\sigma_3 = 0.150 \text{ W/(m} \cdot \text{K)}$, and the thicknesses of three walls are 150, 100 and 150 mm respectively. When the operation is stable, the inner surface temperature and outer surface temperature of the flat wall can be measured as 850°C and 104°C . Try to calculate the heat loss per square meter per hour.

Answer: $2.13 \times 10^6 \text{ J/(h} \cdot \text{m}^2)$.

2. A flat wall with a thickness of 400 mm, the temperature on both sides is maintained at constant values of 950°C and 250°C . The thermal conductivity σ is a function of temperature, which can be expressed as $\sigma = \sigma_0(1 + 0.001 t)$, where the unit of t is K. If the thermal conductivity is calculated as constant (taking the average thermal conductivity) and variable, please calculate the heat flow density and temperature distribution inside the wall respectively.

Answer: Constant: $q_T = 3277\sigma_0(\text{W/m} \cdot \text{K})$, $t = 1223 - 1750x$, Variables: $q_T = 3277\sigma_0(\text{W/m}^2)$, $t = -1000 + 1000\sqrt{4.924 - 6.556x}$.

3. A tubular heat exchanger consists of 30 steel pipes ($\Phi 24 \text{ mm} \times 2 \text{ mm}$ and 2 m of length). The air with 80°C , atmospheric pressure and 12 m/s velocity flows into the inside steel pipe. The outlet temperature of the air is 20°C . Please determine the convection heat transfer film coefficient of air to the pipe wall.

Answer: $58.33 \text{ W/(m}^2 \cdot \text{K)}$.

4. A tubular heat exchanger consists of 80 steel pipes ($\Phi 25 \text{ mm} \times 2.5 \text{ mm}$) with a length of 2 m. Benzene is heated by saturated water steam through the heat exchanger. Benzene flows through the pipe with a flow rate of 15 kg/s. During this process, it is heated from 20°C to 80°C . Try to find the convection heat transfer film coefficient of benzene in the pipe. If the benzene flow rate increases

by 90%, assuming that the original outlet temperature is maintained, what is the convection heat transfer film coefficient at this time? [Known: benzene's physical properties at 50 °C are as follows: $\rho = 860 \text{ kg/m}^3$, $C_p = 1.80 \text{ kJ/(kg} \cdot \text{K)}$, $\mu = 0.45 \times 10^{-3} \text{ Pa} \cdot \text{s}$, $\sigma = 0.14 \text{ W/(m} \cdot \text{K)}$]

Answer: $1119.90 \text{ W/(m}^2 \cdot \text{K)}$, $1871.47 \text{ W/(m}^2 \cdot \text{K)}$.

5. There is a steel pipe wrapped with a layer of insulation material to ensure the temperature of steam inside the pipe. Its thermal conductivity $\sigma = 0.03 \text{ W/(m} \cdot \text{K)}$ and the convection heat transfer film coefficient of the exterior facing the environment $\alpha = 6.5 \text{ W/(m}^2 \cdot \text{K)}$. The inner and outer radii of the insulation layer are r_i and r_o . Let the inner surface temperature of the insulation layer be t_i and the ambient temperature be t_f . Please find the critical diameter of insulation layer and write down the derivation process.

Answer: 9.2 mm.

6. There is a pipeline for carrying steam with an outer diameter of 100 mm. To reduce heat loss, a layer of insulation material ($\sigma = 0.2 \text{ W/(m} \cdot \text{K)}$) is wrapped around the pipe. Given that the temperature of the outer wall of the steam pipe is 200 °C, the heat loss per meter of pipe length is required to be 250 W/m. Please find the temperature distribution in the insulation layer. If the temperature outside the insulation layer is required to be lower than 50 °C, how thick should the insulation layer be?

Answer: The temperature distribution in the thermal insulation layer is $t = -198.94 \ln r - 395.97 \text{ }^\circ\text{C}$, the outside temperature is lower than 50 °C, and the insulation layer thickness is at least 0.056 m.

7. There is a sludge digestion tank, and the tank body is mainly made of reinforced concrete material [$\sigma_1 = 1.55 \text{ W/(m} \cdot \text{K)}$] with a thickness of 400 mm. There is a thick layer of foamed cement outside the tank wall as the insulation material [$\sigma_2 = 0.3 \text{ W/(m} \cdot \text{K)}$] with a thickness of 30 mm. The convection heat transfer film coefficient of the internal surface of the digester (sludge transfer to the reinforced concrete tank wall) is $\alpha_1 = 350 \text{ W/(m}^2 \cdot \text{K)}$, while the convection heat transfer film coefficient of the external surface (the wall of the tank transferred to the air of the external medium) is $\alpha_2 = 6.5 \text{ W/(m}^2 \cdot \text{K)}$. (1) Please find the overall heat transfer coefficient of the digester. (2) For the purpose of reducing the overall heat transfer coefficient, please compare the results and get the conclusion when half value of α_1 or α_2 , respectively, while keeping other conditions unchanged.

Answer: $K_0 = 1.94 \text{ W/(m}^2 \cdot \text{K)}$. Reduce α_1 to a half value, $K = 1.93 \text{ W/(m}^2 \cdot \text{K)}$. Reduce α_2 to a half value, $K = 1.49 \text{ W/(m}^2 \cdot \text{K)}$. Reducing the value of smaller α can make the decreased amplitude of K become greater.

8. A coil heat exchanger consists of copper pipes ($\Phi 20 \text{ mm} \times 2 \text{ mm}$) in a water tank. Carbon dioxide gas flows inside the copper pipes. The cooling water comes from the bottom of the tank and goes away from the top after heat exchange with carbon dioxide. The heat transfer film coefficient of carbon dioxide against the wall $\alpha_1 = 54.5 \text{ W}/(\text{m}^2 \cdot \text{K})$, the heat transfer film coefficient of the pipe wall to water $\alpha_2 = 1400.5 \text{ W}/(\text{m}^2 \cdot \text{K})$ and the wall $\sigma_n = 375.6 \text{ W}/(\text{m} \cdot \text{K})$. Please determine the overall heat transfer coefficient of the heat exchanger.

Answer: $42.27 \text{ W}/(\text{m}^2 \cdot \text{K})$.

9. A tubular heat exchanger are used to preheat water. Water flows in a $\Phi 25 \text{ mm} \times 2.5 \text{ mm}$ steel pipe with a velocity of 0.8 m/s . The length of steel pipe is 2 m . The inlet and outlet temperatures are 30°C and 90°C , respectively. The specific method is to use saturated water vapor to condense between pipes to transfer heat and assume the water vapor condensation heat transfer film coefficient is $9500 \text{ W}/(\text{m}^2 \cdot \text{K})$, the fouling thermal resistance on the water side is $0.5 \times 10^{-3} (\text{m}^2 \cdot \text{K})/\text{W}$, and the thermal resistance of the pipe wall is small, which can be ignored. Please find the overall heat transfer coefficient K .

Answer: $1014.7 \text{ W}/(\text{m}^2 \cdot \text{K})$

10. In a single-shell single-pipe heat exchanger without a baffle, hot fluid is cooled from 120°C to 50°C . The inlet temperature of cooling water is 15°C , and the outlet temperature is 30°C . Please find the mean temperature difference between the countercurrent and parallel flow under this temperature condition.

Answer: Parallel flow: 51.3 K , counter current: 58.2 K .

11. At present, a tubular heat exchanger is used as a full condenser for the gas on the top of a rectification tower, in which heat transfer tube is steel tube with $\sigma = 49 \text{ W}/(\text{m} \cdot \text{K})$, as well as big diameter and thin thickness. Water flows in reverse with organic vapor between pipes during operation. The temperature of organic vapor was measured as 70°C , and the heat transfer rate Φ which makes all organic vapor be condensated is 422.2 kW . The mass flow rate q_{mc} of cooling water is $4 \times 10^4 \text{ kg}$ per hour, and its inlet temperature is taken as 30°C . The specific heat capacity at constant pressure $C_p = 4178 \text{ J}/(\text{kg} \cdot \text{K})$, the heat transfer film coefficient of the organic side surface $\alpha_1 = 1300 \text{ W}/(\text{m}^2 \cdot \text{K})$, and the surface heat transfer film coefficient on the water side $\alpha_2 = 1000 \text{ W}/(\text{m}^2 \cdot \text{K})$ are desirable. Please calculate how much heat transfer area is needed to meet the requirements of heat transfer.

Answer: 21.9 m^2 .

12. There is a combustion furnace whose flat wall is composed of two kinds of materials and whose inner layer is firebrick with a thickness of 400 mm [$\sigma_1 = 1.04 \text{ W}/(\text{m} \cdot \text{K})$], and the outer layer is ordinary brick with a thickness of 200 mm [$\sigma_2 = 0.55 \text{ W}/(\text{m} \cdot \text{K})$]. The measured temperature in the furnace is 1200°C , the temperature outside the furnace is 20°C , the convection heat transfer film coefficient of the combustion gas in the furnace to the wall is $35.7 \text{ W}/(\text{m}^2 \cdot \text{K})$, and

the convection heat transfer film coefficient of the wall to air is $15.5 \text{ W}/(\text{m}^2 \cdot \text{K})$. If the ordinary brick is capable of working at a maximum temperature of 600°C , please determine whether the ordinary brick can be used in this case.

Answer: It cannot be used.

13. A steel pipe of $\Phi 25 \text{ mm} \times 2 \text{ mm}$ and $\sigma = 45 \text{ W}/(\text{m} \cdot \text{K})$ is placed in a waste heat boiler. The outer pipe contains boiling water, and the absolute pressure is 2.55 MPa . The temperature of the synthetic converted gas inside the pipe decreases from 600°C to 450°C . α_i on the gas side $= 300 \text{ W}/(\text{m}^2 \cdot \text{K})$, and water side $\alpha_0 = 10,000 \text{ W}/(\text{m}^2 \cdot \text{K})$. If the thermal resistance caused by fouling on the pipe wall is ignored, please obtain the following:

- (1) Heat transfer rate per meter of pipe length.
- (2) Wall temperature on both sides of the steel pipe.

Answer: The heat transfer rate per meter of pipe length is 5602.37 W/m . The steel pipe outside the wall temperature $t_0 = 232.13^\circ\text{C}$, and the temperature of the inner wall $t_i = 235.58^\circ\text{C}$

14. There is a casing heat exchanger with an inner pipe of $\Phi 25 \text{ mm} \times 2.5 \text{ mm}$, in which heat transfer is conducted by countercurrent flow between solution and water. The specific heat capacity is $4.00 \text{ kJ}/(\text{kg} \cdot \text{K})$ for the solution, which travels outside the pipe at a flow rate of 1000 kg/h . The temperature decreases from 150°C to 100°C . The specific heat capacity is $4.19 \text{ kJ}/(\text{kg} \cdot \text{K})$ for the water, and the import and export temperatures are 60°C and 100°C , respectively. The convection heat transfer film coefficient of the solution is $1000 \text{ W}/(\text{m}^2 \cdot \text{K})$. If the thermal resistance of the pipe wall can be ignored. Please determine the flow rate of water and the overall heat transfer coefficient based on the area of pipe out. If the water flow rate increases by 50% and the original outlet temperature is maintained, what is the convection heat transfer film coefficient of the water?

Answer: The flow rate of the water is 1193 kg/h , and the overall heat transfer coefficient based on area of pipe out is $849.12 \text{ W}/(\text{m}^2 \cdot \text{K})$. When the water flow rate increases by 50% and the original outlet temperature is maintained, the convection heat transfer film coefficient of water is $9708.02 \text{ W}/(\text{m}^2 \cdot \text{K})$.

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Chapter 4

Absorption



Yan Liu, Liang Li, Guang-Tuan Huang, and Chen-Xi Li

4.1 Overview

When a drop of blue ink is added to a still basin of clear water, the blue color gradually diffuses around itself until the whole basin of clear water becomes uniform blue. This indicates that the position of the material (blue pigment) in the water has shifted, and the composition of the material in the liquid phase has also changed; eventually, the concentration of the material has reached equilibrium everywhere. From a microanalysis, it can be seen that some blue pigments move from high concentration to low concentration and from low concentration to high concentration due to the irregular thermal motion of molecules. However, due to the difference in concentration, the total statistical results still show that blue pigments move from high concentrations to low concentrations. In general, the mass transfer from high concentration to low concentration is a mass transfer process in single-phase medium.

In the wastewater biological treatment process, there are also complex mass transfer processes between biofilms and wastewater. There is not only single-phase mass transfer but also interphase mass transfer between different phases, as shown in Fig. 4.1. Biofilm has a large surface area. Due to the adsorption of biological films, a thin and slow-moving layer of water is attached to the film, which is called the attached water layer. The concentration of microbial metabolites in biofilms is higher than that in wastewater, while the concentrations of both organic matter and dissolved oxygen in wastewater are higher than those in biofilms. Therefore, there are many mass transfer processes inside and outside biofilms, as well as between biofilms and water layers. Organic pollutants in wastewater is transferred from the flowing water layer to the attached water layer and then adsorbed by biofilms. Oxygen in the air dissolves in the flowing water layer and is transferred to the biofilm through the

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245

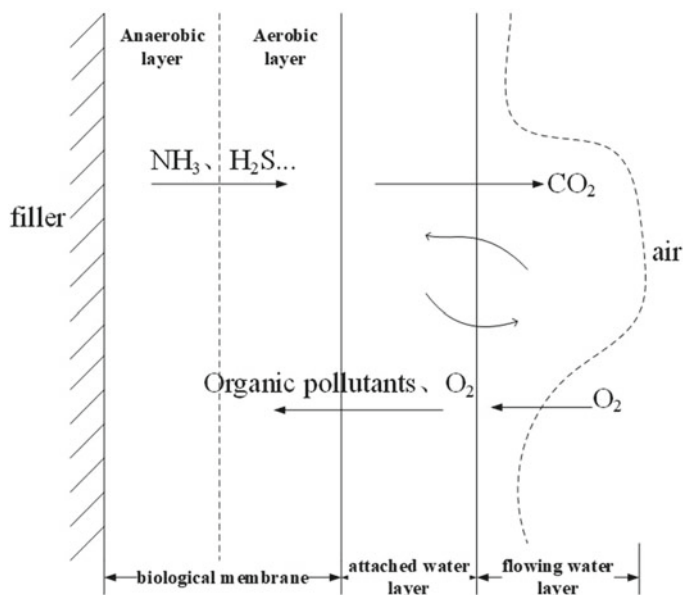


Fig. 4.1 Mass transfer process between biofilm and wastewater (Gao et al., 2018; Zhou and Wang, 2015). Source Reprinted from book “Water Pollution Control Engineering (Volume 1)” with kind permission from Prof. Qi Zhou, as well as from book “Microbiology of Environmental Engineering, Fourth ed.” with kind permission from Prof. Shifen Wang

attached water layer for microorganisms to breathe. Under these conditions, aerobic bacteria oxidize, decompose and synthesize organic matter. Some of the CO_2 and other metabolites produced are dissolved in the attached water layer, and some are released into the air (i.e., discharged into the air from the water layer through the biofilm in the opposite direction). Such recycling makes the organic matter in the wastewater decrease continuously to purify the wastewater. When the biofilm is thicker or the concentration of organic matter in wastewater is higher, the oxygen in the air is quickly consumed by the biofilm on the surface. The biofilm near the filler will not obtain enough oxygen supply to make the anaerobic bacteria grow and produce anaerobic decomposition products such as organic acid, methane (CH_4), ammonia (NH_3) and hydrogen sulfide (H_2S), which also diffuse into the biofilm. The water layer is partially discharged into the air.

In waste gas treatment, if the flue gases of coal-fired power plants and copper smelters containing SO_2 , NO_2 and CO_2 are sprayed with clean water, SO_2 , NO_2 and CO_2 will gradually dissolve into water, resulting in a gradual decrease in SO_2 , NO_2 and CO_2 concentrations in the gas phase and a gradual increase in SO_2 , NO_2 and CO_2 concentrations in water. If the contact time between a certain amount of clean water and a certain amount of flue gas containing SO_2 , NO_2 and CO_2 is long enough, the concentration of SO_2 , NO_2 and CO_2 in the two phases will reach a certain

equilibrium state. This is the mass transfer process of SO_2 , NO_2 and CO_2 from the gas phase to the liquid phase.

In summary, the transfer process of matter can occur both in the same phase or between different phases. When there is a concentration difference between one phase or two directly contacted phases and the equilibrium state is not reached, the position of the material will move, and then the composition of the phase will change. This process is called the diffusion process or mass transfer process.

However, the transfer of materials is not always a mass transfer process. For example, pumping water from underground to a water tower, or conveying harmful gas from a workshop to purifying equipment by a fan, has initiates material transfer, but this is caused by external forces, and in the conveyance process, the phase composition of materials does not change. Thus, these belong to material transport processes replacing mass transport processes.

4.1.1 Mass Transfer Process in Environmental Engineering

Harmful materials are often needed to be separated from wastewater, waste gas or solid waste in environmental engineering treatment. The second law of thermodynamics notes that all spontaneous processes occur as a process of increasing entropy, such as the spontaneous mixing of blue ink and clear water mentioned above. In contrast, the separation process is a process of entropy reduction, which cannot be generated spontaneously. This can only be carried out by external work (or input energy) on the system, so a certain means must be used to separate the mixture.

Mass transfer can be carried out in homogeneous or heterogeneous phases. According to the mass transfer mechanism, mass transfer separation process can be divided into two categories: the equilibrium separation process and the rate-controlled separation process. This book briefly introduces several common mass transfer and separation processes in environmental engineering treatment.

1) Rate-controlled separation process

The rate-controlled separation process uses a specific medium under the action of a certain driving force to produce differences in the mass transfer rate of each component to achieve separation.

Membrane separation technology in the water supply and wastewater treatment process is a rate-controlled separation process. Membrane separation technology includes reverse osmosis, electrodialysis, ultrafiltration, nanofiltration and so on. This approach uses the permeability of specific membranes to separate harmful ions, molecules or colloidal materials in water by pressure difference or potential difference in the presence of concentration difference to realize the further purification of water quality. Membrane separation technology has been widely used in water and wastewater treatment engineering, such as the preparation of high purity water and membrane bioreactors.

2) Equilibrium separation process

Equilibrium separation is a mass transfer process in which a component forms a new phase or migrates to another phase by the addition of an energy separator or a material separator. For example, the introduction of heat (energy separator) in distillation operations, the addition of absorbent or solvent (material separator) in the process of absorption or extraction, and the components of the mixture to be separated, because of their different separation properties, move from one phase of the mixture to another to achieve separation.

(1) Absorption and desorption

A component which dissolves from a gas phase into a liquid phase, is called gas absorption (Fig. 4.2a). This is a mass transfer process between the gas and liquid phases. Absorption is often used for the purification of gas containing harmful pollutants, such as coal-fired power plants and flue gas purification. A component, which is released from a liquid phase into a gas phase, is called gas desorption. Desorption can be used to remove harmful gases in wastewater, such as the removal and recovery of ammonia in wastewater.

(2) Adsorption

The process of adsorbing materials from a gas or liquid phase to a solid surface is called adsorption (Fig. 4.2b) and vice versa is called desorption. This is a mass transfer process between the gas–solid and liquid–solid phases. It is often used to purify waste gas or wastewater, such as removing toxic and harmful organic materials from wastewater with activated carbon and purifying indoor air or air in automobiles with activated carbon.

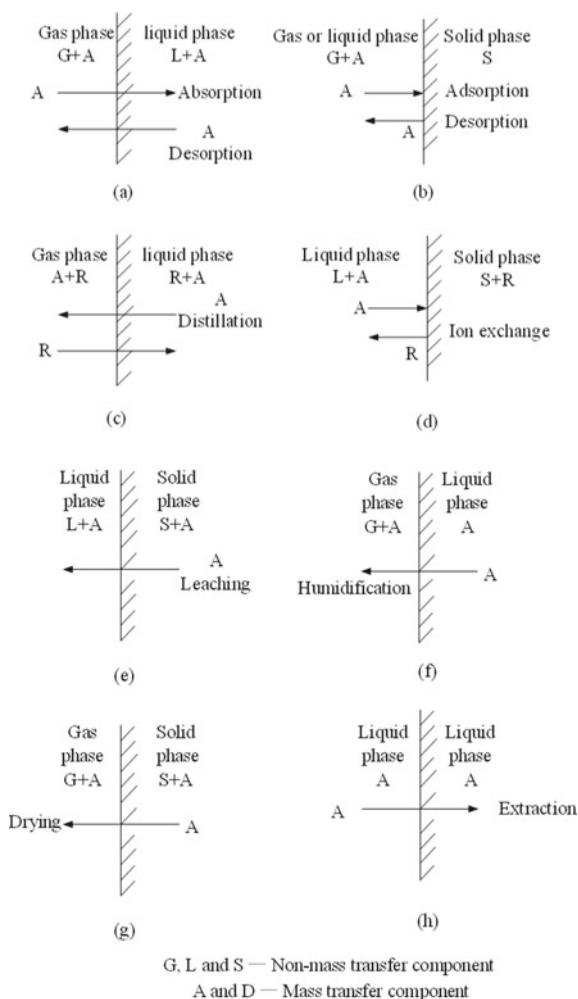
(3) Distillation

Distillation is a process in which materials transfer between liquid and gas phases to enrich volatile components in the gas phase by adding or extracting heat. The process in which volatile components are enriched in the gas phase is called distillation (Fig. 4.2c). For example, in binary distillation, volatile component A enters the gas phase from the liquid phase, and nonvolatile component R enters the liquid phase from the gas phase.

(4) Ion exchange

Exchangeable ions in an ion exchange resin exchange with ions of the same charge in water, thus removing ions from water (Fig. 4.2d). Ion exchange is often used to remove Ca^{2+} and Mg^{2+} to produce softened water and pure water, and remove certain specified pollutants with charge from water, such as heavy metals in electroplating wastewater.

Fig. 4.2 Schematic diagrams of several interphase mass transfer processes belonging to equilibrium separation process



(5) Leaching (solid–liquid extraction)

Leaching refers to the transfer of materials from a solid phase to a liquid phase (Fig. 4.2e). It is a mass transfer between solid and liquid phases and can be used to remove harmful materials from solid waste. Because solid mixtures are usually multiphase systems, leaching is also a separation process of multiphase systems, such as immersing phosphogypsum, and removing fluorine from a solid phase.

(6) Humidification

The process of moving water from a liquid phase to a gas phase (Fig. 4.2f) is a mass transfer process between the liquid and gas phases. For example, when using the electrostatic precipitation method, humidification can be used to change the electrical properties of smoke and dust.

(7) Drying

Drying usually refers to the process in which liquid (mostly water) in a solid material is gasified into a gas phase (Fig. 4.2g). This is a mass transfer between the solid and gas phases, which can be used for drying media or byproducts in environmental engineering treatments, such as activated sludge drying.

(8) Extraction

Extraction usually refers to the process of removing a solute from a liquid phase to another liquid phase that is not compatible with liquid phase (solvent) (Fig. 4.2h). This is a mass transfer process between liquid and liquid phases. This method can be used for the purification of wastewater, such as the removal of phenols from wastewater by xylene.

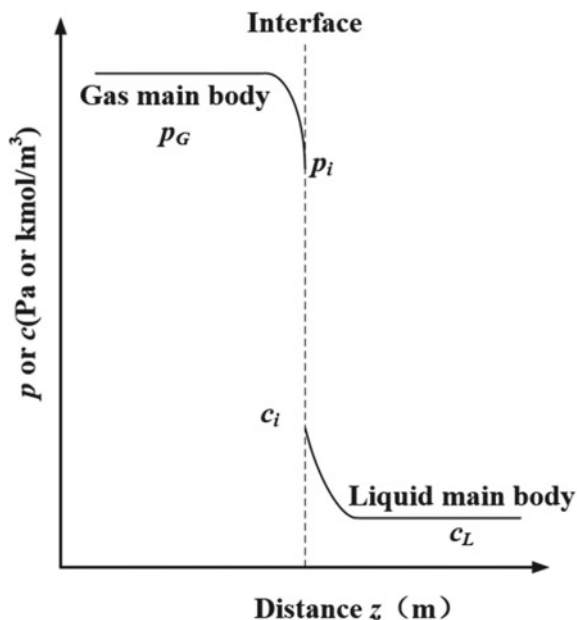
4.2 Mass Transfer Mechanism as well as Mass Transfer Rate and Flux

Phase equilibrium is only a function of the composition of two phases when a system is in equilibrium. In an actual mass transfer process, the system state deviates from the equilibrium state, thus a material can be continuously transferred from one phase to another. Otherwise, if the system state is in equilibrium, the process will terminate. In the mass transfer process, it is more important to study the rate of mass transfer (the amount of material transferred from one phase to another in unit time). If the mass transfer rate is high, the production intensity of the treatment equipment will be high, and a certain mass transfer amount can be completed with less volume equipment. Conversely, if the mass transfer rate is low, equipment with larger volume will be needed.

The relationship between the mass transfer rate and its influencing factors is expressed by the mass transfer rate equation. Therefore, the mechanism of mass transfer should be studied first.

For example, the mass transfer process between two phases, such as absorption, can be decomposed into three basic steps, as shown in Fig. 4.3. 1) The solute is transferred from the gas main body, in which the gas phase partial pressure is p_G , to a two-phase interface with the partial pressure of p_i , which is a mass transfer inside the gas phase. 2) The solute is dissolved into the liquid phase at the interface, and the liquid phase concentration of solute is c_i at the interface. 3) The solute is transferred from the interface to the main body of the liquid phase, where the liquid phase concentration of solute is c_L , which is the transfer inside the liquid phase. Regardless of whether a gas or liquid phase, the mass transfer mechanism in a single phase depends on diffusion. Diffusion can be divided into molecular diffusion and eddy current diffusion, and both are collectively called convection diffusion. The former refers to the diffusion of molecules in stationary fluid or the laminar layer, such as adding a drop of blue ink into a glass of water. The blue ink slowly disperses into the

Fig. 4.3 Schematic diagram of the mass transfer process between two phases



whole glass of water because of the thermal motion of molecules. The latter refers to the diffusion caused by the movement of fluid particles called eddy diffusion. For example, a drop of blue ink is added into a glass of water and stirred immediately. The blue ink quickly disperses into the whole glass. The diffusion in this case is mainly eddy diffusion.

4.2.1 Molecular Diffusion

If there is a concentration difference of a component in a stationary fluid, due to the irregular movement of the molecule, the component will transfer from a high concentration to a low concentration until the concentration in the fluid reaches uniformity. This phenomenon of mass transfer is called molecular diffusion.

4.2.1.1 Fick's Law

The diffusion rate of component A per unit area perpendicular to the direction of mass transfer is called diffusion flux. In a single-phase system, the rate of mass transfer by molecular diffusion follows Fick's law: The diffusion flux is proportional to the concentration gradient, and the direction of mass transfer moves along the direction of the concentration decrease. Its mathematical expression is as follows:

$$N_A = \frac{J_A}{A_d} = -D \frac{dc_A}{dZ} \quad (4.1)$$

where

N_A —Diffusion flux, diffusion rate of component A per unit area, kmol/(m²·s) or kmol/(m²·h)

J_A —Diffusion rate of diffusive component A, kmol/s or kmol/h

A_d —Diffusion area, m²

Z —Distance along the direction of diffusion, m

c_A —Concentration of diffusive component A, kmol/m³

D —Molecular diffusion coefficient, m²/s or m²/h.

The negative sign indicates that the direction of mass diffusion is opposite to that of the concentration increase.

In a steady state, the diffusion rate J_A is constant, the concentration of c_A does not change with time at each point, and the integral form of the molecular diffusion rate is as follows.

$$J_A = \frac{DA_d}{\delta_d} (c_{A1} - c_{A2}) \quad (4.2)$$

where

c_{A1}, c_{A2} —Concentration of diffusive components at points 1 and 2, kmol/m³

δ_d —Thickness of diffusion layer, m.

The above equation can be written in the form of diffusion flux:

$$N_A = \frac{J_A}{A_d} = \frac{D}{\delta_d} (c_{A1} - c_{A2}) \quad (4.3)$$

4.2.1.2 Steady Molecular Diffusion

Simple steady molecular diffusion can be divided into two types: equimolecular counterdiffusion and unidirectional diffusion.

1) Equimolecular counterdiffusion

In the process of molecular counterdiffusion, volatile component transfer from the liquid phase to the gas phase, and nonvolatile component transfer from the gas phase to the liquid phase. If the vaporization heat of the two components is equal, namely, the heat released by the one mol nonvolatile component from gas condensation to liquid phase just is equal to the heat needed by one mole volatile component transfer from liquid to gas phase. In this way, in the gas and liquid phases, the two components form equimolecular counterdiffusion. In the gas phase, the nonvolatile component diffuses from the main body of the gas phase to the gas–liquid interface, and the

volatile component diffuses from the interface to the main body of the gas phase. The diffusion fluxes of the two components are equal and opposite. Similarly, in the liquid phase, the nonvolatile component diffuses from the gas–liquid interface to the main body of the liquid phase, while the volatile components diffuse from the main body of the liquid phase to the interface.

If the mixture of two components A and B is the ideal gas and the total pressure and temperature are the same everywhere, then the total concentration of each point is the same:

$$c_T = c_A + c_B = \text{constant}$$

$$\frac{dc_T}{dZ} = \frac{dc_A}{dZ} + \frac{dc_B}{dZ} = 0$$

Thus,

$$\begin{aligned}\frac{dc_A}{dZ} &= -\frac{dc_B}{dZ} \\ N_A &= -D_{AB} \frac{dc_A}{dZ} \\ N_A &= -N_B\end{aligned}$$

Because

$$N_B = -D_{BA} \frac{dc_B}{dZ} = D_{BA} \frac{dc_A}{dZ} = D_{AB} \frac{dc_A}{dZ}$$

thus, $D_{AB} = D_{BA} = D$.

If the boundary conditions of integration are

$$Z = 0, \quad c_A = c_{A1}$$

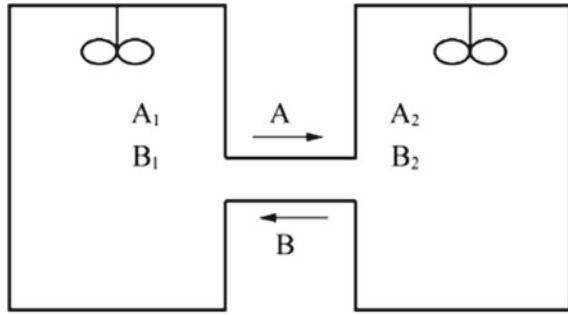
$$Z = \delta_d, \quad c_A = c_{A2}$$

The following equation is obtained

$$N_A = \frac{D}{\delta_d} (c_{A1} - c_{A2}) = \frac{\Delta c}{\delta_d/D} = \frac{\text{driving force}}{\text{resistance}} \quad (4.4)$$

If expressed by partial pressure, then

Fig. 4.4 Schematic diagram of Example 4.1



$$N_A = \frac{D}{RT\delta_d}(p_{A1} - p_{A2}) \quad (4.5)$$

where

R —Gas constant, $8.314\text{J}/(\text{mol}\cdot\text{K})$

T —Temperature, K

p_{A1}, p_{A2} —Partial pressure of component A at points 1 and 2, Pa.

Example 4.1 A certain concentration of ammonia is prepared by using nitrogen as the inert gas. As shown in Fig. 4.4, it is assumed that ammonia (A) and nitrogen (B) transfer by equimolecular counterdiffusion in the connecting pipe. The length of the connecting pipe is 0.2 m , and the diffusion coefficient is $0.23 \times 10^{-4}\text{ m}^2/\text{s}$ when the total pressure is $1.013 \times 10^5\text{ Pa}$ and the temperature is 298 K . At the same temperature, the partial pressures of ammonia gas in the two containers are $p_{A1} = 5.0 \times 10^4\text{ Pa}$ and $p_{A2} = 2.5 \times 10^3\text{ Pa}$, respectively.

Calculate: Under this condition, the diffusion flux of ammonia (A) and nitrogen (B).

Known: $\delta_d = 0.2\text{ m}$, $p_{A1} = 5.0 \times 10^4\text{ Pa}$, $p_{A2} = 2.5 \times 10^3\text{ Pa}$, $D = 0.23 \times 10^{-4}\text{ m}^2/\text{s}$, $T = 298\text{ K}$.

Determine: N_A and N_B .

Solution: According to Eq. (4.5)

$$\begin{aligned} N_A &= \frac{D}{RT\delta_d}(p_{A1} - p_{A2}) \\ &= \frac{0.23 \times 10^{-4}}{8.314 \times 298 \times 0.2} \times (5.0 \times 10^4 - 2.5 \times 10^3) \\ &= 2.204 \times 10^{-3}\text{ mol}/(\text{m}^2\cdot\text{s}) \\ p_{B1} &= p - p_{A1} = 1.013 \times 10^5 - 5.0 \times 10^4 = 5.13 \times 10^4\text{ Pa} \\ p_{B2} &= p - p_{A2} = 1.013 \times 10^5 - 2.5 \times 10^3 = 9.88 \times 10^4\text{ Pa} \\ N_B &= \frac{D}{RT\delta_d}(p_{B1} - p_{B2}) = \frac{0.23 \times 10^{-4}}{8.314 \times 298 \times 0.1} \times (5.13 \times 10^4 - 9.88 \times 10^4) \\ &= -2.204 \times 10^{-3}\text{ mol}/(\text{m}^2\cdot\text{s}) \end{aligned}$$

$N_A = -N_B$ is an equimolecular counterdiffusion process.

Answer: The diffusion fluxes of ammonia and nitrogen are both $2.204 \times 10^{-3} \text{ mol}/(\text{m}^2 \cdot \text{s})$, but in the opposite direction.

2) Unidirectional diffusion

The absorption process is unidirectional diffusion. In the absorption operation, the inert component is insoluble in the liquid; that is, it does not cross the interface between the two phases, while component A is continuously dissolved into the liquid phase, and its concentration distribution is shown in Fig. 4.3. The distribution of inert component B is contrary to component A; that is, the concentration at the interface of the two phases is higher than that of the main gas phase, so component B will diffuse from the interface to gas main body by molecular random movement. If only molecular diffusion exists, the concentration of B at the interface will decrease continuously, and the total pressure at the interface (equal to the sum of the partial pressures of A and B) will be lower than that in the gas phase, which will not occur in fact. When the pressure difference occurs, the A and B components in the gas phase are driven to flow to the interface together, which is called the main body flow. The amount of the transfer resulted from main body flow is $N'_A + N'_B$, and $N'_A:N'_B = c_A:c_B$. The amount of component B carried by the main body flow is exactly the same as that of the molecular diffusion amount of B, and the net transfer of B is kept at zero; that is, the amount of component B carried by the main body flow is equal to that of the molecular diffusion of B.

$$-N'_B = N_B = -D \frac{dc_B}{dz}$$

The main body flow is different from the diffusion flow. Diffusion flow is the macroscopic result of molecular micromotion, and the abovementioned mass transfer belongs to macroscopic flow of material.

The flux of component A in unidirectional diffusion is as follows:

$$\begin{aligned} N''_A &= N'_A + N_A = \frac{c_A}{c_B} N'_B + N_A \\ &= \frac{c_A}{c_B} (-N_B) + N_A = \frac{c_A}{c_B} D \frac{dc_B}{dz} - D \frac{dc_A}{dz} \\ &= -D \frac{c_A}{c_B} \frac{dc_A}{dz} - D \frac{dc_A}{dz} = -D \frac{c_A + c_B}{c_B} \frac{dc_A}{dz} \\ &= -D \frac{c_T}{c_T - c_A} \frac{dc_A}{dz} = \text{constant} \end{aligned}$$

The boundary conditions are

$$Z=0, \quad c_A = c_{A1}; \quad Z = \delta_d, \quad c_A = c_{A2}$$

Because c_T and D are constants, integrals are obtained.

$$\begin{aligned} N_A'' &= \frac{Dc_T}{\delta_d} \ln \frac{c_T - c_{A2}}{c_T - c_{A1}} = \frac{Dc_T}{\delta_d} \frac{\ln \frac{c_T - c_{A2}}{c_T - c_{A1}}}{c_T - c_{A2} - (c_T - c_{A1})} (c_{A1} - c_{A2}) \\ &= \frac{Dc_T}{\delta_d c_{Bm}} (c_{A1} - c_{A2}) \end{aligned} \quad (4.6)$$

The logarithmic mean concentration of c_{Bm} of component B between the interface and the main body of the liquid phase in the equation is as follows:

$$c_{Bm} = \frac{c_{B2} - c_{B1}}{\ln \frac{c_{B2}}{c_{B1}}} = \frac{c_T - c_{A2} - (c_T - c_{A1})}{\ln \frac{c_T - c_{A2}}{c_T - c_{A1}}}$$

If expressed by partial pressure, the mass transfer flux of component A is

$$N_A'' = \frac{D}{RT\delta_d} \frac{p_T}{p_{Bm}} (p_{A1} - p_{A2}) \quad (4.7)$$

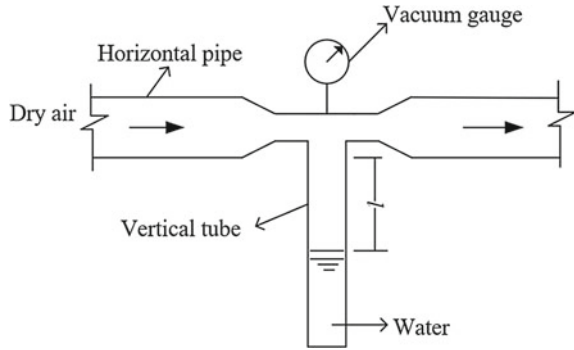
In the equation, p_{Bm} is the logarithmic mean value of the partial pressure difference of inert gas between the main body of the gas phase and the interface.

$$p_{Bm} = \frac{p_{B2} - p_{B1}}{\ln \frac{p_{B2}}{p_{B1}}} = \frac{p_T - p_{A2} - (p_T - p_{A1})}{\ln \frac{p_T - p_{A2}}{p_T - p_{A1}}}$$

Compared with the equimolecular counterdiffusion, in unidirectional diffusion, the mass transfer flux N_A of component A increases by (c_T/c_{Bm}) or (p_T/p_{Bm}) times due to the presence of main body flow. This multiple is called the drift factor, which is always >1 . When the concentration of c_A in the mixture is very low, $c_T \approx c_{Bm}$ (or $p_T \approx p_{Bm}$), and the drift factor is close to 1.

Example 4.2 The diffusion coefficient of water vapor in air is measured by means of the device diagram shown in Fig. 4.5. When the ambient pressure is 1.013×10^5 Pa, the device is placed in a 328 K thermostat. The distance between the water surface in the vertical tube and the upper pipe orifice is 0.125 m. Dry air is quickly introduced into the upper horizontal pipe, and the velocity of dry air is controlled to ensure that the partial pressure (absolute pressure) of the measured gas at the pipe orifice is approximately zero. The distance between the water surface in the vertical tube and the upper pipe orifice is 0.2 m after 3.71×10^6 s. Please calculate the diffusion coefficient of water vapor in air.

Fig. 4.5 Schematic diagram of Example 4.2



Known: $T = 328 \text{ K}$, $p_T = 1.013 \times 10^5 \text{ Pa}$, $l_1 = 0.125$, $l_2 = 0.2 \text{ m}$, $\tau_1 = 0$, $\tau_2 = 3.71 \times 10^6 \text{ s}$

Determine: D

Solution: The water surface drop in the riser is caused by water evaporation resulted from the diffusion of water molecules. When dry air passes the upper nozzle, the diffusion process can be regarded as unidirectional diffusion. When the distance between the water surface and the upper nozzle is l , the mass transfer flux of vapor diffusion is as follows:

$$N_A = \frac{D}{RT\delta_d} \frac{p_T}{p_{Bm}} (p_{A1} - p_{A2}) = \frac{D}{RTl} \frac{p_T}{p_{Bm}} (p_{A1} - p_{A2})$$

In the equation, the partial pressure of water vapor at the interface between water and air (p_{A1}) at 328 K is $1.574 \times 10^4 \text{ Pa}$, and p_{A2} is the partial pressure of water vapor at the outlet of the riser ($p_{A2} = 0$ according to the meaning of the question). The test device is under $1.013 \times 10^5 \text{ Pa}$, so p_T is $1.013 \times 10^5 \text{ Pa}$. Then:

$$\begin{aligned} p_{Bm} &= \frac{p_{B2} - p_{B1}}{\ln \frac{p_{B2}}{p_{B1}}} \\ &= \frac{1.013 \times 10^5 - (1.013 \times 10^5 - 1.574 \times 10^4)}{\ln \frac{1.013 \times 10^5}{1.013 \times 10^5 - 1.574 \times 10^4}} \\ &= 9.32 \times 10^4 \text{ Pa} \end{aligned}$$

The mass transfer flux of water diffusion in the air can be expressed by the rate of water surface descent in the pipe:

$$N_A = \frac{c_A dl}{d\tau}$$

The molar concentration of c_A in the equation is mol/m^3 of water. At 328 K , the density of water is 985.6 kg/m^3 . Then:

$$c_A = \frac{985.6 \times 10^3}{18} = 5.48 \times 10^4 \text{ mol/m}^3$$

$$N_A = \frac{c_A dl}{d\tau} = \frac{D}{RTl} \frac{p_T}{p_{Bm}} (p_{A1} - p_{A2})$$

From $\tau_1 = 0$ s, $l_1 = 0.125$ m to $\tau_2 = 3.71 \times 10^6$ s, $l_2 = 0.20$ m, the upper integral is obtained:

$$\begin{aligned} \int_{0.125}^{0.20} l dl &= \frac{D}{c_A RT} \frac{p_T}{p_{Bm}} (p_{A1} - p_{A2}) \int_0^{3.71 \times 10^6} d\tau \\ \frac{1}{2} \times (0.20^2 - 0.125^2) &= \frac{D}{5.48 \times 10^4 \times 8.314 \times 328} \times \frac{1.013 \times 10^5}{9.32 \times 10^4} \\ &\quad \times (1.574 \times 10^4 - 0) \times 3.71 \times 10^6 \\ D &= 2.87 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$

Answer: The diffusion coefficient of water vapor in air is $2.87 \times 10^{-5} \text{ m}^2/\text{s}$.

4.2.1.3 Diffusion Coefficient

The diffusion coefficient is the physical property constant of a material, which indicates the diffusion ability of the material in a homogeneous medium. According to Eq. (4.1), the physical meaning of the diffusion coefficient D is that when the concentration of the diffusion component decreases by one unit in the unit distance along the direction of diffusion, the amount of mass passing through the unit area in unit time. The unit of diffusion coefficient D is m^2/s or cm^2/s .

The factors affecting the diffusion coefficient are as follows:

- (1) The properties of diffusion components. For example, the D of oxygen in air is $2.06 \times 10^{-5} \text{ m}^2/\text{s}$, and that of ethanol in air is $1.19 \times 10^{-5} \text{ m}^2/\text{s}$.
- (2) The properties of diffusion media. For example, the D of ammonia in air is $2.36 \times 10^{-5} \text{ m}^2/\text{s}$, and that in water is $1.76 \times 10^{-9} \text{ m}^2/\text{s}$.
- (3) Temperature. The diffusion coefficient increases with temperature.
- (4) Pressure. The greater the system pressure is, the greater the diffusion resistance. Pressure has little effect on the diffusion coefficient of materials in liquids but has a great effect on gases.
- (5) Concentration. The diffusion resistance increases with concentration of diffusion components. The influence of the concentration in liquid on the diffusion coefficient is greater than that in gas.

The numerical value of the diffusion coefficient is determined experimentally. Diffusion coefficients of common materials can be found in relevant manuals.

Table 4.1 Diffusion coefficient of vapor and gas in air (298 K, 101.3 kPa) (Hu et al., 2015; He and Li, 2014)

Material	$D/(\times 10^{-5} \text{ m}^2/\text{s})$	Material	$D/(\times 10^{-5} \text{ m}^2/\text{s})$
NH ₃	2.36	Ethanol	1.19
SO ₂	1.64	Propyl alcohol	1.00
H ₂	4.10	Butanol	0.900
O ₂	2.06	Amyl alcohol	0.700
H ₂ O	2.56	Hexanol	0.590
CS ₂	1.07	Formic acid	1.59
Ethyl ether	0.980	Acetic acid	1.33
Methanol	1.59	Propionic acid	0.990

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Table 4.2 Diffusion coefficient of solute in solvent (293 K) (Hu et al. 2015; He and Li, 2014)

Material	Solvent	$D/(\times 10^{-9} \text{ m}^2/\text{s})$	Material	Solvent	$D/(\times 10^{-9} \text{ m}^2/\text{s})$
O ₂	Water	1.80	Acetic acid	Water	0.880
CO ₂	Water	1.50	Methanol	Water	1.28
NH ₃	Water	1.76	Ethanol	Water	1.00
Cl ₂	Water	1.22	Lactose	Water	0.430
Br ₂	Water	1.20	Glucose	Water	0.600
H ₂	Water	5.13	Sodium chloride	Water	1.35
N ₂	Water	1.64	Trichloromethane	Ethanol	1.54
HCl	Water	2.64	Phenol	Benzene	2.11
H ₂ S	Water	1.41	Trichloromethane	Benzene	1.92
H ₂ SO ₄	Water	1.73	Acetic acid	Benzene	0.880
HNO ₃	Water	2.60	Acetone	Water	1.16

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Tables 4.1 and 4.2 show the diffusion coefficients of several common materials in air and solvents.

1) Diffusion coefficients in gases

The diffusion coefficients in gases are related to the system, temperature and pressure. The order of magnitude is $10^{-5} \text{ m}^2/\text{s}$. Generally, for the mutual diffusion of binary gases A and B, the diffusion coefficient of A in B is equal to that of B in A. Therefore, the subscript can be omitted and denoted by the same symbol D , namely, $D_{AB} = D_{BA} = D$.

Because diffusion coefficients are not only related to the nature of materials but also vary with the diffusion media, the data in the manual cannot be complete and often need to be estimated according to empirical equations.

The diffusion coefficient of diffusion component A in gas B is usually estimated by the Maxwell-Gilliland empirical equation:

$$D = \frac{4.36 \times 10^{-5} T^{3/2}}{p \left(V_{mA}^{1/3} + V_{mB}^{1/3} \right)^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}} \quad (4.8)$$

where

D —Molecular diffusion coefficient, m^2/s

p —Total pressure, kPa

T —Temperature, K

M_A, M_B —Molar mass of A and B, g/mol, respectively

V_{mA}, V_{mB} —Molar volumes of A and B at the normal boiling point, m^3/mol , respectively.

The molar volume of components can be calculated by the density of pure liquid components at the normal boiling point. Table 4.3 lists the molar volumes of some common gases. For more complex molecules, their molar volumes can be regarded as the sum of the atomic volumes of the constituent elements. The atomic volumes can generally be found in the relevant manuals. Table 4.4 lists the atomic volumes of some common elements.

Equation (4.8) shows that the diffusion coefficient in gas is directly proportional to the $3/2$ power of absolute temperature and inversely proportional to the total pressure of the system. Namely:

$$D = D_0 \frac{p_0}{p} \left(\frac{T}{T_0} \right)^{3/2} \quad (4.9)$$

where

D —Diffusion coefficient of gas, m^2/s

D_0 —Diffusion coefficient of gas at a pressure of p_0 and T_0 , m^2/s

p —Pressure, Pa

Table 4.3 Molar volumes of some common gases (Lin, 1999)

Gas	Molar volume/(cm^3/mol)	Gas	Molar volume/ cm^3/mol	Gas	Molar volume/(cm^3/mol)
H_2	14.3	CO_2	34.0	H_2O	18.9
O_2	25.6	SO_2	44.8	H_2S	32.9
N_2	31.2	NO	23.6	Cl_2	48.4
Air	29.9	N_2O	36.4	Br_2	53.2
CO	30.7	NH_3	25.8	I_2	71.5

Table 4.4 Atomic volume of some common elements (Lin, 1999)

Element	Atomic volume /(cm ³ /mol)	Element	Atomic volume /(cm ³ /mol)
H	3.7	N	15.6
C	14.8	N in primary amine	10.5
F	8.7	N in secondary amine	12.0
Cl at one end, for example: R—Cl	21.6	O	7.4
Cl in the center, for example R—CHCl—R	24.6	O in methyl ester	9.1
Br	27.0	O in ethyl ester and methyl/ethyl ether	9.9
I	37.0	O in higher esters and ethers	11.0
S	25.6	O in acid	12.0
P	27.0	O combined with N, S, P	8.3

p_0 —Pressure, Pa

T —Temperature, K

T_0 —Temperature, K.

According to Eq. (4.9), the diffusion coefficient D of the gas system at temperature T and pressure p can be calculated from the known diffusion coefficient D_0 of the gas system at temperature T_0 and pressure p_0 .

Example 4.3 In a warehouse, the SO₂ gas storage tank leaks. The ambient pressure is 1.013×10^5 Pa (absolute pressure), and the temperature is 25 °C.

Calculate: Diffusion coefficient of SO₂ gas in air

Known: $p = 1.013 \times 10^5$ Pa, $T = 25^\circ\text{C} = 298$ K, $M_A = M_{\text{SO}_2} = 64$ g/mol, $M_B = M_{\text{air}} = 29$ g/mol

Determine: D

Solution: Let component A be SO₂ and component B be air

$$T = 25 + 273 = 298 \text{ K}$$

Table 4.3 shows that $V_{mA} = V_{m\text{SO}_2} = 44.8 \text{ cm}^3/\text{mol} = 4.48 \times 10^{-5} \text{ m}^3/\text{mol}$, $V_{mB} = V_{m\text{air}} = 29.9 \text{ cm}^3/\text{mol} = 2.99 \times 10^{-5} \text{ m}^3/\text{mol}$.

Calculations based on Eq. (4.8)

$$D = \frac{4.36 \times 10^{-5} T^{3/2}}{p \left(V_{mA}^{1/3} + V_{mB}^{1/3} \right)^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$$

$$\begin{aligned}
 D &= \frac{4.36 \times 10^{-6} T^{3/2}}{p(V_{mA}^{1/3} + V_{mB}^{1/3})^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}} \\
 &= \frac{4.36 \times 10^{-5} \times 298^{3/2}}{1.013 \times 10^5 \times [(4.48 \times 10^{-5})^{1/3} + (2.99 \times 10^{-5})^{1/3}]^2} \sqrt{\frac{1}{64} + \frac{1}{29}} \\
 &= 1.12 \times 10^{-5} \text{ m}^2/\text{s}
 \end{aligned}$$

Answer: The diffusion coefficient of SO₂ in air is $1.12 \times 10^{-5} \text{ m}^2/\text{s}$.

2) Diffusion coefficient in liquids

The diffusion coefficient of a solute in solution is related to the properties of the solute and solvent, the temperature of the solution, and the viscosity and concentration of the solvent. Because the molecules in liquid are much denser than those in gas and the movement of molecules is not as free as those in gas, the diffusion coefficient of liquid is much smaller than that of gas, and its order of magnitude is $10^{-9} \text{ m}^2/\text{s}$. For very dilute non-electrolyte solutions, the diffusion coefficients of materials in liquids can be estimated by the Wilke–Chang empirical equation:

$$D = 1.859 \times 10^{-18} \frac{(\alpha_a M)^{1/2} T}{\mu V_m^{0.6}} \quad (4.10)$$

where

D —Solute diffusion coefficient in liquid, m^2/s

T —Temperature, K

M —Molar mass of solvent, g/mol

μ —Viscosity of the solvent, Pa · s

V_m —Molar volume of solute, cm^3/mol

α_a —Association parameters of a typical solvent.

The association parameters of some commonly used solvents are shown in Table 4.5.

Example 4.4 In natural water, dissolved oxygen content is an important index used to judge the quality of water. According to Eq. (4.10), please deduce the diffusion coefficient of oxygen in water at 20 °C.

Known: $T = 20 \text{ }^\circ\text{C}$,

Determine: D

Table 4.5 Association parameters of some commonly used solvents (Lin, 1999)

Solvent	Water	Methanol	Ethanol	Benzene	Ethyl ether ^a
Association parameters	2.6	1.9	1.5	1.0	1.0

^a Solvents with association parameters of 1.0 are nonassociative solvents

Solution: $T = 20 + 273 = 293 \text{ K}$

For water: $\mu = 100.5 \times 10^{-5} \text{ Pa} \cdot \text{s}$, $\alpha_a = 2.6$, $M = 18$.

From Table 4.3, molar volume of O_2 : $V_m = 25.6 \text{ cm}^3/\text{mol} = 25.6 \times 10^{-6} \text{ m}^3/\text{mol}$.

Calculations based on Eq. (4.10)

$$\begin{aligned} D &= 1.859 \times 10^{-18} \frac{(\alpha_a M)^{1/2} T}{\mu V_m^{0.6}} \\ &= 1.859 \times 10^{-18} \times \frac{(2.6 \times 18)^{1/2} \times 293}{100.5 \times 10^{-5} \times (25.6 \times 10^{-6})^{0.6}} \\ &= 2.12 \times 10^{-9} \text{ m}^2/\text{s} \end{aligned}$$

Answer: The diffusion coefficient of O_2 in water at 20°C is approximately $2.12 \times 10^{-9} \text{ m}^2/\text{s}$.

4.2.2 Convection diffusion

Convection diffusion refers to the diffusion and mass transfer process between the moving fluid and phase interface. The integral equation of molecular diffusion mentioned above applies only to stationary or laminar fluids. In practice, fluids always flow, and the flow of fluids enhances the mass transfer inside the phase.

In turbulent flow, fluid particles move irregularly in all directions, resulting in vortices in the fluid. Because of the existence of vortices, the rate of mass transfer is much greater than that of molecular diffusion alone, which is called eddy diffusion. The mass transfer in real turbulent fluid depends on both molecular diffusion and eddy diffusion, which are called convection diffusion. The eddy diffusion flux is also proportional to the concentration gradient. Therefore, the convection diffusion flux equation can also be written in a form similar to Eq. (4.1):

$$N_A = -D \frac{dc_A}{dZ} - D_w \frac{dc_A}{dZ} = -(D + D_w) \frac{dc_A}{dZ} \quad (4.11)$$

where

N_A —Diffusion flux, $\text{kmol}/(\text{m}^2 \cdot \text{s})$ or $\text{kmol}/(\text{m}^2 \cdot \text{h})$

D —Molecular diffusion coefficient, m^2/s or m^2/h

c_A —Concentration of diffusive component A, kmol/m^3

Z —Distance along the direction of diffusion, m

D_w —Eddy diffusion coefficient, m^2/s .

The negative sign indicates that the direction of mass diffusion is opposite to that of the concentration increase.

Although the eddy diffusion coefficient (D_w) and the molecular diffusion coefficient (D) have the same unit, the molecular diffusion coefficient (D) is the physical property of the material and is related to the properties of the medium in which it is

located, while the eddy diffusion coefficient (D_w) is affected by many factors, such as fluid flow condition, diffusion location, flow velocity, structure size of the fluid system, fluid viscosity, density and so on.

In stationary or laminar fluids, the eddy diffusion is zero, i.e., $D_w = 0$. In highly turbulent flow, the eddy diffusion coefficient D_w is much greater than the molecular diffusion coefficient D , and D_w is almost 100 times D in strong turbulence. Therefore, to increase the diffusion rate in the fluid phase, it is necessary to make the fluid flow in turbulent conditions and increase its turbulence degree as much as possible.

4.2.2.1 Mechanism of Convection–Diffusion Mass Transfer Process

At present, the understanding of convection diffusion is lacking, so a method similar to convection heat transfer is adopted to deal with the problem of convection diffusion. Therefore, some simplified mass transfer models are proposed to clarify the mechanism of convective mass transfer. One of the earliest and most representative models is the stagnant-film model. The basic arguments of the stagnant-film model can be summarized as follows.

There is a virtual stagnant film near the interface of a fluid. The concentration of the turbulent zone outside the film is the same. The resistance of convective mass transfer in the phase is concentrated in the virtual stagnant film, as shown in Fig. 4.6. The virtual stagnant film of the gas phase δ'_G is thicker than that of the laminar sublayer of gas δ_G . The thickness of the virtual stagnant film of the liquid phase δ'_L is thicker than that of the laminar sublayer of liquid δ_L . Assuming that the stagnant

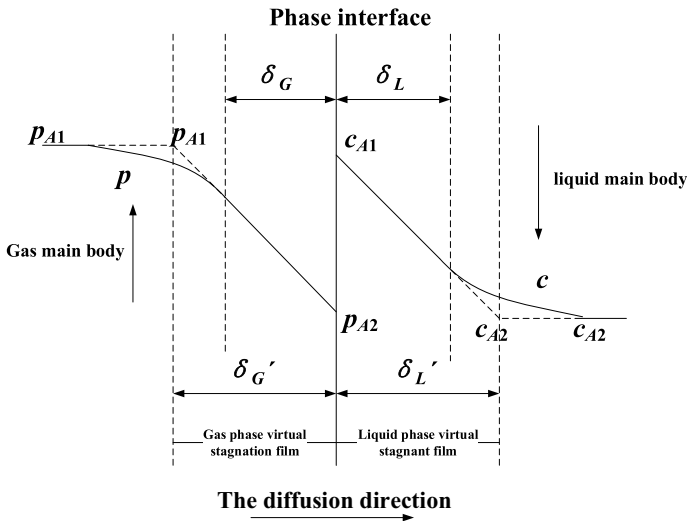


Fig. 4.6 Schematic diagram of stagnant-film model

film is very thin, the capacity of solute in the film at any time is negligible compared with the mass transfer rate, it can be considered that there is no accumulation of solute in the film, and solute diffuses through the stagnant film in a steady state. Therefore, the convection–diffusion flux of one component through another can be written as follows.

In gaseous phase:

$$N_A = \frac{D}{RT\delta'_G} \frac{p_T}{p_{Bm}} (p_{A1} - p_{A2}) \quad (4.12)$$

In liquid phase:

$$N_A = \frac{Dc_T}{\delta'_L c_{Bm}} (c_{A1} - c_{A2}) \quad (4.13)$$

In the equation, δ'_G and δ'_L are the thicknesses of the virtual stagnant film in the gas phase and liquid phase, respectively.

4.2.2.2 Convection–Diffusion Mass Transfer Flux Equation

According to the stagnant film model, the mass transfer flux through the virtual stagnant film is shown in Eq. (4.12), but δ'_G is a function of the gas flow condition, which is difficult to measure. In fact, following Newton's cooling law of the convective heat transfer rate, the above relationship is expressed by the mass transfer flux equation.

In order

$$k_G = \frac{D}{RT\delta'_G} \frac{p_T}{p_{Bm}} \quad (4.14)$$

Equation (4.12) becomes

$$N_A = k_G (p_{A1} - p_{A2}) \quad (4.15)$$

In the equation, k_G is the gas phase mass transfer partial coefficient expressed by driving force of the partial pressure difference, $\text{kmol}/(\text{s} \cdot \text{m}^2 \cdot \text{kPa})$.

It is also difficult to determine δ'_L in Eq. (4.13).

$$k_L = \frac{Dc_T}{\delta'_L c_{Bm}} \quad (4.16)$$

Then, Eq. (4.13) becomes

$$N_A = k_L (c_{A1} - c_{A2}) \quad (4.17)$$

In the equation, k_L is the liquid mass transfer partial coefficient expressed by the driving force of the concentration difference, $\text{kmol}/(\text{s} \cdot \text{m}^2 \cdot \text{kmol}/\text{m}^3)$ or m/s .

The stagnant film model is used to classify all the complex factors affecting mass transfer flux into the mass transfer coefficient. Equations (4.16) and (4.17) show that there are two ways to increase the mass transfer flux: (1) increase the mass transfer partial coefficients, k_G or k_L ; (2) increase the mass transfer driving forces, $p_{A1} - p_{A2}$ or $c_{A1} - c_{A2}$.

4.2.2.3 Correlation of Dimensionless Numbers for Convection–Diffusion Mass Transfer Coefficient

Mass transfer partial coefficient in gas or liquid phase can be directly determined according to the actual system, equipment and specific operating conditions, or the correlation of dimensionless numbers between the mass transfer partial coefficient and various influencing factors can be found through model tests, which can be used for similar amplification design calculations. For the case of gas flowing through the liquid film on the pipe wall, when $Re > 2100$ and $Sc = 0.6 \sim 3000$ ($Sc = 0.5 \sim 3$ for gas and $Sc > 100$ for liquid), the typical correlation equation of dimensionless numbers obtained from the experiment is as follows:

$$Sh = 0.023 Re^{0.83} Sc^{0.33} \quad (4.18)$$

where

Sh —Sherwood number, $Sh = \frac{k_L d}{D} \cdot \frac{c_{Bm}}{c_T} = \frac{k_G d RT}{D} \cdot \frac{p_{Bm}}{p_T}$, dimensionless

Sc —Schmidt number, $Sc = \frac{\mu}{\rho D}$, dimensionless

Re —Reynolds number, $Re = \frac{du\rho}{\mu}$, dimensionless.

4.3 Gas-Liquid Phase Equilibrium

4.3.1 Overview

At a certain temperature and pressure, when the gas and liquid phases are fully contacted, the absorbate in the gas phase is transferred to the liquid phase absorbent and absorbed. At the same time, the absorbate absorbed in the liquid phase may also be desorbed by transferring from the liquid phase to the gas phase. At the beginning, absorption is the main process. With the increasing concentration of absorbate in the liquid phase, the mass transfer rate of the absorbate from the gas phase to the liquid phase gradually slows down, and the desorption rate from the liquid phase to the gas

phase gradually accelerates. Finally, the absorption and desorption rates are equal, and a dynamic equilibrium of gas and liquid phases is achieved. At this time, the concentration of absorbate in the gas phase and liquid phase does not change any more. The concentration of solution is the equilibrium concentration, which is also the maximum solubility of absorbate in solution, also known as equilibrium solubility. The partial pressure of absorbate gas above the solution is called equilibrium partial pressure.

Under certain conditions, the equilibrium between the two phases is governed by the phase law: Degree of freedom = Components number – Phase number + 2. For example, the gas–liquid system formed by NH_3 –air–water has two phases and three components (air is regarded as an inert component), so it can be obtained from the equation.

$$\begin{aligned}\text{Degree of freedom} &= \text{Components number} - \text{Phase number} + 2 \\ &= 3 - 2 + 2 = 3\end{aligned}$$

That is, the degree of freedom is equal to 3. If the total pressure and temperature are constant, then there is only one variable left to choose at will. If the concentration of NH_3 in the gas phase is constant, the concentration of NH_3 in the liquid phase is also constant.

However, the phase law cannot give the concentration of absorbate in the liquid phase at a certain gas phase concentration, which must be measured by experiments or calculated by relevant equations. Equilibrium experimental data are necessary to predict the composition of absorbates in two phases at equilibrium. If the two phases are unequilibrium, the absorbate mass will be transferred between them. The mass transfer flux is proportional to the mass transfer driving force, namely, the degree of deviation from the equilibrium, so the calculation of the driving force also needs equilibrium data. Temperature, pressure and concentration are the main variables affecting the gas–liquid equilibrium of absorbates.

4.3.2 Solubility Curve

The gas–liquid phase equilibrium relationship can be expressed by table lists, graphs or equations. The gas-liquid phase equilibrium curve drawn in two-dimensional coordinates is called a solubility curve, which can be found in relevant manuals. Figures 4.7, 4.8 and 4.9 show the solubility curves of SO_2 , NH_3 and O_2 in water at different temperatures, respectively.

It can be seen in the graphs that the solubility of each gas in water increases with increasing partial pressure and decreases with increasing temperature, which is the general trend of the solubility change of gas in liquid. It can also be seen in the figures that at a certain temperature, the partial pressure of soluble gas (such as NH_3) is low and that of insoluble gas (such as O_2) is high, corresponding to solutions of the same concentration. In other words, to obtain the same concentration of solution, the partial pressure required for soluble gases is low, and is high for insoluble gases.

Fig. 4.7 Solubility curve of SO_2 in water (Hu et al., 2018; Cardarelli, 2018; Lin et al., 2003). *Source* Reprinted from book “*Principles of Environmental Engineering*” with kind permission from Prof. Hongying Hu, as well as from book “*Materials Handbook*” with kind permission from Springer Nature

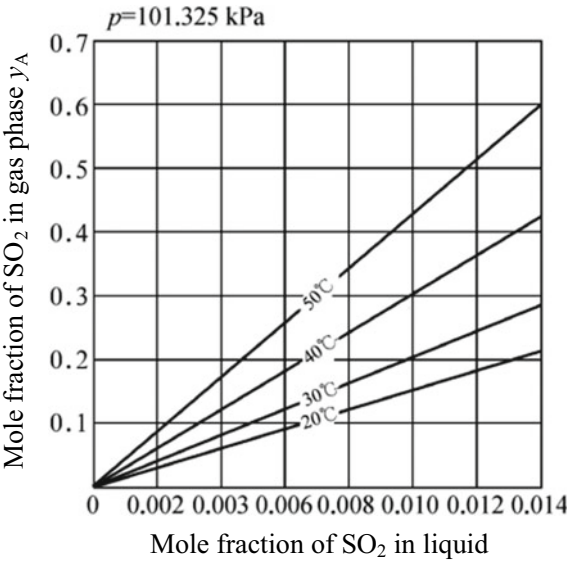


Fig. 4.8 Solubility curve of O_2 in water (Hu et al., 2018; Cardarelli, 2018; Lin et al., 2003). *Source* Reprinted from book “*Principles of Environmental Engineering*” with kind permission from Prof. Hongying Hu, as well as from book “*Materials Handbook*” with kind permission from Springer Nature

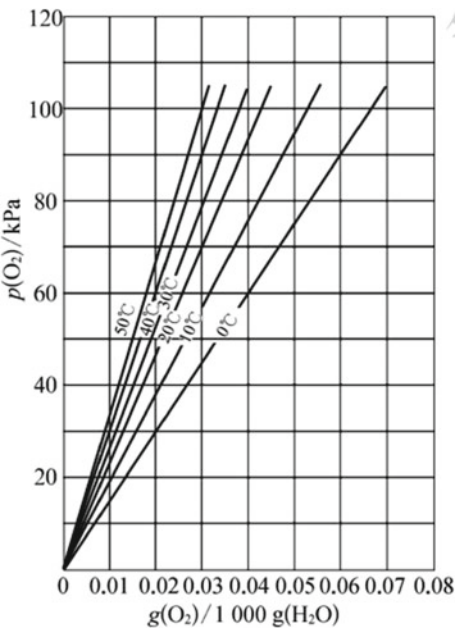
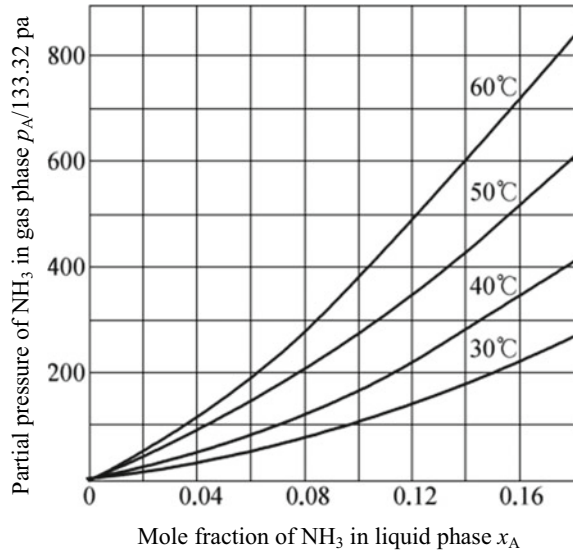


Fig. 4.9 Solubility curve of NH_3 in water (Hu et al. 2018; Cardarelli, 2018; Lin et al., 2003). Source Reprinted from book “*Principles of Environmental Engineering*” with kind permission from Prof. Hongying Hu, as well as from book “*Materials Handbook*” with kind permission from Springer Nature



4.3.3 Henry's Law

At a certain temperature and low total gas pressure, the concentration of solute in the liquid phase is proportional to its equilibrium partial pressure in the gas phase insoluble gases and most gases with a dilute solution. This law is called Henry's law, and its mathematical expression is as follows.

$$p_A^* = H_{pm} x_A \quad (4.19)$$

where

p_A^* —Equilibrium partial pressure of solute A in the gas phase, kPa

x_A —Mole fraction of solute A in solution, dimensionless

H_{pm} —Henry's coefficient, kPa.

$$\text{Mole fraction } x = \frac{\text{mass of solute/molar mass of solute}}{\frac{\text{mass of solute}}{\text{molar mass of solute}} + \frac{\text{mass of solvent}}{\text{molar mass of solvent}}} = \frac{X}{1 + X}$$

$$\text{Molar ratio } X = \frac{\text{mass of solute/molar mass of solute}}{\text{mass of solvent/molar mass of solvent}} = \frac{x}{1 - x}$$

Henry’s coefficient depends on the properties of the material and the temperature of the system. If the solutes or solvents are different, or the system is different, the H_{pm} is different. The value of H_{pm} indicates the degree of difficulty or easiness of gas absorption. The higher the value of H_{pm} is, the more difficult it is for the gas to be absorbed, and vice versa. For a certain system, Henry’s coefficient increases with temperature.

Henry’s coefficient H_{pm} of some gases in different solvents can be found in manuals and related books and periodicals. Table 4.6 lists the H_{pm} values of some gases in water. The Henry’s coefficient is an empirical equation based on experimental results. The linear relationship between gas and liquid only exists in a certain concentration range, as shown in Fig. 4.9.

Henry’s law $p_A^* = H_{pm}x_A$ can also be expressed in the following three common forms because of the different expressions of the composition of solutes in gas and liquid phases:

Table 4.6 Henry’s coefficient H_{pm} of some gases in water (Hu et al., 2015; He and Li, 2014)

Gas	Temperature/°C								
	0	10	20	30	40	50	60	80	100
	$H_{pm} \times 10^{-6}/\text{kPa}$								
H ₂	5.87	6.44	6.92	7.38	7.61	7.75	7.73	7.65	7.55
N ₂	5.36	6.77	8.15	9.36	10.54	11.45	12.16	12.77	12.77
Air	4.37	5.56	6.73	7.81	8.82	9.56	10.23	10.84	10.84
CO	3.57	4.48	5.43	6.28	7.05	7.71	8.32	8.56	8.57
O ₂	2.58	3.31	4.06	4.81	5.42	5.96	6.37	6.96	7.10
CH ₄	2.27	3.01	3.81	4.55	5.27	5.85	6.34	6.91	7.10
NO	1.71	2.21	2.68	3.14	3.57	3.95	4.24	4.54	4.60
C ₂ H ₆	1.28	1.92	2.67	3.47	4.29	5.07	5.72	6.70	7.01
C ₂ H ₄	0.56	0.78	1.03	1.29	—	—	—	—	—
	$H_{pm} \times 10^{-4}/\text{kPa}$								
N ₂ O	—	14.29	20.26	26.24	—	—	—	—	—
CO ₂	7.38	10.54	14.39	18.85	23.61	28.68	34.55	—	—
C ₂ H ₂	7.30	9.73	12.26	14.79	—	—	—	—	—
H ₂ S	2.72	3.72	4.89	6.17	7.55	8.96	10.44	13.68	15.00
Br ₂	0.22	0.37	0.60	0.92	1.35	1.94	2.54	4.09	—
SO ₂	0.17	0.24	0.35	0.48	0.65	0.86	1.1	1.68	—
NH ₃	0.0208	0.0240	0.0277	0.0321	—	—	—	—	—

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(1) Expressed by molar concentration

$$p_A^* = \frac{c_A}{H_{pc}} \quad (4.20)$$

where

p_A^* —Equilibrium partial pressure of solute A in gas phase, kPa

c_A —Molar concentration of solute A in solution, kmol/m³

H_{pc} —Solubility coefficient, kmol/(m³·kPa).

In contrast with H_{pm} , the higher the H_{pc} value is, the higher the gas solubility, and the easier the solute can be absorbed by the solvent. The solubility coefficient H_{pc} decreases with increasing temperature.

The conversion relationship between H_{pc} and H_{pm} is deduced as follows.

Comparisons (4.19) and sums (4.20) are obtained.

$$H_{pm}x_A = \frac{c_A}{H_{pc}}$$

The molar concentration of the solute in the solution is

$$c_A = c_T x_A = \frac{\rho_T}{Mx_A + (1 - x_A)M_0} x_A$$

For dilute solutions, x_A is very small, and ρ_T is approximately equal to ρ_0 , so

$$c_A = \frac{\rho_0}{M_0} x_A$$

Thus,

$$\begin{aligned} H_{pm}x_A &= \frac{\frac{\rho_0}{M_0} x_A}{H_{pc}} \\ H_{pc} &= \frac{1}{H_{pm}} \frac{\rho_0}{M_0} \end{aligned} \quad (4.21)$$

where

c_T —Total molar concentration of solution, kmol/m³

ρ_T —Density of solution, kg/m³

M —Molar mass of solute A, kg/kmol

M_0 —Molar mass of solvent, kg/kmol

ρ_0 —Solvent density, kg/m³.

(2) Expressed in mole fraction

$$y_A^* = m_e x_A \quad (4.22)$$

where

y_A^* —Equilibrium concentration of solute A in gas phase, mole fraction, dimensionless

x_A —Mole fraction of solute A in solution, dimensionless

m_e —Phase equilibrium constant, dimensionless.

The conversion relationship between m_e and H_{pm} is deduced as follows:

The total pressure p_T , for ideal gas, according to Dalton's law of partial pressure:

$$p_A = p_T y_A$$

Substitute into the previous Eq. (4.19) and obtain:

$$\begin{aligned} p_T y_A^* &= H_{pm} x_A \\ y_A^* &= \frac{H_{pm}}{p_T} x_A \end{aligned}$$

In comparing above equation with Eq. (4.22),

$$m_e = \frac{H_{pm}}{p_T} \quad (4.23)$$

The higher the m_e value is, the smaller the solubility of the solute. The value of m_e decreases with increasing total pressure or decreasing temperature. Therefore, higher total pressure and lower temperature are beneficial for absorption and vice versa for desorption.

(3) Expressed in molar ratio

$$Y_A^* = \frac{m_e X_A}{1 + (1 - m_e) X_A} \quad (4.24a)$$

The composition of gas and liquid phases is expressed by the molar ratio of materials. Equation (4.24a) can be obtained by transforming y_A^* and x_A in Eq. (4.22) as follows.

$$\begin{aligned} y_A^* &= \frac{Y_A^*}{1 + Y_A^*} \\ x_A &= \frac{X_A}{1 + X_A} \end{aligned}$$

For dilute solutions, if the X_A value is very low and the denominator of Eq. (4.24a) approaches 1, Henry's law can be approximately expressed as

$$Y_A^* = m_e X_A \quad (4.24b)$$

where

Y_A^* —Molar ratio of solute A to inert component in gas phase, dimensionless

X_A —Molar ratio of solute A to solvent in solution, dimensionless

m_e —Phase equilibrium constant, dimensionless.

The Henry's coefficient H_{pm} , solubility coefficient H_{pc} , and phase equilibrium constant m_e in the above equations were determined experimentally. Although Eq. (4.24b) is the most convenient method for the calculation of absorption, only the Henry's coefficient H_{pm} is listed in physical or chemical manuals in general. The value of the phase equilibrium constant m_e can be obtained by Eq. (4.23). For gases that do not conform to Henry's law, to meet the needs of engineering, experiments are usually made by determining final solute absorption equilibrium concentration in liquid phase and its partial pressure in gas phase under different conditions such as initial partial pressure or concentration. The gas–liquid equilibrium data are obtained and then listed or plotted for practical application.

Example 4.5 Under normal temperature and pressure, the dissolved oxygen in water can be rapidly increased by aeration with a blower. The known temperature is 25 °C, and the pressure of the air blower is 3 atm, in which the mole fraction of O_2 is 0.21. It is known that the dissolution of oxygen in water under this condition conforms to Henry's law.

Calculate: Under this condition, with the solubility coefficient of O_2 in water and the phase equilibrium constant, how many grams of dissolved oxygen per cubic meter water can be obtained?

$$\text{known } T = 25^\circ\text{C}, p_T = 3.03 \times 10^5 \text{ Pa}, y = 0.21, p_{O_2} = 0.21 p_T,$$

$$H_{pm} = 4.44 \times 10^9 \text{ Pa}$$

Determine the following: H_{pc} , m_e , $C_{A\max}$.

Solution: The Henry's coefficient H_{pm} of O_2 is $4.44 \times 10^9 \text{ Pa}$ at 25 °C, as obtained from Table 4.6. The density of the solution is approximately equal to the density of water. When $\rho_0 = 996.95 \text{ kg/m}^3$, the solubility coefficient H_{pc} is as follows:

$$\begin{aligned} H_{pc} &= \frac{1}{H_{pm}} \frac{\rho_0}{M_0} \\ &= \frac{1}{4.44 \times 10^9} \times \frac{996.95}{18} = 1.25 \times 10^{-8} \text{ kmol}/(\text{m}^3 \cdot \text{Pa}) \end{aligned}$$

The phase equilibrium constant m_e is:

$$\begin{aligned} m_e &= \frac{H_{pm}}{p_T} \\ &= \frac{4.44 \times 10^9}{3.03 \times 10^5} = 1.47 \times 10^4 \end{aligned}$$

The partial pressure of oxygen in the gas phase is:

$$p_A = p_T y_A = 3.03 \times 10^5 \times 0.21 = 6.36 \times 10^4 \text{ Pa}$$

Because the solubility of oxygen in water is very low, the partial pressure p_A of oxygen in the gas phase is approximately equal to the equilibrium partial pressure p_A^* .

$$\begin{aligned} p_A^* &= \frac{c_{A\max}}{H_{pc}} \\ c_{A\max} &= p_A^* \cdot H_{pc} = p_A \cdot H_{pc} = 6.36 \times 10^4 \times 1.25 \times 10^{-8} \\ &= 7.95 \times 10^{-4} \text{ kmol/m}^3 \end{aligned}$$

The maximum amount of oxygen dissolved in per cubic meter of aqueous solution:

$$7.95 \times 10^{-4} \times 10^3 \times 32 = 25.44 \text{ g}$$

Answer: The solubility coefficient H_{pc} of O_2 in water is 1.251×10^{-8} kmol/(m³·Pa), and the phase equilibrium constant m_e is 1.47×10^4 , the maximum amount of oxygen dissolved in per cubic meter of aqueous solution is 25.44 g.

4.3.4 Phase Equilibrium and Absorption Process

1) Phase equilibrium can determine the direction of the absorption process

When gas–liquid two-phase contact occurs, the composition of one phase and another can be determined by the gas–liquid phase equilibrium relationship. Compared with the actual composition, the direction of the process can be judged, that is, absorption or desorption. As shown in Fig. 4.10(a), the gas and liquid phases in a absorption tower are in close contact. The mole fraction of the gas phase containing absorbate is y_A and that of the liquid phase containing absorbate is x_A at a certain cross section of the absorption tower. In the y_A – x_A coordinate chart shown in Fig. 4.10(c) ~ (e), point A is above the equilibrium line, and the gas phase composition, which is in equilibrium with liquid phase concentration x_A , is y_A^* . Since $y_A > y_A^*$, the absorbate A should be transferred from the gas to liquid phase, and the absorption process occurs. Similarly, the difference between liquid phase concentration and equilibrium concentration can also be judged. In Fig. 4.10(c) ~ (e), it can be seen that the liquid phase composition equilibrium with gas phase composition y_A is x_A^* , while $x_A < x_A^*$, so absorbate A should be transferred from the gas to the liquid phase. In contrast, point B in Fig. 4.10(c) ~ (e) will be desorbed from the liquid to gas phase because $y_B < y_B^*$ or $x_B > x_B^*$.

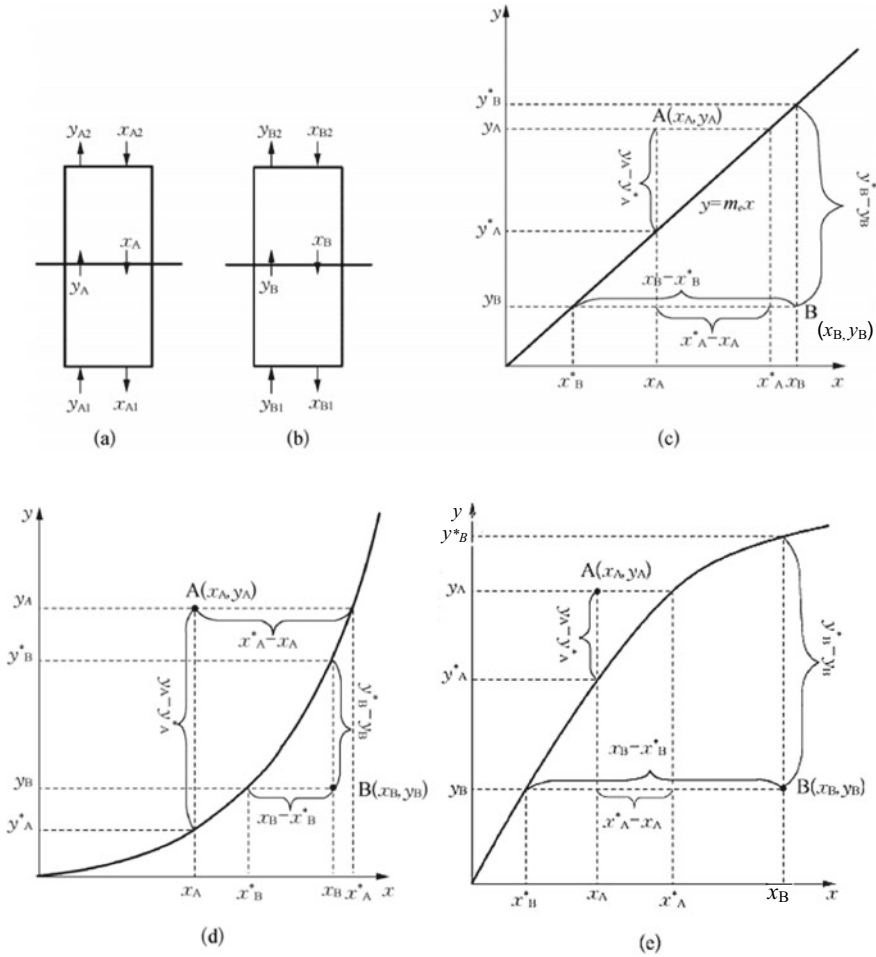


Fig. 4.10 Relationship between phase equilibrium and absorption process

2) **Phase equilibrium can be used to calculate the driving force of the absorption process**

Equilibrium is the limit state of the mass transfer process. Gas absorption or desorption occurs when gas and liquid do not reach equilibrium when they contact each other. The farther the actual concentration deviates from the equilibrium concentration, the greater the driving force of the absorption process and the faster the absorption rate. In the absorption process, the driving force of absorption is expressed as the difference between the actual concentration and the equilibrium concentration. For A in Fig. 4.10(c) ~ (e), when the gas phase concentration is y_A and the liquid phase concentration is x_A , the liquid equilibrium concentration related to y_A is x_A^* , and the gas phase equilibrium concentration related to x_A is y_A^* . Then, $y_A - y_A^*$ is the absorption driving force expressed by the gas phase concentration difference, and

$x_A^* - x_A$ is the absorption driving force expressed by the liquid phase concentration difference. Similarly, for B in Fig. 4.10(c) ~ (e), when the gas phase concentration is y_B and the liquid phase concentration is x_B , the liquid equilibrium concentration related to y_B is x_B^* , and the gas phase equilibrium concentration related to x_B is y_B^* . The gas phase concentration difference between $y_B^* - y_B$ is the desorption driving force expressed by the gas concentration difference, and $x_B - x_B^*$ is the desorption driving force expressed by the liquid phase concentration difference.

3) Phase equilibrium can determine the limit of the absorption process

The equilibrium state is the limit of the process. For a countercurrent absorption process (Fig. 4.10(a)), with increasing tower height and absorbent dosage, the composition y_{A2} of absorbate A in the outlet gas will decrease accordingly. However, even if the tower is infinitely high and the absorbent dosage is infinitely great, the composition of y_{A2} in the outlet gas will not be lower than that of y_{A2}^* in equilibrium with the composition of x_{A2} in the inlet of the absorbate in liquid. For the gas phase, according to Henry's law, the minimum value of gas at the top of the tower is:

$$y_{A2 \min} = y_{A2}^* = m_e x_{A2}$$

Similarly, the composition of absorbate A (x_A) in the solution at the outlet of the tower bottom has a maximum value x_{A1}^* . For the liquid phase, according to Henry's law, the maximum absorption liquid phase concentration flowing from the bottom of the tower is:

$$x_{A1 \max} = x_{A1}^* = \frac{y_{A1}}{m_e}$$

Example 4.6 The air contains SO_2 of 0.5 (mole fraction, the same below) leaked from a gas cylinder contacted and dissolved into a solution with the SO_2 of 0.01 in a tank inside a hermetic warehouse. It is known that the gas–liquid phase equilibrium relationship of SO_2 is $y_A^* = 34.6x_A$ at 101.3 kPa and 20 °C. Please determine the direction of the phase equilibrium process when the polluted air contacted with the solution.

Known: $y_A^* = 34.6x_A$, $x_A = 0.01$, $y = 0.5$

Determine: Process direction

Solution: The gas phase equilibrium concentration y_A^* of SO_2 related to x_A in solution is as follows:

$$y_A^* = 34.6x_A = 34.6 \times 0.01 = 0.346 < 0.5 = y_A$$

Thus, 0.346 is less than the actual concentration of the gas phase 0.5. Therefore, as a result of gas–liquid contact, SO_2 will diffuse from the gas to the liquid, i.e., absorption.

$$y = 34.6x_A^* \quad x_A^* = y/34.6 = 0.5/34.6 = 0.0145 > 0.01 = x_A$$

Answer: SO_2 will diffuse from the gas phase to the liquid phase.

4.4 Kinetic Basis of Absorption

4.4.1 Overview

Absorption is a unit operation process aimed at mass transfer and separation. The absorption process is widely used in industry for ① the preparation and refinement of products, such as HCl absorption by water to produce hydrochloric acid; ② the purification of feed gas, such as removal of impurities H_2S , CO_2 , and CO from feed gas during synthetic ammonia industry; and ③ the separation of gas components, such as oil absorption of petroleum cracking gas to separate the C_2 component from CH_4 and H_2 . In the field of environmental engineering, the absorption process is applied to waste gas treatment, such as the absorption of odor from sewage treatment plants, which can remove H_2S , NH_3 , mercaptan and so on. In the waste gas treatment process, the absorption process can not only remove harmful pollutants in the waste gas but also transform some pollutants into useful products for comprehensive utilization. For example, the desulfurization of flue gas from coal-fired power plants can produce ammonium sulfate, gypsum and other products. Diethanolamine solution can absorb hydrogen sulfide from petroleum tail gas and further produce sulfur. Therefore, the absorption process has broad application prospects in the field of environmental engineering, especially in the treatment of air pollution. In environmental engineering, absorption is applied not only to waste gas treatment but also to sewage treatment plants. For example, the oxygen needed for the growth of aerobic microorganisms in the aerobic activated sludge process usually comes from the absorption of oxygen in compressed air into wastewater. Ozone is only absorbed firstly in wastewater, and then it can oxidize pollutants in wastewater during ozone oxidation treatment for wastewater.

Absorption is a unit operation process of mass transfer and separation of gas mixtures by dissolving the components of gas mixtures with appropriate solvents with different solubilities in solvents. The solvent used in the absorption operation is called absorbent, and the gas component that can be dissolved significantly in the solvent is called absorbate or solute. The almost insoluble component is called inert component, the solution with or without absorbate or solute is called absorption liquid or liquid, as well as absorption solution or solution, and the gas completing absorption process is called absorption tail gas or purified gas.

The absorption process can be divided into physical absorption and chemical absorption according to the type of interaction between the absorbent and absorbate. In physical absorption, the absorbate is dissolved only in the absorbent and does not react with the absorbent. For example, water absorbs O_2 , carbon dioxide, ethanol vapor, acetone vapor and so on in gas phase. If there is a chemical reaction between

the absorbate and the absorbent in the absorption process, this absorption is called chemical absorption, such as absorption of CO_2 , SO_2 and H_2S in gas by NaOH solution.

Gas components dissolved in liquids are often accompanied by heat resulted from solution or reaction. The absorption process can be divided into isothermal absorption and non-isothermal absorption according to the change in solution temperature. If the thermal effect is large, the temperature of the solution will rise. Such an absorption process is called non-isothermal absorption. If the thermal effect is small or the thermal effect is large but the amount of absorbent is large, the solution temperature remains basically unchanged, then this kind of absorption is called isothermal absorption.

The absorption process can be divided into single-component absorption and multicomponent absorption according to the number of gas components. When only one component is absorbed in the absorption process, it is called single-component absorption. For example, water is used to absorb HCl in gas to produce hydrochloric acid. The absorption of two or more components is called multicomponent absorption, such as CO_2 , SO_2 , NO_2 , and NO in flue gas absorption by water.

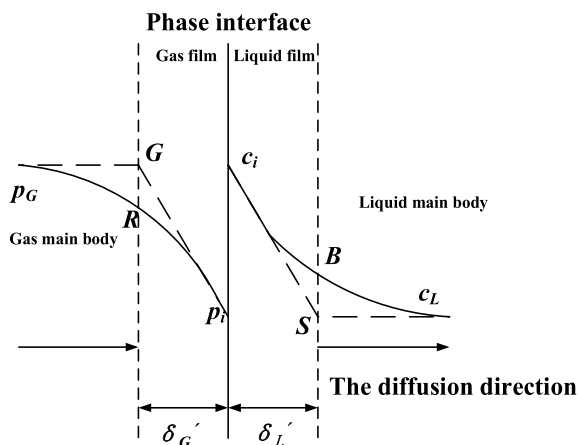
After the absorption process is completed, the absorbent dissolves the useful components or pollutants in the gas mixture and becomes the absorption solution. To obtain useful components or remove pollutants completely, it is often necessary to desorb the absorption solution. Desorption is the operation of releasing solute dissolved in absorbent from liquid to gas phase. For example, high-concentration ammonia water is desorbed by steam stripping to obtain pure ammonia in gas. Wastewater containing ammonia is treated by air stripping. Desorption can not only obtain high purity gas components but also regenerate and recycle the absorbent. Therefore, the combined process of absorption and desorption is adopted in industry.

4.4.2 Two-Film Theory

Gas absorption is a complex process of interphase mass transfer. In this process, solutes are transferred from the main body of the gas phase to the interface of the two phases, dissolved through the interface to the liquid phase, and finally transferred from the liquid at the interface to the main body of the liquid phase. Among them, the mass transfer at the interface of two phases is faced to the equilibrium between gas and liquid, while inside a single phase (liquid or gas phase), the mass transfer is faced to only the mass diffusion in a homogeneous phase (liquid or gas phase). Figure 4.11 shows the mass transfer process and concentration distribution between the gas and liquid phases.

In absorption equipment, the gas and liquid phases are usually turbulent, and the solute is transferred from high concentration to low concentration through convection diffusion. Convection diffusion includes molecular diffusion and eddy current diffusion. This is related not only to the physical properties of gas and liquid phases but also to the operating parameters (such as flow rate, pressure and temperature)

Fig. 4.11 Schematic diagram of two-film theory model



of the system and the characteristics of the equipment. Moreover, the situation of the interface between two phases changes constantly in the process of gas-liquid mass transfer. Therefore, for such a complex case, the mass transfer flux equation of the absorption process should be established according to the simplified physical model. The two-film theory model is the simplest and most practical model. The basic assumptions for establishing the model are as follows.

- (1) There is a stable phase interface between gas and liquid, and there is a virtual stagnant gas film and stagnant liquid film on both sides of the interface. Solute components diffuse through the two films in a steady state.
- (2) At the phase interface, the gas and liquid phases reach equilibrium once they contact, i.e., $p_i = c_i/H_{pc}$, and there is no mass transfer resistance at the interface.
- (3) Beyond the two virtual stagnant films at the phase interface, the gas and liquid fluids are fully turbulent with uniform composition and no mass transfer resistance. The mass transfer resistance of the solute in each phase is concentrated in the virtual stagnant film.

Based on the above assumptions, the gas-liquid interphase mass transfer mechanism is simplified as a stable molecular diffusion process of solute components through gas-liquid two-layer virtual films, as shown in Fig. 4.11. This process conforms to Fick's law and a steady molecular diffusion process, including molecular counterdiffusion and unidirectional diffusion. It is assumed that broken lines $p_G G p_i$ and $c_i S c_L$ represent actual concentration changes $p_G R p_i$ and $c_i B c_L$. The film thickness is assumed to be δ_G' and δ_L' . Since it has been assumed that the gas-liquid two-phase equilibrium at the interface can be expressed as $p_i = f(c_i)$, the relative position of p_i and c_i is determined by the phase equilibrium relationship.

The two-film model is suitable for mass transfer between two fluids with a fixed phase interface and a free interface with low velocity, but it has some limitations for mass transfer between two fluids with high turbulence. However, as far as engineering mass transfer is concerned, the two-film theory and the interphase mass transfer flux

thereby determined are the main basis for the analysis and calculation of the mass transfer process and equipment design.

4.4.3 Absorption Mass Transfer Flux Equation

1) Gas-liquid equilibrium under general (usual) conditions

Based on the two-film theory and the single-phase mass transfer flux equation, the equation of gas and liquid film absorption mass transfer flux can be written out.

$$N_{A,G} = k_G(p_G - p_i) \quad (4.25)$$

$$N_{A,L} = k_L(c_i - c_L) \quad (4.26)$$

where

$N_{A,G}$, $N_{A,L}$ —Gas film, liquid film absorption mass transfer flux, $\text{kmol}/(\text{m}^2 \cdot \text{s})$

p_G —Partial pressure of absorbate in the main body of gas phase, kPa

p_i —Partial pressure of absorbate at interface, kPa

c_i —Concentration of absorbate at the interface, kmol/m^3

c_L —Concentration of absorbate in the main body of liquid phase, kmol/m^3

k_G —Absorption mass transfer partial coefficient in gas film, $\text{kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$

k_L —Absorption mass transfer partial coefficient in liquid film, $\text{kmol}/(\text{m}^2 \cdot \text{s} \cdot (\text{kmol}/\text{m}^3))$ or m/s .

2) Gas-liquid phase equilibrium conforming to Henry's law

To avoid the interface partial pressure p_i and concentration c_i , which are difficult to measure, when the gas-liquid phase equilibrium relationship conforms to Henry's law, the interface partial pressure p_i can be replaced by the gas phase equilibrium partial pressure (p_L^*), which is equilibrated with the liquid phase concentration c_L . The interface liquid concentration c_i can be replaced by the liquid equilibrium concentration (c_G^*), which is equilibrated with the partial pressure of the absorbate in the main body of the gas phase p_G . Because the gas-liquid equilibrium conforms to Henry's law, $p_i = c_i/H_{pc}$, $p_L^* = c_L/H_{pc}$, and $N_A = N_{A,G} = N_{A,L}$ in the steady state. Substitute them into Eq. (4.26), it can be obtained:

$$N_A = \frac{p_i - p_L^*}{\frac{1}{H_{pc}k_L}} \quad (4.27)$$

Simultaneously, Eq. (4.25) is transformed to

$$N_A = \frac{p_G - p_i}{\frac{1}{k_G}} \quad (4.28)$$

According to the additivity of the driving force and the drag force in the series process, the absorption mass transfer flux is equal to the sum of the driving forces of the gas and liquid films and the ratio of the corresponding drag force. Equations (4.27) and (4.28) are synthesized.

$$N_A = \frac{p_G - p_L^*}{\frac{1}{k_G} + \frac{1}{H_{pc}k_L}} \quad (4.29)$$

In order,

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{1}{H_{pc}k_L} \quad (4.29a)$$

The absorption mass transfer flux equation expressed by the gas phase partial pressure difference is obtained.

$$N_A = K_G(p_G - p_L^*) \quad (4.30)$$

where

K_G —Overall absorption mass transfer coefficient of the driving force expressed by the partial pressure difference of the gas phase, $\text{kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$

p_G —Partial pressure of absorbate in the main body of gas phase, kPa

p_L^* —Partial pressure of absorbate in gas phase equilibrated with the main body concentration of liquid phase c_L , kPa .

Similarly, substitute $p_G = c_G^*/H_{pc}$ and $p_i = c_i/H_{pc}$ into Eqs. (4.27) and (4.28), combine the two equations, it can be obtained:

$$N_A = \frac{c_G^* - c_L}{\frac{H_{pc}}{k_G} + \frac{1}{k_L}} \quad (4.31)$$

In order,

$$\frac{1}{K_L} = \frac{H_{pc}}{k_G} + \frac{1}{k_L} \quad (4.31a)$$

The total absorption mass transfer flux equation expressed by the liquid phase concentration difference is obtained as

$$N_A = K_L(c_G^* - c_L) \quad (4.32)$$

where

K_L —Overall absorption mass transfer coefficient of the driving force expressed by the liquid phase concentration difference, m/s

c_L —Concentration of absorbate in the main body of liquid phase, kmol/m³

c_G^* —Liquid phase concentration in equilibrium with the partial pressure p_G of the absorbate in the main body gas phase, kmol/m³.

Equations (4.30) and (4.32) are two mass transfer flux equations, which only use different expressions for the driving force but describe the same process, so they are equivalent. From Eqs. (4.29a) and (4.31a),

$$K_G = H_{pc} K_L \quad (4.33)$$

When the gas film resistance is much greater than the liquid film resistance and the solubility coefficient H_{pc} is very great, for example, when water absorbs NH₃ or HCl gas, i.e., $\frac{1}{k_G} \gg \frac{1}{H_{pc} k_L}$ (The driving force is expressed by the partial pressure difference of the gas phase.) or $\frac{H_{pc}}{k_G} \gg \frac{1}{k_L}$ (The driving force is expressed by the concentration difference of the liquid phase), the process of interphase mass transfer is gas film control, then $K_G \approx k_G$ or $K_L \approx \frac{k_G}{H_{pc}}$.

When the liquid film resistance is much greater than that of the gas film and H_{pc} is very small, for example, when water absorbs O₂ or CO₂ gas, i.e., as $\frac{1}{H_{pc} k_L} \gg \frac{1}{k_G}$ (The driving force is expressed by the partial pressure difference of the gas phase.) or $\frac{1}{k_L} \gg \frac{H_{pc}}{k_G}$ (The driving force is expressed by the concentration difference of the liquid phase.), the mass transfer process between phases is controlled by the liquid film, and then $K_G \approx H_{pc} k_L$ or $K_L \approx k_L$.

When the mass transfer resistance of gas and liquid films is of the same order of magnitude, neither of them can be neglected. The total absorption mass transfer flux is controlled by the combined resistance of the two films. Gas absorption with moderate solubility, such as absorption of SO₂ by water, is the result.

3) Absorption and mass transfer flux equation with molar ratio difference as driving force

In many cases, it is more convenient to use the molar ratio difference to represent the driving force than the partial pressure difference or concentration difference in the calculation. At this time, the gas and liquid films and the total absorption mass transfer flux equation can be obtained by similar deduction as before.

Gas–liquid equilibrium under general (usual) conditions.

The equation of gas film absorption mass transfer flux is as follows:

$$N_A = k_Y(Y - Y_i) \quad (4.34)$$

The equation of liquid film absorption mass transfer flux is as follows:

$$N_A = k_X (X_i - X) \quad (4.35)$$

where

k_Y, k_X —Absorption mass transfer partial coefficient driven by the gas film molar ratio difference ($Y - Y_i$) and liquid film molar ratio difference ($X_i - X$) respectively, $\text{kmol}/(\text{m}^2 \cdot \text{s})$

Y, X —Molar ratio of absorbate in the main body of the gas phase and liquid phase, respectively, dimensionless

Y_i, X_i —Molar ratio of absorbate at the interface respectively, dimensionless.

If gas-liquid equilibrium conforms to Henry's law, total absorption mass transfer flux equation:

$$N_A = K_Y (Y - Y^*) \quad (4.36)$$

$$N_A = K_X (X^* - X) \quad (4.37)$$

In the equation:

K_Y, K_X —Overall absorption mass transfer coefficients driven by molar ratio difference ($Y - Y^*$) and ($X^* - X$), respectively, $\text{kmol}/(\text{m}^2 \cdot \text{s})$

X^*, Y^* —Molar ratios of absorbate materials in equilibrium with gas Y and liquid X , respectively, dimensionless.

4) Transformational relationships among absorption mass transfer coefficients in different flux equations

The transformational relationship among the mass transfer coefficients of gas film absorption:

Substitute $y = \frac{Y}{1 + Y}$ and $p = p_T y$ into Eq. (4.25) to obtain

$$\begin{aligned} N_A &= k_G (p_G - p_i) \\ &= k_G p_T \frac{Y - Y_i}{(1 + Y)(1 + Y_i)} \end{aligned}$$

Comparing Eq. (4.34) with the equations above:

$$k_Y = \frac{k_G p_T}{(1 + Y)(1 + Y_i)} \quad (4.38)$$

Similarly, substitute $x = \frac{X}{1 + X}$ and $c = c_T x$ into Eq. (4.26) to obtain:

$$N_A = k_L (c_i - c_L)$$

$$= k_L c_T \frac{X_i - X}{(1 + X)(1 + X_i)}$$

The transformational relationship among the absorption mass transfer coefficients of liquid films obtained by comparing above equation with Eq. (4.35):

$$k_X = \frac{k_L c_T}{(1 + X)(1 + X_i)} \quad (4.39)$$

Similarly, the transformational relationship among the overall absorption mass transfer coefficients can be obtained.

$$K_Y = \frac{K_G p_T}{(1 + Y)(1 + Y^*)} \quad (4.40)$$

$$K_X = \frac{K_L c_T}{(1 + X)(1 + X^*)} \quad (4.41)$$

If the concentration of gas and liquid phases is very small, the denominator in the above transformational equations of absorption mass transfer coefficients tends to be 1, which can be simplified as follows:

$$k_Y = k_G p_T \quad (4.42)$$

$$k_X = k_L c_T \quad (4.43)$$

$$K_Y = K_G p_T \quad (4.44)$$

$$K_X = K_L c_T \quad (4.45)$$

5) Physical significance of the absorption mass transfer coefficient

The physical significance of absorption mass transfer coefficients (k_G , k_L , K_G , K_L , k_Y , k_X , K_Y , K_X , k_y , k_x , K_y , K_x) is the diffusion amount (kmol) of mass transfer per unit time passing through the unit diffusion area in the gas or liquid phase, or from the gas to the liquid phase, when the absorption driving force corresponds to a unit partial pressure difference, concentration difference, molar ratio difference or mole fraction difference. The commonly used absorption mass transfer flux equations and mass transfer coefficients are listed in Table 4.7. All forms of mass transfer flux are equivalent, and their units same, namely, kmol/(m²·s).

Table 4.8 lists those relationship equations among overall mass transfer coefficients and mass transfer partial coefficients in liquid or gas film. There are corresponding relationships between k_L and k_X , K_L and K_X , which represent the liquid phase mass transfer coefficients; k_G and k_Y , K_G and K_Y , which represent the gas phase mass transfer coefficients.

Table 4.7 Mass transfer flux equations

Mass transfer flux equation	Driving force		Mass transfer coefficient		Expression forms of material content of gas and liquid	Corresponding phase equilibrium equation
	Expression	Unit	Symbol	Unit		
$N_A = k_G(p_G - p_i)$	$p_G - p_i$	kPa	k_G	$\text{kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$	Partial pressure	General equation
$N_A = k_L(c_i - c_L)$	$c_i - c_L$	kmol/m^3	k_L	m/s	concentration	
$N_A = K_G(p_G - p_G^*)$	$p_G - p_G^*$	kPa	K_G	$\text{kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$	Partial pressure	Conform to Henry's Law
$N_A = K_L(c_L^* - c_L)$	$c_L^* - c_L$	kmol/m^3	K_L	m/s	concentration	
$N_A = k_Y(Y - Y_i)$	$Y - Y_i$	Dimensionless	k_Y	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	molar ratio	General equation
$N_A = k_X(X_i - X)$	$X_i - X$	Dimensionless	k_X	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	molar ratio	
$N_A = K_Y(Y - Y^*)$	$Y - Y^*$	Dimensionless	K_Y	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	molar ratio	Conform to Henry's Law
$N_A = K_X(X^* - X)$	$X^* - X$	Dimensionless	K_X	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	molar ratio	
$N_A = k_y(y - y_i)$	$y - y_i$	Dimensionless	k_y	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	mole fraction	General equation
$N_A = k_x(x_i - x)$	$x_i - x$	Dimensionless	k_x	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	mole fraction	
$N_A = K_y(y - y^*)$	$y - y^*$	Dimensionless	K_y	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	mole fraction	Conform to Henry's Law
$N_A = K_x(x^* - x)$	$x^* - x$	Dimensionless	K_x	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	mole fraction	

$$Y^* = \frac{m_e X}{1 + (1 - m_e)}$$

$$y^* = m_e x$$

$$y^* = m_e x + b$$

Table 4.8 Interconversion equations among mass transfer coefficients

Expressions of gas and liquid material contents			Partial pressure or concentration of components	Mole fraction or molar ratio
Phase equilibrium relation			$p_G^* = c_L/H_{pc} + a$ or $p_G^* = c_L/H_{pc}$	$y^* = m_e x$ or $y^* = m_e x + b$
The relationship between overall absorption mass transfer coefficients and absorption mass transfer partial coefficients in gas or liquid film	Only if relationship conforms to Henry's Law can these equations be applied	–	$K_G = \frac{1}{\frac{1}{k_G} + \frac{1}{H_{pc}k_L}}$ $K_L = \frac{1}{\frac{H_{pc}}{k_G} + \frac{1}{k_L}}$ $K_G = H_{pc}K_L$	$K_y = \frac{1}{\frac{1}{k_y} + \frac{m_e}{k_x}}$ $K_x = \frac{1}{\frac{1}{m_e k_y} + \frac{1}{k_x}}$
The relation equation of number of film mass transfer molecules and mass transfer coefficient in gas or liquid film	Not conforms to Henry's law, these equations are also applicable	General equation	$k_Y = \frac{k_G p_T}{(1+Y)(1+Y_i)} \quad (34)$ $k_X = \frac{k_L c_T}{(1+X)(1+X_i)} \quad (35)$	$k_Y = k_y \cdot \frac{1}{(1+Y)(1+Y_i)}$ $k_X = k_x \cdot \frac{1}{(1+X)(1+X_i)}$
		Dilute solution	$k_Y = k_G p_T \quad (38)$ $k_X = k_L c_T \quad (39)$ $k_Y = k_G p_T$ $k_X = k_L c_T$	$k_y = k_Y$ $k_x = k_X$
The relation equation of total adsorption mass transfer coefficient	Only if relationship conforms to Henry's Law can these equations be applied	General equation	$K_Y = \frac{K_G p_T}{(1+X)(1+Y^*)} \quad (36)$ $K_X = \frac{K_L c_T}{(1+X)(1+X^*)} \quad (37)$ $K_G = H_{pc}K_L \quad (20)$	$K_Y = K_y \cdot \frac{1}{(1-y)(1-y^*)}$ $K_X = K_x \cdot \frac{1}{(1-x)(1-x^*)}$ $K_y m_e = K_x$
		Dilute solution	$K_Y = K_G p_T \quad (40)$ $K_X = K_L c_T \quad (41)$ $K_Y = \frac{p_T H_{pc}}{c_T} K_X$	$K_y = K_Y$ $K_x = K_X$

6) Analysis of the total absorption mass transfer flux equation

Figure 4.12(a) shows a schematic diagram of an absorption system. The coordinates of point M are p_G and c_L , which are composed of gas and liquid phases at a cross section of the absorption tower. p_G and c_L are generally unequilibrated, and the M point is above the equilibrium curve OE. The liquid phase composition in equilibrium with the partial pressure p_G of the absorbate in the main body of the gas phase is at point B, $c = c_G^* = H_{pc}p_G$, and the gas phase partial pressure in equilibrium with the concentration c_L of the absorbate in the main body of the liquid phase is at point A, $p = p_L^* = c_L/H_{pc}$. The MA line represents the total driving force $(p_G - p_L^*)$ expressed by the gas phase partial pressure difference, and the MB line represents the total

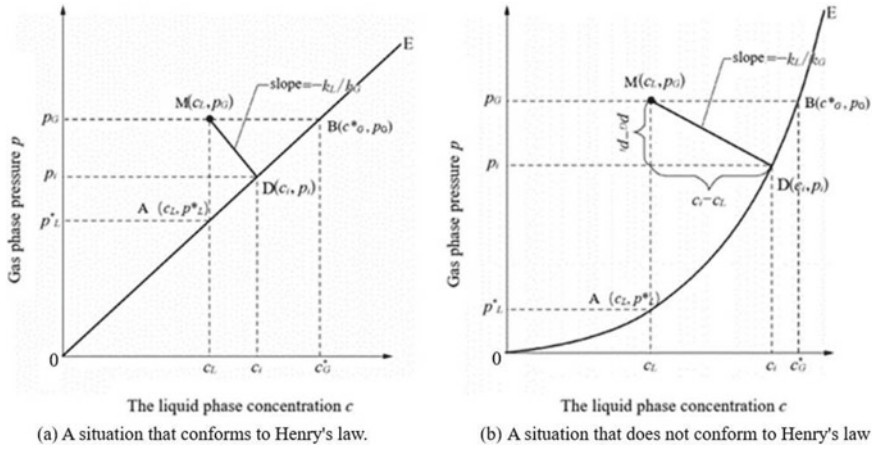


Fig. 4.12 Phase interface composition and mass transfer driving force

driving force $(c_G^* - c_L)$ expressed by the liquid phase concentration difference. The more the actual concentration (c_L, p_G) deviates from the equilibrium state, the farther the M point is above the equilibrium line, the greater the mass transfer driving force $(p_G - p_L^*)$ or $(c_G^* - c_L)$. The closer the actual concentration is to the equilibrium state, the closer the M point is to the equilibrium line, and the smaller the mass transfer driving force. If the actual concentration has reached equilibrium, the M point falls on the equilibrium line, and the mass transfer does not occur. When the M point falls below the equilibrium line, the process becomes desorption.

7) Composition of interface

When the composition of gas and liquid phases does not conform to Henry's law, the phase equilibrium line shown in Fig. 4.12(b) is a curve. At this time, because H_{pc} cannot be determined, the mass transfer flux cannot be calculated according to the total absorption mass transfer Eq. (4.30) or Eq. (4.32). Only the interface composition (c_i, p_i) can be calculated, which can be calculated by using the equation of gas film absorption mass transfer flux (4.25) or the liquid film absorption mass transfer flux equation (4.26).

- ① Under steady conditions, $N_{A,G} = N_{A,L}$, and the following equation can be obtained according to Eqs. (4.25) and (4.26):

$$N_{A,G} = k_G(p_G - p_i) = N_{A,L} = k_L(c_i - c_L)$$

- ② Then,

$$-\frac{k_L}{k_G} = \frac{p_i - p_G}{c_i - c_L} \quad (4.46)$$

If the gas–liquid two-phase composition is c_L and p_G at a section M of the absorption tower, and the mass transfer partial coefficient in gas film and the mass transfer partial coefficient in liquid film are known, the interface composition $D(c_i, p_i)$ can be obtained by a graphic method. MD is a straight line through point M with a slope of $-k_L/k_G$, and it intersects point D with the equilibrium curve.

Figure 4.12 also shows the driving forces of convective mass transfer:

$$\text{Gas phase: } \Delta p = p_G - p_i$$

$$\text{Liquid phase: } \Delta c = c_i - c_L$$

The total driving force of interphase mass transfer is:

$$\Delta p = p_G - p_L^*, \Delta c = c_G^* - c_L$$

Example 4.7 Ammonia is recovered by an absorption-desorption system in a factory. The composition of ammonia in the gas and liquid phase is $y = 0.02$, $x = 0.05$ (both mole fractions), the mass transfer coefficient in gas and liquid film are $k_y = 0.03 \text{ kmol}/(\text{m}^2 \cdot \text{s})$, $k_x = 0.05 \text{ kmol}/(\text{m}^2 \cdot \text{s})$ in a monitoring section of the desorption tower. Under this operating condition, the gas–liquid equilibrium equation of ammonia $y = 0.8x$. Please determine the total driving force, total resistance, mass transfer flux, distribution of the total driving force on both sides and composition of the gas–liquid interface of two-phase mass transfer in this section.

Known: $y = 0.02$, $x = 0.05$, $k_y = 0.03 \text{ kmol}/(\text{m}^2 \cdot \text{s})$, $k_x = 0.05 \text{ kmol}/(\text{m}^2 \cdot \text{s})$, $y = 0.8x$, $m_e = 0.8$

Determine the following: Δy , $\frac{1}{K_y}$, N_A , x_i , y_i , driving force in the gas or liquid film

Solution: Total mass transfer driving force

$$\Delta y = y^* - y = m_e x - y = 0.8 \times 0.05 - 0.02 = 0.02 > 0$$

This process is desorption.

Total resistance:

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m_e}{k_x} = \frac{1}{0.03} + \frac{0.8}{0.05} = 49.33 \text{ m}^2 \cdot \text{s}/\text{kmol}$$

Mass transfer flux:

$$N_A = K_y(y^* - y) = \frac{1}{49.33} \times (0.8 \times 0.05 - 0.02) = 4.05 \times 10^{-4} \text{ kmol}/(\text{m}^2 \cdot \text{s})$$

Simultaneous with

$$N_A = k_y(y_i - y) = k_x(x - x_i)$$

$$y_i = m_e x_i$$

$$4.05 \times 10^{-4} = 0.03 \times (y_i - 0.02) = 0.05 \times (0.05 - x_i)$$

$$y_i = 0.8x_i$$

The interface concentrations $x_i = 0.0419$ and $y_i = 0.0335$ are obtained.

Therefore, driving force in the gas film $= y_i - y = 0.0335 - 0.02 = 0.0135$

$$\text{Driving force in the liquid film} = x - x_i = 0.05 - 0.0419 = 0.0081$$

Answer: The total driving force of two-phase mass transfer is 0.02, and the total resistance is $49.33(\text{m}^2 \cdot \text{s})/\text{kmol}$, and the mass transfer flux is $4.05 \times 10^{-4} \text{ kmol}/(\text{m}^2 \cdot \text{s})$, and the driving force of the gas film is 0.0135, and the driving force of the liquid film is 0.0081, and the mole fraction of the gas phase interface is 0.0335, and the mole fraction of the liquid phase interface is 0.0419.

Example 4.8 On a cross section of an absorption tower operating at 20°C and a total pressure of 202.6 kPa, the gas containing 0.04 (mole fraction) of component A meets a solution with a concentration of $0.9 \text{ kmol}/\text{m}^3$ of component A. The gas film mass transfer coefficient (k_G) is $5 \times 10^{-6} \text{ kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$, and the liquid film mass transfer coefficient (k_L) is $1.5 \times 10^{-4} \text{ m/s}$. The equilibrium relationship of component A can be expressed by Henry's law, and the solubility coefficient is $0.73 \text{ kmol}/(\text{m}^3 \cdot \text{kPa})$.

Please calculate:

- (1) The composition of two phases on the gas-liquid interface.
- (2) The driving force of the gas film and liquid film is expressed by the pressure difference and molar concentration difference.
- (3) The total driving force and molar transfer coefficient are expressed by the pressure difference and molar concentration difference.
- (4) The overall mass transfer coefficient with the driving force expressed by the molar ratio difference.
- (5) Mass transfer fluxes calculated in at least six ways.
- (6) Gas film resistance, liquid film resistance and total resistance of mass transfer with driving force expressed by a pressure difference.
- (7) Gas film resistance, liquid film resistance and total resistance of mass transfer with driving force expressed by molar concentration differences.

Known: $p_T = 202.6 \text{ kPa}$, $y_G = y_A = 0.04$, $T = 273 + 20 = 293 \text{ K}$,

$$c_L = c_A = 0.9 \text{ kmol}/\text{m}^3, k_G = 5 \times 10^{-6} \text{ kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa}), k_L = 1.5 \times 10^{-4} \text{ m/s},$$

$$H_{pc} = 0.73 \text{ kmol}/(\text{m}^3 \cdot \text{kPa}),$$

$$p_G = p_T y_G = 202.6 \times 0.04 = 8.104 \text{ kPa},$$

$$\text{Density of water at } 20^\circ\text{C } \rho_0 = 998 \text{ kg}/\text{m}^3, M_0 = 18 \text{ kg}/\text{kmol}$$

$$c_{\text{absorbent}} = \frac{\rho_0}{M_0} = \frac{998}{18} \text{ kmol}/\text{m}^3 = 55.44 \text{ kmol}/\text{m}^3$$

$$c_T = c_{\text{absorbent}} + c_{\text{absorbate}} = 55.44 + c_L = 55.44 + 0.9 = 56.34 \text{ kmol}/\text{m}^3$$

$$x_A = \frac{c_L}{c_T} = \frac{0.9}{56.34} = 0.016$$

$$H_{pm} = \frac{1}{H_{pc}} \frac{\rho_0}{M_0} = 998/(0.73 \times 18) = 75.95 \text{ kPa}$$

Determine the following:

- (1) p_i, c_i
- (2) $\Delta p, \Delta c$
- (3) total Δp , total $\Delta c, K_G, K_L$
- (4) K_Y, K_X
- (5) More than 6 kinds of expression of N_A
- (6) Gas film resistance, liquid film resistance and total resistance of mass transfer driven by a partial pressure difference, as well as their ratio
- (7) Gas film resistance, liquid film resistance and total resistance of mass transfer driven by molar concentration difference, as well as their ratio

Solution:

- (1) The composition of the gas–liquid interface:

$$N_{A,G} = k_G(p_G - p_i)$$

$$N_{A,L} = k_L(c_i - c_L)$$

$$\frac{p_G - p_i}{c_i - c_L} = \frac{k_L}{k_G}$$

$$\frac{8.104 - p_i}{c_i - 0.9} = \frac{1.5 \times 10^{-4}}{5 \times 10^{-6}}$$

$$c_i = H_{pc} p_i = 0.73 p_i$$

Obtain: $p_i = 1.53 \text{ kPa}$

$$c_i = H_{pc} p_i = 0.73 \times 1.53 = 1.12 \text{ kmol}/\text{m}^3$$

Therefore, the composition (c_i, p_i) of component A in the gas-liquid interface is (1.12 kmol/m³, 1.53 kPa).

- (2) The driving force expressed by the pressure difference in the gas film:

$$\Delta p = p_G - p_i = 8.104 - 1.53 = 6.57 \text{ kPa}$$

The driving force expressed by the molar concentration difference in the liquid film:

$$\Delta c = c_i - c_L = 1.12 - 0.9 = 0.22 \text{ kmol/m}^3$$

- (3) Total driving force expressed by pressure difference:

$$\text{Total } \Delta p = p_G - p_L^* = p_G - \frac{c_L}{H_{pc}} = 8.104 - \frac{0.9}{0.73} = 6.87 \text{ kPa}$$

Total driving force expressed by the molar concentration difference:

$$\text{Total } \Delta c = c_G^* - c_L = H_{pc} p_G - c_L = 0.73 \times 8.104 - 0.9 = 5.02 \text{ kmol/m}^3$$

The overall mass transfer coefficient expressed by the pressure difference:

$$\begin{aligned} K_G &= \frac{1}{\frac{1}{k_G} + \frac{1}{H_{pc} k_L}} = \left(\frac{1}{5 \times 10^{-6}} + \frac{1}{0.73 \times 1.5 \times 10^{-4}} \right)^{-1} \\ &= 4.78 \times 10^{-6} \text{ kmol/(m}^2 \cdot \text{s} \cdot \text{kPa)} \end{aligned}$$

The overall mass transfer coefficient expressed by the molar concentration difference:

$$K_L = \frac{K_G}{H_{pc}} = \frac{4.78 \times 10^{-6}}{0.73} = 6.55 \times 10^{-6} \text{ m/s}$$

- (4) The overall mass transfer coefficients with the driving force expressed by the molar ratio difference:

$$\begin{aligned} X_L &= \frac{c_{\text{absorbate}}}{c_{\text{absorbent}}} = \frac{0.9}{55.44} = 0.0162 \\ Y_G &= \frac{y_G}{1 - y_G} = \frac{0.04}{1 - 0.04} = 0.0417 \\ m_e &= \frac{H_{pm}}{p_T} = \frac{75.95}{202.6} = 0.375 \end{aligned}$$

If it is not a dilute solution:

$$\begin{aligned}
 K_Y &= \frac{K_G p_T}{(1+Y)(1+Y^*)} = \frac{K_G p_T}{(1+Y)(1+m_e X_L)} \\
 &= \frac{4.78 \times 10^{-6} \times 202.6}{(1+0.0417)(1+0.375 \times 0.0162)} \\
 &= 9.24 \times 10^{-4} \text{ kmol}/(\text{m}^2 \cdot \text{s}) \\
 K_X &= \frac{K_L c_T}{(1+X_L)(1+X_L^*)} = \frac{K_L c_T}{(1+X_L)\left(1+\frac{Y_G}{m_e}\right)} \\
 &= \frac{6.55 \times 10^{-6} \times 56.34}{(1+0.0162)\left(1+\frac{0.0417}{0.375}\right)} \\
 &= 3.27 \times 10^{-4} \text{ kmol}/(\text{m}^2 \cdot \text{s})
 \end{aligned}$$

If it is a dilute solution:

$$\begin{aligned}
 K_Y &= K_G \times p_T = 4.78 \times 10^{-6} \times 202.6 \\
 &= 9.68 \times 10^{-4} \text{ kmol}/(\text{m}^2 \cdot \text{s}) \\
 K_X &= K_L \times c_T = 6.55 \times 10^{-6} \times 56.34 \\
 &= 3.69 \times 10^{-4} \text{ kmol}/(\text{m}^2 \cdot \text{s})
 \end{aligned}$$

(5) Mass transfer fluxes calculated in at least six ways:

- ① $N_A = k_G(p_G - p_i) = 5 \times 10^{-6} \times (8.104 - 1.53)$
 $= 3.29 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})$
- ② $N_A = k_L(c_i - c_L) = 1.5 \times 10^{-4} \times (1.12 - 0.9)$
 $= 3.3 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})$
- ③ $N_A = K_G(p_G - p_L^*) = K_G\left(p_G - \frac{c_L}{H_{pc}}\right)$
 $= 4.78 \times 10^{-6} \times \left(8.104 - \frac{0.9}{0.73}\right)$
 $= 3.28 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})$

$$\begin{aligned}
 \textcircled{4} \quad N_A &= K_L(c_G^* - c_L) = K_L(H_{pc}p_G - c_L) \\
 &= 6.55 \times 10^{-6} \times (0.73 \times 8.104 - 0.9) \\
 &= 3.29 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \quad N_A &= K_Y(Y - Y^*) = K_Y(Y_G - Y_L^*) = K_Y(Y_G - m_e X_L) \\
 &= 9.24 \times 10^{-4} (0.0417 - 0.375 \times 0.0162) \\
 &= 3.29 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad N_A &= K_X(X^* - X) = K_X(X_G^* - X_L) = K_X\left(\frac{Y_G}{m_e} - X_L\right) \\
 &= 3.27 \times 10^{-4} \left(\frac{0.0417}{0.375} - 0.0162\right) \\
 &= 3.11 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})
 \end{aligned}$$

If it is a dilute solution:

$$\begin{aligned}
 \textcircled{7} \quad N_A &= K_Y(Y - Y^*) = K_Y(Y_G - Y_L^*) = K_Y(Y_G - m_e X_L) \\
 &= 9.68 \times 10^{-4} (0.0417 - 0.375 \times 0.0162) \\
 &= 3.45 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \quad N_A &= K_X(X^* - X) = K_X(X_G^* - X_L) = K_X\left(\frac{Y_G}{m_e} - X_L\right) \\
 &= 3.69 \times 10^{-4} \times \left(\frac{0.0417}{0.375} - 0.0162\right) \\
 &= 3.51 \times 10^{-5} \text{ kmol}/(\text{m}^2 \cdot \text{s})
 \end{aligned}$$

- (6) Gas film resistance, liquid film resistance and total resistance of mass transfer with driving force expressed by a pressure difference

$$\text{gas film resistance} = \frac{1}{k_G} = \frac{1}{5 \times 10^{-6}} = 2 \times 10^5 (\text{m}^2 \cdot \text{s} \cdot \text{kPa})/\text{kmol}$$

$$\begin{aligned}\text{liquid film resistance: } \frac{1}{H_{pc}k_L} &= \frac{1}{0.73 \times 1.5 \times 10^{-4}} \\ &= 9.13 \times 10^3 \text{ (m}^2 \cdot \text{s} \cdot \text{kPa)/kmol}\end{aligned}$$

$$\begin{aligned}\text{total resistance} &= \text{gas film resistance} + \text{liquid film resistance} \\ &= 2 \times 10^5 + 9.13 \times 10^5 \\ &= 2.09 \times 10^5 \text{ (m}^2 \cdot \text{s} \cdot \text{kPa)/kmol}\end{aligned}$$

$$\begin{aligned}\frac{\text{gas film resistance}}{\text{total resistance}} &= \frac{2 \times 10^5}{2.09 \times 10^5} = 95.6\% \\ \frac{\text{liquid film resistance}}{\text{total resistance}} &= \frac{9.13 \times 10^3}{2.09 \times 10^5} = 4.4\%\end{aligned}$$

This is a gas film control process.

- (7) The gas film resistance, liquid film resistance and total resistance of mass transfer with driving force expressed by the molar concentration:

$$\text{gas film resistance} = \frac{H_{pc}}{k_G} = \frac{0.73}{5 \times 10^{-6}} = 1.46 \times 10^5 \text{ s/m}$$

$$\text{liquid film resistance: } \frac{1}{k_L} = \frac{1}{1.5 \times 10^{-4}} = 6.67 \times 10^3 \text{ s/m}$$

$$\begin{aligned}\text{total resistance} &= \text{gas film resistance} + \text{liquid film resistance} \\ &= 1.46 \times 10^5 + 6.67 \times 10^3 \\ &= 1.527 \times 10^5 \text{ s/m}\end{aligned}$$

$$\begin{aligned}\frac{\text{gas film resistance}}{\text{total resistance}} &= \frac{1.46 \times 10^5}{1.527 \times 10^5} = 95.6\% \\ \frac{\text{liquid film resistance}}{\text{total resistance}} &= \frac{6.67 \times 10^3}{1.527 \times 10^5} = 4.4\%\end{aligned}$$

It is a gas film control process.

Answer:

- (1) The composition of gas–liquid interface p_i is 1.53 kPa, and c_i is 1.12 kmol/m³
- (2) The driving forces of the gas film and liquid film expressed by the pressure difference and molar concentration difference are 6.57 kPa and 0.22 kmol/m³, respectively.
- (3) The total driving force and overall mass transfer coefficient expressed by the pressure difference are 6.87 kPa and 4.78×10^{-6} kmol/(m²·s·Pa), respectively.

The total driving force and overall mass transfer coefficient expressed by the molar concentration difference are 5.02 kmol/m^3 and $6.55 \times 10^{-6} \text{ m/s}$, respectively.

- (4) The overall mass transfer coefficient K_Y with molar ratio difference as the driving force is $9.24 \times 10^{-4} \text{ kmol/(m}^2\cdot\text{s)}$, and K_X is $3.27 \times 10^{-4} \text{ kmol/(m}^2\cdot\text{s)}$.

If it is a dilute solution, the overall mass transfer coefficient (K_Y) with molar ratio difference as the driving force is $9.68 \times 10^{-4} \text{ kmol/(m}^2\cdot\text{s)}$, and K_X is $3.69 \times 10^{-4} \text{ kmol/(m}^2\cdot\text{s)}$.

- (5) The mass transfer flux calculated by the six ways are as follows:

- ① $N_A = k_G(p_G - p_i) = 3.29 \times 10^{-5} \text{ kmol/(m}^2\cdot\text{s)}$
- ② $N_A = k_L(c_i - c_L) = 3.3 \times 10^{-5} \text{ kmol/(m}^2\cdot\text{s)}$
- ③ $N_A = K_G(p_G - p_L^*) = 3.28 \times 10^{-5} \text{ kmol/(m}^2\cdot\text{s)}$
- ④ $N_A = K_L(c_G^* - c_L) = 3.29 \times 10^{-5} \text{ kmol/(m}^2\cdot\text{s)}$
- ⑤ $N_A = K_Y(Y - Y^*) = 3.29 \times 10^{-5} \text{ kmol/(m}^2\cdot\text{s)}$
- ⑥ $N_A = K_X(X^* - X) = 3.11 \times 10^{-5} \text{ kmol/(m}^2\cdot\text{s)}$

If it is a dilute solution:

- ⑦ $N_A = K_Y(Y - Y^*) = 3.45 \times 10^{-5} \text{ kmol/(m}^2\cdot\text{s)}$
 - ⑧ $N_A = K_X(X^* - X) = 3.51 \times 10^{-5} \text{ kmol/(m}^2\cdot\text{s)}$
- (6) The gas film resistance with driving force expressed by the pressure difference is $2 \times 10^5 (\text{m}^2\cdot\text{s}\cdot\text{kPa})/\text{kmol}$, the liquid film resistance is $9.13 \times 10^3 (\text{m}^2\cdot\text{s}\cdot\text{kPa})/\text{kmol}$, and the total resistance is $2.09 \times 10^5 (\text{m}^2\cdot\text{s}\cdot\text{kPa})/\text{kmol}$. The gas film resistance accounted for 95.6% of the total resistance, and the liquid film resistance accounted for 4.4% of the total resistance.
- (7) The gas film resistance driving force expressed by the molar concentration difference is $1.46 \times 10^5 \text{ s/m}$, the liquid film resistance is $6.67 \times 10^3 \text{ s/m}$, and the total resistance is $1.527 \times 10^5 \text{ s/m}$. The gas film resistance accounts for 95.6% of the total resistance, and the liquid film resistance accounts for 4.4% of the total resistance.

4.5 Design and Operation Calculation of the Absorption Process

4.5.1 Overview

In industrial production, tower equipment is often used in the absorption process, which is called an absorption tower. According to the different contact modes of the gas and liquid phases, absorption towers are mainly divided into two types: packed

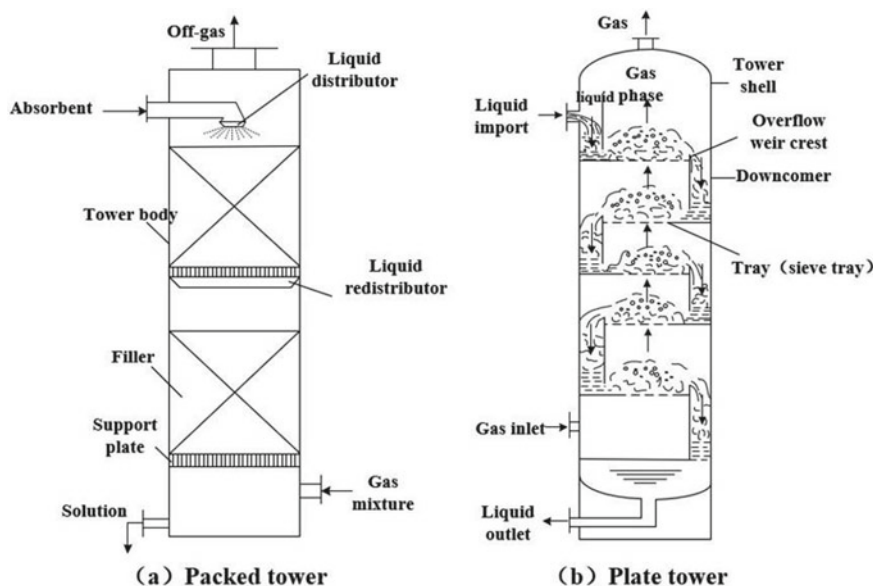


Fig. 4.13 Schematic diagram of packed tower and plate tower

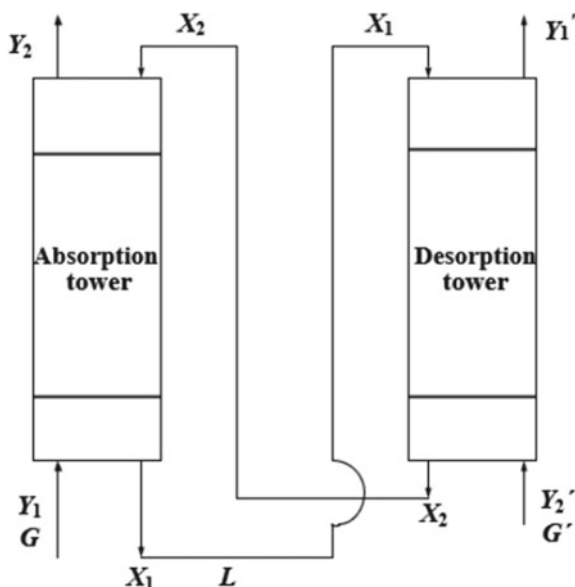
tower and plate tower. The gas and liquid phases contact continuously in the packed tower, while gas and liquid phases contact step by step in the plate tower, as shown in Fig. 4.13.

There are two main flow modes of gas and liquid in an absorption tower, i.e., cocurrent and countercurrent. In general, countercurrent flow is more efficient than cocurrent flow, so countercurrent operations are typically used, as shown in Fig. 4.14. For absorption tower, absorbent with the molar ratio of absorbate X_2 is added from the top of the tower, flows from top to bottom, contacts the gas mixture flowing from bottom to top, and absorbs the solute in the gas. The absorption solution (X_1) with the absorbed solute from gas phase flows out from the bottom of the tower. Most of the solutes are absorbed by the absorbent, and the tail gas (Y_2) discharges from the top of the tower.

Usually, absorbents are recycled. The absorption solution leaving the absorption tower is sent to the desorption tower for desorption so that the concentration of solute in the absorption solution is reduced from X_1 to X_2 and then sent back to the absorption tower as an absorbent, as shown in Fig. 4.14.

According to the given conditions, requirements and tasks, the calculation of the absorption process can be divided into design calculations and operational calculations. The designed calculation designs an absorption tower to meet certain separation requirements under given conditions. Operational calculations analyze and calculate the relationship between the operational conditions and absorption effect of existing absorption towers.

Fig. 4.14 Schematic diagram of countercurrent absorption/desorption



The following design conditions are generally given in advance for the design calculation of absorption towers:

- (1) Flow rate and absorbate composition of mixed gases to be separated.
- (2) The type of absorbent as well as the operating temperature and pressure, which can determine the phase equilibrium relationship of absorption.
- (3) Initial concentration of absorbate in absorbent or regeneration efficiency of known absorbent.
- (4) Separation requirements to be met, i.e., tail gas discharge concentration or absorption efficiency.

The process calculation of the absorption tower determines the amount of absorbent L (kmol/h) or the ratio of liquid to gas L/V on the basis of selecting the absorbent and then calculates the main dimensions of the tower, such as the tower height and diameter.

4.5.2 Material Balance and Operating Line Equation of the Absorption Tower

In the countercurrent packed tower shown in Fig. 4.15a, the composition of liquid and gas phases on any cross section is set as X and Y (molar ratio), and the composition of liquid and gas phases on the other cross section dh is set as $X + dX$ and $Y + dY$. Under steady conditions, the mass balance calculation for the absorbate in the

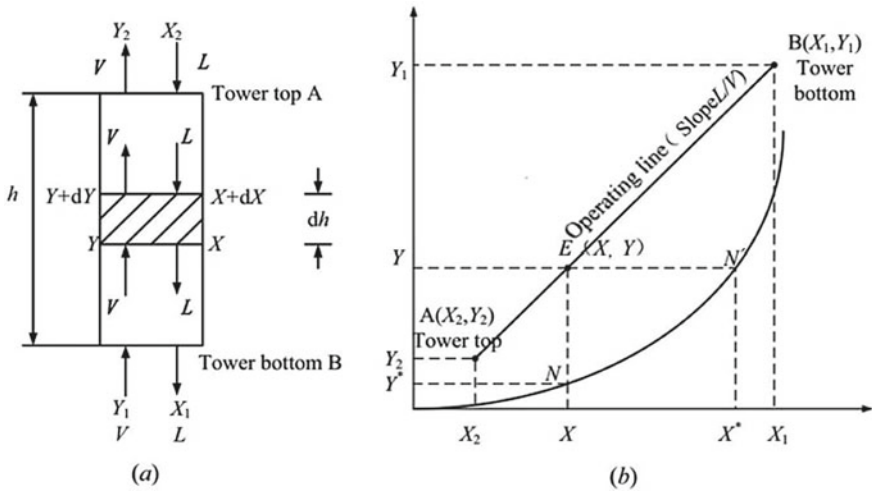


Fig. 4.15 Countercurrent absorption operation of packed tower

microelement height of dh is conducted as:

$$VY + L(X + dX) = V(Y + dY) + LX$$

Therefore,

$$VdY = LdX \quad (4.47)$$

where

V —Molar flow rate of inert gas, kmol/h

L —Molar flow rate of absorbent, kmol/h.

The integral equation between the selected cross section and the bottom of the tower is (4.47):

$$\int_Y^{Y_1} V dY = \int_X^{X_1} L dX$$

$$V(Y_1 - Y) = L(X_1 - X)$$

Thus:

$$Y = \frac{L}{V}X + \left(Y_1 - \frac{L}{V}X_1\right) \quad (4.48)$$

Likewise, the integral Eq. (4.47) between the selected cross section and the top of the tower is as follows.

$$Y = \frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2\right) \quad (4.49)$$

The material balance for the whole tower is as follows:

$$V(Y_1 - Y_2) = L(X_1 - X_2) \quad (4.50)$$

In Fig. 4.15b, it can be seen that Eq. (4.50) is a straight line with the slope of L/V , through two points (X_1, Y_1) and (X_2, Y_2) . This line is called an operating line. This equation is an operating line equation. Any point on the operating line represents the relationship between the gas–liquid two-phase composition X and Y at the corresponding cross section of the absorption tower.

Explanations for the operating line of the absorption tower are offered as follows:

- (1) Point B, one of the two endpoints on operating line, represents the bottom gas–liquid composition (X_1, Y_1) . Compared with the whole tower, the values of X_1 and Y_1 are the highest, which is called the concentrated end; while at the other endpoint A (X_2, Y_2) , concentration is the lowest, which is called the dilute end.
- (2) Any point M (X, Y) on the operating line represents the composition of the gas–liquid two phases in the absorption tower at a corresponding cross section, which is called the operating point (X, Y) .
- (3) The vertical distance (EN) between any point (X, Y) on the operating line and the equilibrium line is the driving force in gas phase $(Y - Y^*)$ of the cross section in the tower, while the horizontal distance (EN') between point E and the equilibrium line is the liquid phase driving force $(X^* - X)$ of the cross section in the tower. The larger the distance between the operating line and the equilibrium line is, the greater the deviation of the operating line from the equilibrium line, the greater the mass transfer driving force and the higher the absorption flux. Therefore, when determining the technological conditions of absorption operation, reducing the temperature of the absorbent and choosing the absorbent with high solubility for absorbate (or changing to chemical absorption) are effective ways to improve the absorption flux.
- (4) The operating line equation of an absorption tower is derived from the material balance calculation. Therefore, the operation line equation is only related to the flow rate and composition of gas and liquid phases, not to other factors, such as the equilibrium relationship between absorbate and absorbent, operating temperature, pressure, packing structure, etc.
- (5) During absorption, the partial pressure of the absorbate in the gas phase is always higher than that of the liquid phase equilibrium at any cross section of the packing layer, so the absorption operating line is always above the equilibrium line.

The main factors for selecting the absorbent are ① solubility, ② selectivity, ③ volatility, ④ stability, ⑤ viscosity, ⑥ regeneration, and ⑦ corrosiveness.

4.5.3 Calculation of the Absorbent Dosage or Molar Flow Rate (L) and the Specific Absorbent Dosage (L/V)

The specific absorbent dosage (L/V) refers to the molar flow rate of absorbent (L) over molar flow rate of inert gas (V) during the gas mixture treatment process, also known as the ratio of liquid over gas (L/V) or liquid-gas ratio (L/V). In general, in the calculation of the absorption tower, the molar flow rate V of inert gas to be treated gas mixture is known, the composition Y_1 and Y_2 of the gas phase inlet and outlet, respectively and the composition X_2 of the absorbent inlet are known, and the absorbent dosage molar flow rate L needs to be determined.

Total absorption mass transfer flux Equations (4.36) and (4.37) can be expressed as

$$J' = N_A A = K_Y (Y - Y^*) A = K_X (X^* - X) A$$

where

A —Effective area of gas-liquid contact (A is related to those factors, such as the cross-sectional area of the tower and the tower height, etc.), m^2

J' —Absorption mass transfer rate, $kmol/h$.

In the case of a certain cross-sectional area of the tower, from the above equation, it can be seen that the tower height can be reduced if the driving force is large; otherwise, the tower height increases. In Fig. 4.16, it can be seen that the operating line is far away from the equilibrium line when the driving force is great, that is, the liquid-gas ratio (L/V) is large. This shows that the reduction of tower height leads to the decrease of investment cost, but to follow the increase of liquid-gas ratio (L/V), that is, increased the absorbent molar flow rate (L), which resulted in the increase of operation cost, and vice versa. Therefore, the choice of absorbent molar flow rate (L) affects both equipment investment and operating costs.

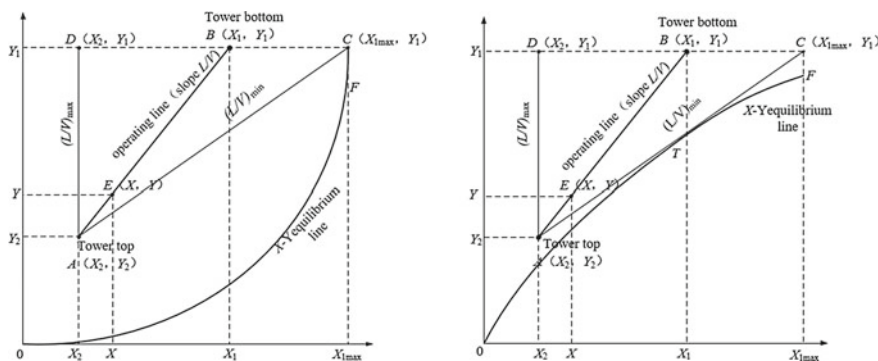


Fig. 4.16 Calculation of amount of absorbent (L) and liquid-gas ratio (L/V)

According to absorption operating line equation (4.50), the liquid–gas ratio L/V can be calculated by the following equation:

$$L/V = \frac{Y_1 - Y_2}{X_1 - X_2} \quad (4.51)$$

If the absorbent molar flow rate is gradually decreased, i.e., the liquid–gas ratio (L/V) is gradually reduced, then the operating line approaches the equilibrium line until the operating line intersects with the equilibrium line (Fig. 4.16a) or is tangent (Fig. 4.16b) of the AC line with the equilibrium line. Then, the point of $B(X_1, Y_1)$, the cross section at the bottom of the tower, the absorption driving force is zero, the absorbent molar flow rate is called the minimum absorbent molar flow rate, expressed as L_{\min} , and the liquid–gas ratio is called the minimum liquid–gas ratio, expressed as $(L/V)_{\min}$. The corresponding concentration of the absorption solution outlet tower reaches the maximum, expressed by $X_{1\max}$. Under these conditions, the mass transfer area required to meet the separation requirements is infinite, that is, the tower height is infinite. In contrast, if the absorbent molar flow rate (L) increases gradually, the operating line will shift away from the equilibrium line, and the distance between the operating line and the equilibrium line will increase; that is, the driving force of the process will increase, and the mass transfer area required will decrease; that is, the tower height will decrease, but the operation cost will increase. When the liquid to gas ratio (L/V) increases to infinity, the slope of the operating line increases to the maximum $(L/V)_{\max}$, as shown by the AD line in Fig. 4.16. At this time, the concentration of the absorption solution corresponding to the tower is the smallest; that is, $X_{1\min} = X_2$, the driving force is the largest, and the required mass transfer area and the tower height are the smallest. Although this limit condition has no practical significance for absorption operation, it provides a analysing basis for determining the appropriate liquid–gas ratio (L/V).

From the above analysis, it can be seen that a liquid–gas ratio (L/V) which is too small or too large is not appropriate. Both the equipment investment cost and operation cost should be considered at the same time to minimize the total cost in selecting the appropriate liquid–gas ratio (L/V). In practice, to ensure the processing capacity of the absorption tower, it is generally accepted that

$$L/V = (1.1 \sim 2)(L/V)_{\min}$$

The minimum liquid–gas ratio $(L/V)_{\min}$ can be calculated by the following equation:

$$(L/V)_{\min} = \frac{Y_1 - Y_2}{X_{1\max} - X_2} \quad (4.52)$$

1) The phase equilibrium relationship conforms to Henry's law

The phase equilibrium relationship conforms to Henry's law in that the equilibrium line is a straight line, and if the equilibrium line is $Y = m_e X$, the minimum liquid–gas

ratio $(L/V)_{\min}$ is

$$(L/V)_{\min} = \frac{Y_1 - Y_2}{X_{1\max} - X_2} = \frac{Y_1 - Y_2}{\frac{Y_1}{m_e} - X_2} \quad (4.53)$$

When the absorbent is a pure solvent, that is, $X_2 = 0$, the calculation of the minimum liquid–gas ratio can be further simplified as follows.

$$(L/V)_{\min} = \frac{Y_1 - Y_2}{\frac{Y_1}{m_e} - 0} = \frac{Y_1 - Y_2}{Y_1} m_e \quad (4.54)$$

The concepts, absorption efficiency of absorbate or recovery of absorbate, are often used in absorption calculations.

$$\begin{aligned} \eta_A &= \frac{\text{the amount of absorbate absorbed (kmol)}}{\text{the amount of absorbate in gas mixture (kmol)}} \\ &= \frac{VY_1 - VY_2}{VY_1} = \frac{Y_1 - Y_2}{Y_1} \end{aligned} \quad (4.55)$$

Substitution into Eq. (4.54) gives

$$(L/V)_{\min} = \eta_A m_e \quad (4.56)$$

2) The phase equilibrium line is a curve

When the phase equilibrium line is a concave (or convex) curve (as shown in Fig. 4.16), the calculation of $(L/V)_{\min}$ is obtained by a graphical method. When the phase equilibrium line is a concave curve, as shown in Fig. 4.16a, the intersection point $C(X_{\max}, Y_1)$ of the $Y = Y_1$ horizontal line and OF equilibrium line is obtained first, and then $X_{1\max}$ is obtained. Then, the minimum liquid–gas ratio $(L/V)_{\min}$ is obtained by substituting $X_{1\max}$ into Eq. (4.52).

If the phase equilibrium line is the convex curve shown in Fig. 4.16b, then the tangent AC of the phase equilibrium line OF crossing point A is tangent to point T . At tangent point T , $Y = Y^*$, $\Delta Y = Y - Y^* = 0$. The point $C(X_{1\max}, Y_1)$ is the intersection point of target AC and the $Y = Y_1$ horizontal line, and corresponding $X_{1\max}$ is the highest absorbate concentration of absorption solution outlet at the bottom of the tower. Finally, the minimum liquid–gas ratio $(L/V)_{\min}$ can be obtained by substituting $X_{1\max}$ in Eq. (4.52).

When choosing the absorbent molar flow rate, it is also necessary to consider whether the packed tower can be fully wetted. In general, the spray density of liquid should be at least greater than $5 \text{ m}^3/(\text{m}^2 \cdot \text{h})$.

Example 4.9 The factory B discharge the gas with recoverable material A during the production process, so an absorption tower is used to recover the absorbate A in gas mixture. It is known that the mole fraction of absorbate A in gas mixture entering

the absorption tower is 0.2, and absorption is conducted by using an absorbent with 0.01 mole fraction of absorbate A. In the absorption process, the phase equilibrium curve between absorbate A and absorbent is $Y = 0.9X$, and the liquid–gas ratio is $L/V = 0.5$. The tower height is unrestricted, and no chemical reaction occurs in the absorption process.

Determine the following:

- (1) In countercurrent operation, the maximum absorption efficiency and the compositions at the bottom and the top of the tower. If the liquid–gas ratio is $L/V = 1.8$, what are the maximum efficiency as well as the compositions at the bottom and the top of the tower at this time?
- (2) When the operation is changed to concurrent flow, please calculate the absorption efficiency as well as the compositions at the bottom and the top of the tower at this time.

Known: $y_1 = 0.2$, $x_2 = 0.01$, $Y = 0.9X$, $L/V = 0.5$, $m_e = 0.9$

Determine: Maximum absorption efficiency as well as the compositions at the bottom and the top of the tower in countercurrent and cocurrent

Solution: (1) Known for countercurrent

$$X_2 = \frac{x_2}{1 - x_2} = \frac{0.01}{1 - 0.01} \approx 0.01, Y_1 = \frac{y_1}{1 - y_1} = \frac{0.2}{1 - 0.2} = 0.25$$

When $L/V = 0.5 < m_e = 0.9$ and tower height is unrestricted, two-phase equilibrium state is achieved at the bottom of the tower (as shown line ① in Fig. 4.17b), and $X_{1\max} = X_1^* = Y_1/m_e = 0.25/0.9 = 0.278$. According to material balance,

$$Y_2 = Y_1 - \frac{L(X_{1\max} - X_2)}{V} = 0.25 - 0.5 \times (0.278 - 0.01) = 0.116$$

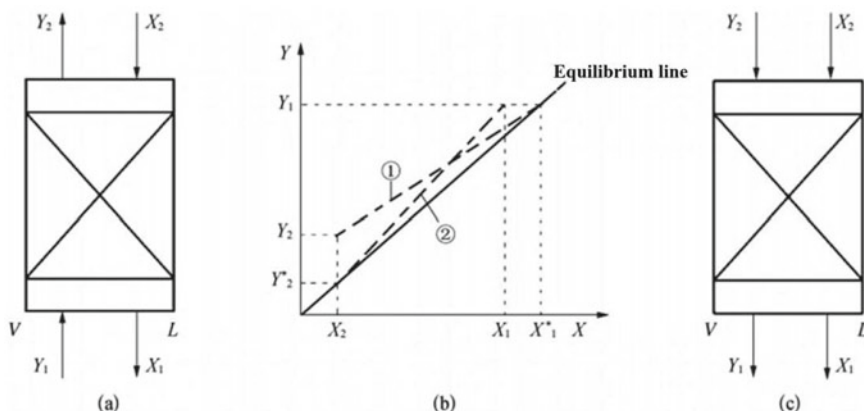


Fig. 4.17 Schematic diagram of Example 4.9

At this point, the absorption efficiency is

$$\eta_A = \frac{Y_1 - Y_2}{Y_1} = \frac{0.25 - 0.116}{0.25} \times 100\% = 53.6\%$$

The composition is (0.278, 0.25) at the bottom and (0.01, 0.116) at the top of the tower.

When $L/V = 1.8 > m_e = 0.9$ and tower height is not restricted, absorption equilibrium state is reached at the top of the tower (as shown line ② in Fig. 4.17b). $Y_2 = Y_{2\min} = Y_2^* = m_e X_2 = 0.9 \times 0.01 = 0.009$, absorption efficiency can be obtained directly.

$$\eta_A = \frac{Y_1 - Y_2}{Y_1} = \frac{0.25 - 0.009}{0.25} \times 100\% = 96.4\%$$

According to material balance,

$$L/V = \frac{Y_1 - Y_2}{X_1 - X_2}$$

$$1.8 = \frac{0.25 - 0.009}{X_1 - 0.01}$$

$$X_1 = 0.143$$

The composition is (0.143, 0.25) at the bottom and (0.01, 0.009) at the top of the tower.

(2) Concurrent operation and $L/V = 0.5$ (as shown in Fig. 4.17c)

$$X_2 = \frac{0.01}{1 - 0.01} \approx 0.01, Y_2 = \frac{0.2}{1 - 0.2} = 0.25$$

Because the tower height is unrestricted, there are

$$Y_1 = m_e X_1, \text{ namely, } Y_1 = 0.9X_1 \quad (①)$$

According to the operating line relationship, there are

$$V(Y_2 - Y_1) = L(X_1 - X_2)$$

$$\frac{Y_2 - Y_1}{X_2 - X_1} = -\frac{L}{V}, \text{ namely, } 0.5 = \frac{0.25 - Y_1}{X_1 - 0.01} \quad (②)$$

Unite Eqs. ① and ② to obtain:

$$X_1 = 0.182, Y_1 = 0.164$$

Therefore,

$$\eta_A = \frac{Y_2 - Y_1}{Y_2} = \frac{0.25 - 0.164}{0.25} \times 100\% = 34.4\%$$

The composition is (0.182, 0.164) at the bottom and (0.01, 0.25) at the top of the tower.

Analysis: In countercurrent absorption, when the slope of the operating line is greater than that of the equilibrium line, the gas–liquid may reach equilibrium state at the top of the tower, and the absorption efficiency is the highest, but the concentration of absorption solution outlet at the bottom of the tower is not the highest. When the slope of the operating line is smaller than that of the equilibrium line, gas–liquid equilibrium occurs at the bottom of the tower, the concentration of absorption solution outlet at the bottom of the tower is the highest, but the absorption efficiency is not the highest.

Answer: In countercurrent operation, the maximum absorption efficiency of the tower is 53.6%; the composition is (0.278, 0.25) at the bottom and (0.01, 0.116) at the top of the tower. If $L/V = 1.8$, the maximum absorption efficiency of the tower is 96.4%; the composition is (0.143, 0.25) at the bottom and (0.01, 0.009) at the top of the tower. In concurrent flow operation, the absorption efficiency is 34.4%; the composition is (0.182, 0.164) at the bottom and (0.01, 0.25) at the top of the tower.

4.5.4 Calculation of Packing Layer Height of Absorption Tower

4.5.4.1 Fundamental Equations for Calculating the Height of the Packing Layer

The calculation of the packing height of the absorption tower is essentially the calculation of the contact area in the absorption process, which involves the material balance, mass transfer flux equation and phase equilibrium relationship in a specific process. The relationship between packing height and gas–liquid contact area is as follows:

$$A = a A_T H' \quad (4.57)$$

where

A —Actual gas–liquid contact area, m^2

H' —Packing layer height, m

A_T —Cross section area of packed tower, m^2

a —Effective contact area between gas and liquid phases in unit volume of packing, m^2/m^3 .

Under general operating conditions, gas–liquid contact occurs on the wet surface of the packings. If the packings is completely wetted by liquid, then a is approximately the specific surface area of the packings.

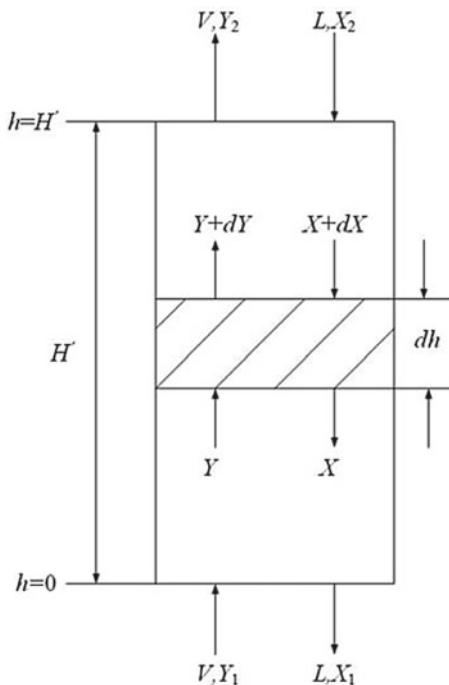
In a packed tower with continuous countercurrent contact, as shown in Fig. 4.18, for the steady isothermal absorption process, the flow rate of the gas mixture and absorbent does not change with time or with the height of the packing layer, but the composition and driving force of the gas and liquid phases change continuously with the height of the packing layer. Therefore, to calculate the height of the packing layer, a microelement height of dh should be intercepted at any cross section of the packing layer, the differential equation of material balance and the differential equation of absorption rate should be listed, and then these two equations should be combined and integrated in the whole tower to derive the calculation equation of the height of the packing layer.

The absorption mass transfer rate equation of the microelement height of dh in packing layer is as follows:

$$dJ = K_Y(Y - Y^*)dA = K_Y(Y - Y^*)a A_T dh \quad (4.58)$$

$$dJ = K_X(X^* - X)dA = K_X(X^* - X)a A_T dh \quad (4.59)$$

Fig. 4.18 Material balance of the microelements height of dh in packing layer



Calculation of absorbate material balance in the dh microelement packing layer

$$dJ = V dY = L dX \quad (4.60)$$

Simultaneous Eqs. (4.58) ~ (4.60), then

$$V dY = K_Y (Y - Y^*) a A_T dh \quad (4.61)$$

$$L dX = K_X (X^* - X) a A_T dh \quad (4.62)$$

In the same absorption tower and under certain operating conditions, K_Y and K_X can be assumed to be constants. Equations (4.61) and (4.62) are integrated along the whole tower height.

$$\begin{aligned} \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} &= \int_0^{H'} \frac{K_Y a A_T dh}{V} = \frac{K_Y a A_T}{V} \cdot H' \\ H' &= \frac{V}{K_Y a A_T} \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} \end{aligned} \quad (4.63)$$

$$\begin{aligned} \int_{X_2}^{X_1} \frac{dX}{X^* - X} &= \int_0^{H'} \frac{K_X a A_T dh}{L} = \frac{K_X a A_T}{L} \cdot H' \\ H' &= \frac{L}{K_X a A_T} \int_{X_2}^{X_1} \frac{dX}{X^* - X} \end{aligned} \quad (4.64)$$

where

$K_Y a$ —Overall volume gas mass transfer coefficient, $\text{kmol}/(\text{m}^3 \cdot \text{s})$

$K_X a$ —Overall volume liquid mass transfer coefficient, $\text{kmol}/(\text{m}^3 \cdot \text{s})$.

From the derivation process of Eqs. (4.63) and (4.64), it can be seen that similar equations for calculating the height of the packing layer can be obtained by applying different absorption flux equations. If the gas-film absorption mass transfer flux Eq. (4.34) or liquid-film absorption mass transfer flux Eq. (4.35) and material balance equation are combined, the solution can be obtained.

$$H' = \frac{V}{k_Y a A_T} \int_{Y_2}^{Y_1} \frac{dY}{Y - Y_i} \quad (4.65)$$

and

$$H' = \frac{L}{k_X a A_T} \int_{X_2}^{X_1} \frac{dX}{X_i - X} \quad (4.66)$$

where

$k_Y a$ —Gas film volume mass transfer coefficient, $\text{kmol}/(\text{m}^3 \cdot \text{s})$

$k_X a$ —Liquid film volume mass transfer coefficient, $\text{kmol}/(\text{m}^3 \cdot \text{s})$.

a is the effective contact area between the gas and liquid phases in a unit volume of packing. a is not only related to the shape, size and filling condition of packings but also affected by the physical properties and flow conditions of fluids. It is difficult to determine the value of a . Therefore, the product of a and the absorption mass transfer coefficient is often regarded as a whole, which is called the volume mass transfer coefficient. Its physical meaning is the absorbed amount of absorbent for absorbate in packing layer at unit time and volume under unit driving force.

4.5.4.2 Number and Height of Mass Transfer Units

Let H_{OG} be equal to an item in Eq. (4.63):

$$H_{OG} = \frac{V}{K_Y a A_T} \quad (4.67)$$

N_{OG} equals another term in Eq. (4.63):

$$N_{OG} = \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} \quad (4.68)$$

Then, the height H' of the packing layer is

$$H' = H_{OG} \times N_{OG} \quad (4.69)$$

where

H_{OG} —Height of overall gas phase transfer unit, m

N_{OG} —Number of overall gas phase transfer units, dimensionless.

The mass transfer unit refers to the mass transfer through a certain height of the packed layer so that the change in the phase composition is exactly equal to the mean driving force. For any j gas phase, the overall gas phase mass transfer unit is as follows (Fig. 4.19):

$$\int_{Y_j}^{Y_{j+1}} \frac{dY}{Y - Y^*} = 1 \quad (4.70)$$

Namely,

$$\frac{Y_j - Y_{j+1}}{(Y - Y^*)_{\text{average}, j \rightarrow j+1}} = 1 \quad (4.71)$$

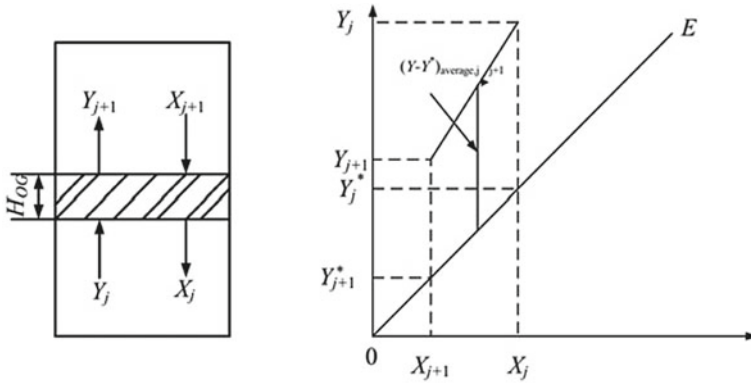


Fig. 4.19 Schematic diagram of gas phase overall mass transfer unit and the height of overall mass transfer unit

where

$(Y - Y^*)_{\text{average}, j \rightarrow j+1}$ —Mean driving force of mass transfer based on a overall mass transfer unit in the gas phase, dimensionless

$Y_j - Y_{j+1}$ —Variation of gas phase composition, dimensionless.

The number of transfer units represents the number of transfer units required to complete all separation tasks in the absorption process. It only depends on the composition and phase equilibrium relationship of the gas and liquid phases before and after separation and has nothing to do with the equipment type. Its value indicates the difficulty of the separation task.

The height of transfer unit is the height of the packing layer needed to complete the separation task of a mass transfer unit. It mainly depends on the type of absorption equipment (such as packing type and size), system characteristics and operating conditions. The value reflects the quality of mass transfer dynamics of the packed layer. For low concentration gas absorption, the height of transfer unit corresponding to each mass transfer unit can be regarded as equal.

Similarly, Eq. (4.64) can be written as follows:

$$H' = H_{OL} \times N_{OL} \quad (4.72)$$

$$H_{OL} = \frac{L}{K_X a A_T} \quad (4.73)$$

$$N_{OL} = \int_{X_2}^{X_1} \frac{dX}{X^* - X} \quad (4.74)$$

where

H_{OL} —Height of overall mass liquid phase transfer unit, m

N_{OL} —Number of overall liquid phase transfer units, dimensionless.

Equations (4.65) and (4.66) can be written as follows:

$$H' = H_G \times N_G \quad (4.75)$$

$$H' = H_L \times N_L \quad (4.76)$$

where

H_G, H_L —Height of gas phase mass transfer unit $\left(\frac{V}{k_y a A_T}\right)$ and height of liquid phase mass transfer unit $\left(\frac{L}{k_x a A_T}\right)$, respectively, m

N_G, N_L —Number of gas phase transfer units $\int_{Y_2}^{Y_1} \frac{dY}{Y-Y_i}$ and number of liquid phase transfer units $\int_{X_2}^{X_1} \frac{dX}{X_i-X}$, respectively, dimensionless.

4.5.4.3 Methods for Calculating the Number of Overall Transfer Units

Y^* (or X^*) in the calculating expression equation of number of overall transfer units is an equilibrium concentration of the liquid phase absorbate concentration X (or gas phase absorbate concentration Y), which need to be determined by the equilibrium relationship. According to the equilibrium relationship, there are four methods for calculating the number of overall transfer units: ① Logarithmic mean driving force method, this method is only applicable when the gas–liquid equilibrium relationship conforms to Henry's law; ② Absorption factor method, this method is also only applicable when the gas–liquid equilibrium relationship conforms to Henry's law; ③ Direct integration method using calculator, this method is applicable when the gas–liquid equilibrium relationship conforms to Henry's law or doesn't conform to Henry's law but with a determined functional relationship which can be expressed by mathematical equations; ④ Numerical integration method, this method is suitable for any gas–liquid equilibrium relationship. However, due to its poor accuracy and troublesome calculations, it is usually applied to gas–liquid equilibrium that does not conform to Henry's Law, and without determined functional relationship which can be expressed by mathematical equations.

1) Logarithmic mean driving force method

In the absorption process, when the equilibrium line is a straight line, the number of overall mass transfer units can be calculated by the mean driving force method, as shown in Fig. 4.20. Since the gas–liquid equilibrium relationship is a straight line and the operating line is also a straight line, the driving force on any cross section of the tower $\Delta Y = Y - Y^*$ is also linear with the gas phase absorbate concentration Y . The slope of the line is:

$$\frac{d(\Delta Y)}{dY} = \frac{\Delta Y_1 - \Delta Y_2}{Y_1 - Y_2}$$

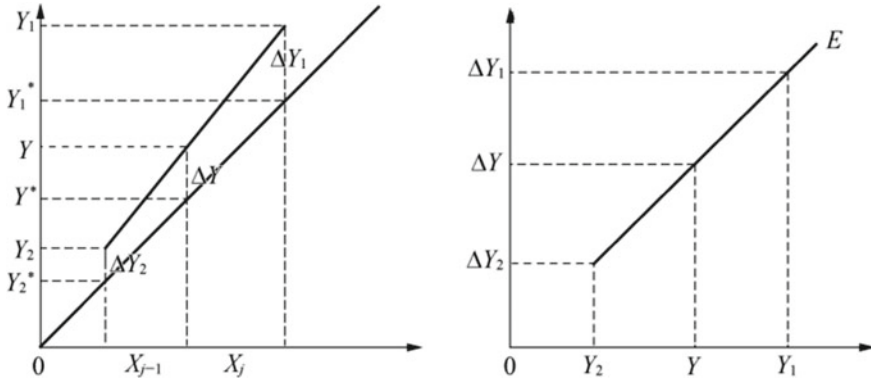


Fig. 4.20 The driving force shown on the diagram $Y-X$

Transposition:

$$dY = \frac{Y_1 - Y_2}{\Delta Y_1 - \Delta Y_2} (d\Delta Y)$$

By substituting the previous Eq. (4.68),

$$\begin{aligned} N_{OG} &= \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} = \int_{Y_2}^{Y_1} \frac{dY}{\Delta Y} \\ &= \frac{Y_1 - Y_2}{\Delta Y_1 - \Delta Y_2} \int_{\Delta Y_2}^{\Delta Y_1} \frac{d(\Delta Y)}{\Delta Y} \\ &= \frac{Y_1 - Y_2}{\Delta Y_1 - \Delta Y_2} \ln \frac{\Delta Y_1}{\Delta Y_2} \end{aligned}$$

That is, the number of overall gas phase transfer units is

$$N_{OG} = \frac{Y_1 - Y_2}{\frac{\Delta Y_1 - \Delta Y_2}{\ln \frac{\Delta Y_1}{\Delta Y_2}}} = \frac{Y_1 - Y_2}{\Delta Y_m} \quad (4.77)$$

Similarly, the equation for calculating the number of overall liquid phase transfer units can be derived.

$$N_{OL} = \frac{X_1 - X_2}{\frac{\Delta X_1 - \Delta X_2}{\ln \frac{\Delta X_1}{\Delta X_2}}} = \frac{X_1 - X_2}{\Delta X_m} \quad (4.78)$$

Mean driving force in Eqs. (4.77) and (4.78)

$$\Delta Y_m = \frac{Y_1 - Y_1^* - (Y_2 - Y_2^*)}{\ln \frac{Y_1 - Y_1^*}{Y_2 - Y_2^*}} \quad (4.79)$$

$$\Delta X_m = \frac{X_1^* - X_1 - (X_2^* - X_2)}{\ln \frac{X_1^* - X_1}{X_2^* - X_2}} \quad (4.80)$$

2) Absorption factor method

If the phase equilibrium relationship conforms to Henry's law, when X is very small, $Y^* = m_e X$; that is, the gas-liquid equilibrium relationship is a straight line through the origin, and the number of overall mass transfer units can be calculated by the absorption factor method. The calculation equation is deduced as follows.

The material balance between any cross section and the top of the absorption tower is:

$$X = \frac{V}{L}(Y - Y_2) + X_2$$

Substitute the previous Eq. (4.68) and obtain:

$$\begin{aligned} N_{OG} &= \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} \\ &= \int_{Y_2}^{Y_1} \frac{dY}{Y - m_e \left[\frac{V}{L}(Y - Y_2) + X_2 \right]} \\ &= \int_{Y_2}^{Y_1} \frac{dY}{\left(1 - \frac{m_e V}{L}\right)Y + \frac{m_e V}{L}Y_2 - m_e X_2} \\ &= \frac{1}{1 - \frac{m_e V}{L}} \ln \left[\frac{\left(1 - \frac{m_e V}{L}\right)Y_1 + \frac{m_e V}{L}Y_2 - m_e X_2}{\left(1 - \frac{m_e V}{L}\right)Y_2 + \frac{m_e V}{L}Y_2 - m_e X_2} \right] \\ &= \frac{1}{1 - \frac{m_e V}{L}} \ln \left[\frac{\left(1 - \frac{m_e V}{L}\right)Y_1 + \frac{m_e V}{L}Y_2 - m_e X_2}{Y_2 - m_e X_2} \right] \\ &= \frac{1}{1 - \frac{m_e V}{L}} \ln \left[\frac{\left(1 - \frac{m_e V}{L}\right)Y_1 - \left(1 - \frac{m_e V}{L}\right)m_e X_2 + \left(1 - \frac{m_e V}{L}\right)m_e X_2 + \frac{m_e V}{L}Y_2 - m_e X_2}{Y_2 - m_e X_2} \right] \\ &= \frac{1}{1 - \frac{m_e V}{L}} \ln \left[\left(1 - \frac{m_e V}{L}\right) \frac{Y_1 - m_e X_2}{Y_2 - m_e X_2} + \frac{m_e V}{L} \right] \end{aligned}$$

Let $\frac{m_e V}{L} = \frac{1}{A_a}$, and substitute it in above equation, then:

$$N_{OG} = \frac{1}{1 - \frac{1}{A_a}} \ln \left[\left(1 - \frac{1}{A_a} \right) \frac{Y_1 - m_e X_2}{Y_2 - m_e X_2} + \frac{1}{A_a} \right] \quad (4.81)$$

In the equation, $A_a = \frac{L}{m_e V}$ is called as the absorption factor, and its geometric meaning is the ratio of the slope L/V of the operating line to the slope m_e of the equilibrium line. The greater the A_a value is, the better the absorption effect is in absorption tower. The reciprocal of the absorption factor, $\frac{1}{A_a} = \frac{m_e V}{L}$, is called as the desorption factor. The greater the $\frac{1}{A_a}$ value is, the better the desorption effect is in desorption tower.

For the absorption in absorption tower, the smaller the $\frac{m_e V}{L}$ is, that is, the greater the A_a , then the smaller the N_{OG} is, and the smaller the height of packing layer needed. Therefore, the greater A_a is beneficial for absorption. On the contrary, the greater the $\frac{1}{A_a}$ is, the greater N_{OG} is, and the higher the height of the packing needed, the more difficult for absorption. Therefore, the greater of the $\frac{1}{A_a}$ is not beneficial for absorption.

The absorption factor A_a and desorption factor $\frac{1}{A_a}$ are important operating parameters for absorption or desorption towers. However, the equilibrium constant m_e in $A_a = \frac{L}{m_e V}$ is determined by the system and operating temperature. Increasing A_a means to increase L/V . Therefore, an appropriate A_a should be selected in design and operation.

Similarly, the equation for calculating the number of overall liquid phase transfer units can be derived.

$$\begin{aligned} N_{OL} &= \frac{1}{1 - \frac{L}{m_e V}} \ln \left[\left(1 - \frac{L}{m_e V} \right) \frac{Y_1 - m_e X_2}{Y_1 - m_e X_1} + \frac{L}{m_e V} \right] \\ &= \frac{1}{1 - A_a} \ln \left[(1 - A_a) \frac{Y_1 - m_e X_2}{Y_1 - m_e X_1} + A_a \right] \end{aligned} \quad (4.82)$$

3) Direct integration method using calculator

If the phase equilibrium relationship conforms to Henry's law or doesn't conform to Henry's law but with a determined functional relationship, it can be obtained by the direct integration method using a calculator.

First, a material balance is carried out for any two cross sections of the absorption tower. Here, taking the arbitrary interface and the top of the tower as an example, $Y^* = f(X)$,

$$Y = \frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2\right)$$

Substitute in (4.68):

$$N_{OG} = \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} = \int_{X_2}^{X_1} \frac{d\left[\frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2\right)\right]}{\frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2\right) - f(X)}$$

Using a calculator to calculate the equation, the specific steps are as follows:

- ① Click the calculator function key “ \int ”
- ② Input the integral equation “ $\int_{X_2}^{X_1} \frac{d\left[\frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2\right)\right]}{\frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2\right) - f(X)}$,”
- ③ Press “=”; the value of the required value N_{OG} can be obtained.

Example 4.10 Factory B adopted a countercurrent packed absorption tower, which operated under standard conditions to absorb absorbate A in the gas mixture. The gas flow rate is 2000 m³/h (standard state), the concentration of gas A in gas mixture is 200 g/m³, the molecular weight of gas A is 50, the absorption equilibrium relationship is $y = 0.8x$, the recovery efficiency of gas A is 98%, the overall mass transfer coefficient of gas phase K_y is 0.5 kmol/(m²·h) and the absorbent used in liquid phase is pure water. The concentration of absorbate A in outlet solution at the bottom of the tower is 75% equilibrium concentration of A in feed gas due to the limitation of tower height. The effective specific surface area of the packings in the tower is 200 m²/m³, and the flow velocity of the gas in the empty tower is 0.5 m/s. It is assumed that gas A does not react with water, and other components in the gas are inert gases.

Calculate the following:

- (1) Water molar flow rate
- (2) Tower diameter
- (3) Packing layer height, please calculate it by three methods, including logarithmic mean driving force, absorption factor, direct integration using calculator, and compare the results.

Known: $q_g = 2000$ m³ (standard)/h, $\rho_y = 200$ g/m³, $x_1 = 75\%x_1^*$, $y = 0.8x$, $K_y = 0.5$ kmol/(m²·h), $u = 0.5$ m/s, $a = 200$ m²/m³, $X_2 = 0$, $\eta_A = 98\%$.

Determine the following: L , d , H' .

Solution: In the following calculation, subscript 1 represents the bottom of the tower, and subscript 2 represents the top of the tower. According to the known operating conditions, the following can be obtained.

$$V = \frac{2000}{22.4} - \frac{2000 \times 200}{50 \times 10^3} = 89.28 - 8 = 81.28 \text{ kmol/h}$$

$$Y_1 = \frac{8}{81.28} = 0.0984, \quad Y_2 = (1 - 98\%)Y_1 = 0.00197$$

$$X_2 = 0, \quad Y_2^* = 0$$

$$\begin{aligned}
 x_1^* &= \frac{y_1}{0.8} = \frac{Y_1}{1 + Y_1} \times \frac{1}{0.8} = 0.112 \\
 x_1 &= 75\%x_1^* = 75\% \times 0.112 = 0.084, X_1 = \frac{x_1}{1 - x_1} \\
 &= \frac{0.112}{1 - 0.112} = 0.0917
 \end{aligned}$$

- (1) Water consumption can be obtained from the total material balance equation for absorbate A

$$\begin{aligned}
 L(X_1 - X_2) &= V(Y_1 - Y_2). \\
 L &= \frac{V(Y_1 - Y_2)}{X_1 - X_2} = \frac{81.28 \times (0.0984 - 0.00197)}{0.0917 - 0} \\
 &= 85.47 \text{ kmol/h}
 \end{aligned}$$

- (2) Tower diameter

$$d_t = \sqrt{\frac{4q_g}{\pi u}} = \sqrt{\frac{4 \times 2000/3600}{\pi \times 0.5}} = 1.19 \text{ m}$$

- (3) Packing height

Due to the low concentration of absorbate, $y = 0.8x$ can be approximated as $Y = 0.8X$, and the following calculations can be performed:

Packing height

$$H' = N_{OG}H_{OG}$$

Total height of gas phase transfer unit

$$H_{OG} = \frac{V}{K_Y a A_T} = \frac{81.28}{0.5 \times 200 \times \frac{\pi}{4} \times 1.19^2} = 0.731 \text{ m}$$

- ① Mean driving method

First, the number of overall gas phase transfer units is calculated.

$$N_{OG} = \frac{Y_1 - Y_2}{\Delta Y_m} \quad \Delta Y_m = \frac{\Delta Y_1 - \Delta Y_2}{\ln \frac{\Delta Y_1}{\Delta Y_2}}$$

$$\Delta Y_1 = Y_1 - Y_1^* = Y_1 - m_e X_1 = 0.0984 - 0.8 \times 0.0917 = 0.025$$

$$\Delta Y_2 = Y_2 - Y_2^* = 0.00197 - 0 = 0.00197$$

$$\Delta Y_m = \frac{\Delta Y_1 - \Delta Y_2}{\ln \frac{\Delta Y_1}{\Delta Y_2}} = \frac{0.025 - 0.00197}{\ln \frac{0.025}{0.00197}} = 0.00906$$

$$N_{OG} = \frac{Y_1 - Y_2}{\Delta Y_m} = \frac{0.0984 - 0.00197}{0.00906} = 10.64$$

Packing height: $H' = N_{OG}H_{OG} = 10.64 \times 0.731 = 7.78$ m.

② Absorption factor method

$$\frac{L}{V} = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0984 - 0.00197}{0.0917 - 0} = 1.05$$

$$\frac{1}{A_a} = \frac{m_e V}{L} = \frac{0.8}{1.05} = 0.762$$

$$N_{OG} = \frac{1}{1 - \frac{1}{A_a}} \ln \left[\left(1 - \frac{1}{A_a} \right) \frac{Y_1 - m_e X_2}{Y_2 - m_e X_2} + \frac{1}{A_a} \right]$$

$$= \frac{1}{1 - 0.762} \times \ln \left[(1 - 0.762) \times \frac{0.0984}{0.00197} + 0.762 \right]$$

$$= 10.66$$

Packing height: $H' = N_{OG}H_{OG} = 10.66 \times 0.731 = 7.79$ m.

③ Direct integration method using calculator

$$\frac{L}{V} = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0984 - 0.00197}{0.0917 - 0} = 1.05$$

The material balance is carried out between any cross section and the top of the tower:

$$Y = \frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2 \right)$$

Put $\frac{L}{V} = 1.05$, $Y_2 = 0.00197$, $X_2 = 0$ in:

$$Y = 1.05X + (0.00197 - 1.05 \times 0) = 1.05X + 0.00197$$

Put into (4.68):

$$N_{OG} = \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} = \int_{X_2}^{X_1} \frac{d\left[\frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2\right)\right]}{\frac{L}{V}X + \left(Y_2 - \frac{L}{V}X_2\right) - f(X)}$$

$$= \int_{X_2}^{X_1} \frac{d[1.05X + (0.00197 - 0)]}{1.05X + 0.00197 - 0.8X} = \int_0^{0.0917} \frac{1.05dX}{1.05X + 0.00197 - 0.8X}$$

$$= \int_0^{0.0917} \frac{1.05dX}{0.25X + 0.00197}$$

Click the calculator function key “ \int ”

Input $\int_0^{0.0917} \frac{1.05dX}{0.25X + 0.00197}$.

Press “=”.

Then, $N_{OG} = 10.65$.

Packing layer height: $H' = N_{OG}H_{OG} = 10.65 \times 0.731 = 7.79$ m.

Thus, it can be seen that the number of overall gas phase transfer units N_{OG} by the three methods is 10.64, 10.66 and 10.65, respectively, which are basically equal. Therefore, the heights of packing layer are also basically equal within the allowable range of error.

Answer: The dosage of water is 85.47 kmol/h, the diameter of the tower is 1.15 m, and the height of the packing layer is 7.79 m.

4) Numerical integration method

When the equilibrium line is not a straight line, Eq. (4.68) is difficult to solve directly with an integral equation and can only be evaluated by various numerical integration methods. The steps of the numerical integration calculation in equation $N_{OG} = \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*}$ (4.68) are as follows: (1) According to the solubility data and operating conditions of the system, equilibrium curves and operating lines are drawn on the $X - Y$ diagram, as shown in Fig. 4.21a. (2) Choose a Y value between the bottom gas concentration Y_1 and the top gas phase concentration Y_2 , determine the corresponding Y^* value from the equilibrium curve, and calculate the value of $1/(Y - Y^*)$. (3) With Y as the abscissa and $1/(Y - Y^*)$ as the ordinate, plot the values of Y and $1/(Y - Y^*)$ in the above groups, as shown in Fig. 4.21b. The area surrounded by vertical lines and curves made by Y_1 and Y_2 in the graph, i.e., the shaded part of the graph, is the required integral value N_{OG} . The trapezoidal method, Simpson method

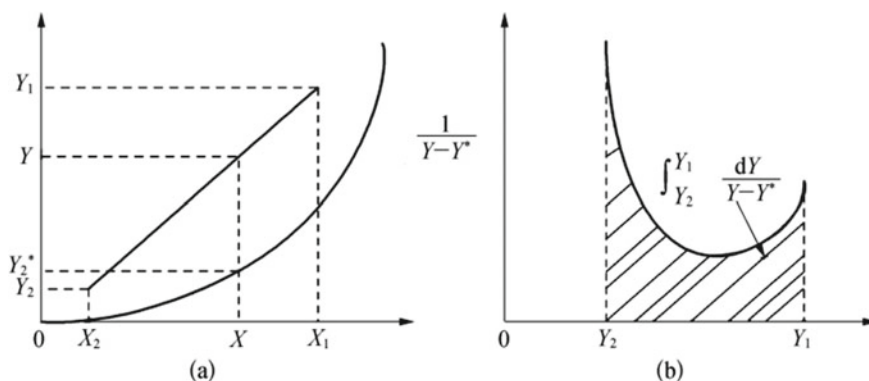


Fig. 4.21 Numerical integration legend

or other methods can be used for numerical calculation, in which the Simpson method is as follows:

$$N_{OG} = \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} = \frac{h_B}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \cdots + 2f_{n-2} + 4f_{n-1} + f_n)$$

In the equation, step size $h_B = \frac{Y_1 - Y_2}{n}$, n is an arbitrarily even number, and the greater the value of n is, the more accurate the upper integral.

Example 4.11 Wash oil is used to absorb the component A in the coke oven gas in a packed tower operated in a countercurrent. The operating pressure of the absorption tower is 101 kPa, and the temperature is 293 K. The content of component A in the gas entering the tower is 0.03 (mole fraction, the same below), the absorption efficiency of component A is required to be 97%, and the wash oil entering the top of the tower contains 0.004 of component A. If the content of component A in the liquid leaving the absorption tower is 0.107. The equilibrium relationship under operating conditions is a curve which can not be expressed by an equation. The values are listed in table as follows. Please find the number of overall transfer units in the gas phase using numerical integration.

The equilibrium relationship between component A and wash oil.

X_A	0.004016	0.023317	0.042618	0.061919	0.081220	0.100521	0.119822
Y_A^*	0.000500	0.002856	0.005136	0.007342	0.009479	0.011549	0.013556

Known: $y_{A,1} = 0.03$, $\eta_A = 0.97$, $x_{A,2} = 0.004$, $x_{A,1} = 0.107$.

Determine: Calculate the number of overall gas phase transfer units using numerical integration.

Solution: Because the equilibrium line is a curve which cannot be expressed by an equation, it can be solved by numerical integration.

$$Y_{A,1} = \frac{y_{A,1}}{1 - y_{A,1}} = \frac{0.03}{1 - 0.03} = 0.0309$$

$$Y_{A,2} = Y_{A,1}(1 - \eta_A) = 0.0309 \times (1 - 97\%) = 0.000928$$

$$X_{A,1} = \frac{x_{A,1}}{1 - x_{A,1}} = \frac{0.107}{1 - 0.107} = 0.120$$

$$X_{A,2} = \frac{x_{A,2}}{1 - x_{A,2}} = \frac{0.004}{1 - 0.004} = 0.00402$$

Make six equal parts between $Y_{A,1}$ and $Y_{A,2}$; the step size is

$$h_{BY} = \frac{Y_{A,1} - Y_{A,2}}{6} = \frac{0.0309 - 0.000928}{6} = 0.005$$

Make corresponding six equal parts between $X_{A,1}$ and $X_{A,2}$; the step size is

$$h_{\text{BX}} = \frac{X_{\text{A},1} - X_{\text{A},2}}{6} = \frac{0.12 - 0.00402}{6} = 0.0193$$

The calculation of $\frac{1}{Y_{\text{A}} - Y_{\text{A}}^*}$ corresponding to each Y_{A} is listed in the following table:

Number	Y_{A}	X_{A}	Y_{A}^*	$\frac{1}{Y_{\text{A}} - Y_{\text{A}}^*}$
0	0.000928	0.004016	0.000500	2336.449
1	0.005928	0.023317	0.002856	325.521
2	0.010928	0.042618	0.005136	172.652
3	0.015928	0.061919	0.007342	116.469
4	0.020928	0.081220	0.009479	87.344
5	0.025928	0.100521	0.011549	69.546
6	0.030928	0.119822	0.013556	57.571

$$\begin{aligned}
 N_{\text{OG}} &= \int_{Y_{\text{A},2}}^{Y_{\text{A},1}} f(Y_{\text{A}}) dY_{\text{A}} = \int_{Y_{\text{A},2}}^{Y_{\text{A},1}} \frac{dY_{\text{A}}}{Y_{\text{A}} - Y_{\text{A}}^*} \\
 &= \frac{h_{\text{B}}}{3} (f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + 4f_5 + f_6) \\
 &= \frac{0.005}{3} (2336.449 + 4 \times 325.521 + 2 \times 172.652 + 4 \times 116.469 + 2 \\
 &\quad \times 87.344 + 4 \times 69.546 + 57.571) \\
 &= 8.27
 \end{aligned}$$

Answer: The number of overall gas phase transfer units in the gas phase obtained by numerical integration is 8.27.

4.5.5 Calculation of Theoretical Plates and Tower Height

The theoretical plate means that when gas and liquid phases meet on a plate, the contact time is long enough and the mass transfer is sufficient, then the composition of gas and liquid phases reaches equilibrium when they leave the plate. As shown in Fig. 4.22a or b, when the gas with the Y_{k+1} gas phase concentration meets the liquid with the X_{k-1} liquid phase concentration from top to bottom on a certain plate, the contact time is long enough, and mass transfer is sufficient, the gas-liquid two-phase composition (X_k, Y_k) reaches equilibrium. Namely, the composition (X_k, Y_k) of gas and liquid phase must belong to a point on the gas-liquid equilibrium line (X_k^*, Y_k^*) when the gas and liquid leave the plate. If the gas-liquid phase equilibrium

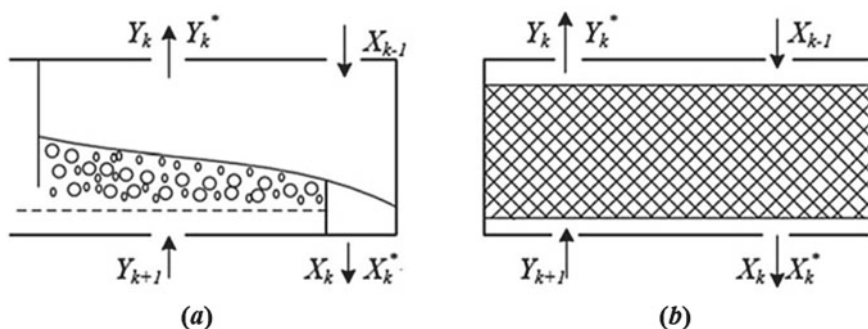


Fig. 4.22 The concept of theoretical plate

relationship conforms to Henry's law, $Y_k^* = m_e X_k^*$. In this way, if absorption is carried out in a plate tower, the number of equivalents of theoretical plate (*NETP*) needed can be calculated first, and then the actual number of plates can be obtained by selecting the total plate efficiency. If the space (distance or height) between the plates is selected, the effective tower height can be determined. For a packed absorption tower, after calculating the *NETP*, the concept of height equivalent to theoretical plate (*HETP*) is introduced; that is, the height of the packing layer required to complete the separation task of a theoretical plate. The height of the packed absorption tower is also determined.

Two basic equations are applied to the calculation of the theoretical plate: the equilibrium relationship (equilibrium line) of the absorbate between the gas and liquid phases and the equation of the operating line. As shown in Fig. 4.23, the gas composition in the inlet tower is Y_{N+1} , the gas composition in the outlet tower is Y_1 , and the absorption solution composition in the inlet tower is X_0 (all molar ratios). The inert gas flow rate is V kmol/h, and the absorbent flow rate is L kmol/h. The operating line equation of the absorption tower shown in Fig. 4.23 is obtained by the following steps: the material balance between the first plate and the top of the tower is made, and the operating line equation can be obtained:

$$Y_2 = \frac{L}{V}(X_1 - X_0) + Y_1$$

The upper equation is a straight line with a pass point (X_0, Y_1) and a slope of L/V . Then, the material balance from the k plate to the top of the tower is calculated, and the equation of the operating line can be obtained.

$$Y_{k+1} = \frac{L}{V}(X_k - X_0) + Y_1 \quad (4.83)$$

The upper equation is also a straight line through point (X_0, Y_1) , and the slope is L/V . This is the general equation of the operating line equation of k plate. It illustrates the relationship between Y_{k+1} and X_k , the two-phase composition of gas phase and liquid phase which meet without contact in a plate. Therefore, unlike the continuous

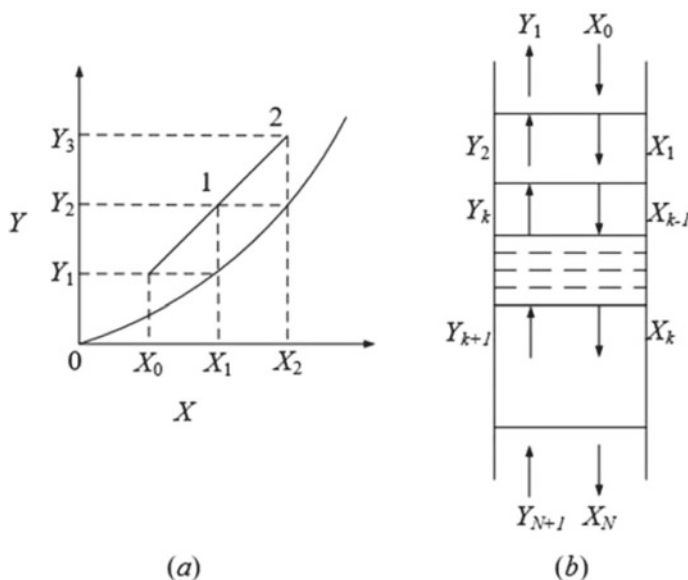


Fig. 4.23 Schematic diagram of multistage countercurrent theoretical model

contact packed tower, the initial driving force of gas–liquid phase contact on the plate tower is $(Y_{k+1} - Y_{k-1}^*)$, not the vertical distance between the operating line and the equilibrium line $(Y_k - Y_{k-1}^*)$.

From the above analysis, it can be seen that the gas–liquid two-phase composition reaches equilibrium when gas and liquid leave the same theoretical plate, its state points, such as (X_k, Y_k) , are on the equilibrium line, and the gas–liquid composition relationship between adjacent plates conforms to the operating line equation. Therefore, according to the above two points, the *NETP* N_T can be calculated by the plate-by-plate method or the graphical calculation method, and then the actual number of plates can be calculated by the following equation:

$$N_P = N_T / \eta_T \quad (4.84)$$

where

N_T —The number of equivalent of a theoretical plate, *NETP*, dimensionless

N_P —Actual number of plates, dimensionless

η_T —Total plate efficiency, dimensionless.

If the phase equilibrium relationship conforms to Henry's law, the *NETP* can be obtained by Kremser's equation:

$$N_T = \frac{1}{\ln A_a} \ln \left[\left(1 - \frac{1}{A_a} \right) \frac{Y_1 - m_e X_2}{Y_2 - m_e X_2} + \frac{1}{A_a} \right] \quad (4.85)$$

where

A_a —Absorption factor, $A_a = \frac{L/V}{m_e}$, dimensionless

Y_1 —Inlet gas composition, dimensionless

X_2 —Liquid composition into tower, dimensionless

Y_2 —Outlet gas composition, dimensionless.

In comparing the equations for calculating the number of mass transfer units (4.81) with the equations for calculating theoretical series (4.85), the following is obtained:

$$N_T = \frac{A_a - 1}{A_a \ln A_a} N_{OG} \quad (4.86)$$

Therefore, the N_{OG} can also be obtained from (4.81) or (4.68), and then the theoretical series can be obtained from Eq. (4.86).

According to the definition, the equation for calculating the equivalent height of the theoretical plate can be obtained.

$$HETP = \frac{H'}{N_T} \quad (4.87)$$

where

H' —Packing layer height, m

N_T —Theoretical series.

Example 4.12 When SO_2 is recovered in factory B, the countercurrent absorption tower is used to absorb it with clean water. It is known that the flow rate of the mixed gas is $2000 \text{ m}^3/\text{h}$ (standard state), and the mole fraction of SO_2 in the gas is 0.15. Under the operating conditions of the absorber, the equilibrium relationship of sulfur dioxide is $Y = 33X$. To maximize the recovery of resources, the manufacturer requires that the recovery efficiency of sulfur dioxide in the absorber be at least 95%.

Determine the following:

- (1) Minimum water consumption.
- (2) When the water consumption is 2 times the minimum water consumption, compare the number of theoretical plate that need to be solved by the graphical method and Kremser's equation.
- (3) If the number of theoretical plate in (2) is adopted, more water is needed to further increase the absorption efficiency to 97.5%.

Known: $q_g = 2000 \text{ m}^3/\text{h}$ (standard), $y_1 = 0.15$, $Y = 33X$, $\eta_A = 95\%$, $L = 2 L_{\min}$, $X_2 = 0$, $\eta'_A = 97.5\%$.

Determine the following: L_{\min} , N_T , ΔL .

Solution: The molar flow rate of inert components can be calculated according to known conditions.

$$V = \frac{pq_g \times 85\%}{RT} = \frac{101.3 \times 2000 \times 0.85}{8.314 \times 273} = 75.87 \text{ kmol/h}$$

(1) Minimum liquid–gas ratio

$$\left(\frac{L}{V}\right)_{\min} = \frac{Y_1 - Y_2}{\frac{Y_1}{m_e} - 0} = \frac{Y_1 - Y_2}{Y_1} m_e = m_e \eta_A = 33 \times 95\% = 31.35$$

Minimum water consumption:

$$L_{\min} = (L/V)_{\min} \times V = 31.35 \times 75.87 = 2378 \text{ kmol/h}$$

(2) Known

$$Y_1 = \frac{y_1}{1 - y_1} = \frac{0.15}{1 - 0.15} = 0.176$$

$$Y_2 = (1 - \eta_A)Y_1 = (1 - 95\%) \times 0.176 = 0.0088$$

and

$$\frac{L}{V} = 2 \left(\frac{L}{V}\right)_{\min} = 2 \times 31.35 = 62.7$$

According to the material balance equation $\frac{Y_1 - Y_2}{X_1 - X_2} = \frac{L}{V}$, $\frac{0.176 - 0.0088}{X_1 - 0} = 62.7$, the solution of $X_1 = 0.00267$ can be obtained.

Draw the operating line through two points (0.00267, 0.176) and (0, 0.0088), as shown in Fig. 4.24. The *NETP* $N_T = 3.59$ is obtained by stepping between the equilibrium line and the operating line in the graph.

Or, by using this equation

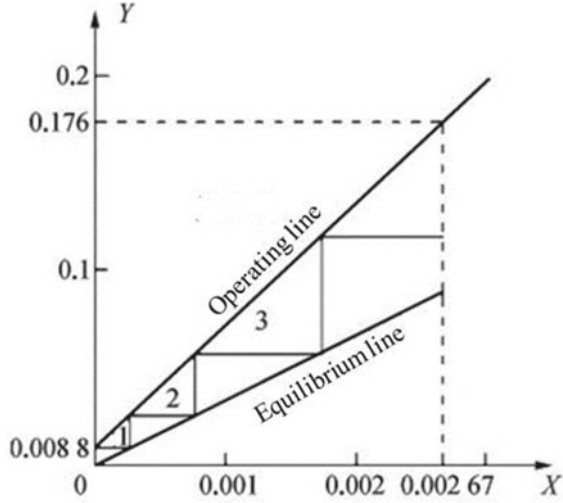
$$N_T = \frac{1}{\ln A_a} \ln \left[\left(1 - \frac{1}{A_a}\right) \frac{Y_1 - m_e X_2}{Y_2 - m_e X_2} + \frac{1}{A_a} \right]$$

in which

$$A_a = \frac{L/V}{m_e} = \frac{62.7}{33} = 1.9$$

and substituting with the data, then:

Fig. 4.24 Schematic diagram of Example 4.12 for calculating the number of theoretical plate by graphic method



$$N_T = \frac{1}{\ln 1.9} \ln \left[\left(1 - \frac{1}{1.9} \right) \frac{0.176}{0.0088} + \frac{1}{1.9} \right] = 3.59$$

Thus, $N_T = 3.59$.

(3) When the number of theoretical plates $N_T = 3.59$, if $\eta'_A = 97.5\%$, then

$$Y'_2 = (1 - \eta'_A) Y_1 = (1 - 97.5\%) \times 0.176 = 0.0044$$

Substituted into:

$$N_T = \frac{1}{\ln A'_a} \ln \left[\left(1 - \frac{1}{\ln A'_a} \right) \frac{Y_1 - m_e X_2}{Y'_2 - m_e X_2} + \frac{1}{\ln A'_a} \right]$$

Obtain

$$3.59 = \frac{1}{\ln A'_a} \ln \left[\left(1 - \frac{1}{A'_a} \right) \frac{0.176}{0.0044} + \frac{1}{A'_a} \right]$$

It can be obtained that $A'_a = 2.42 = \frac{L'}{V m_e}$; then,

$$L' = A'_a V m_e = 2.42 \times 75.87 \times 33 = 6059 \text{ kmol/h}$$

$$\Delta L = L' - L = 6059 - 2378 = 3681 \text{ kmol/h}$$

Answer: The minimum water consumption is 2378 kmol/h, and the *NETP* required is 3.59, and the absorption efficiency increased from 95% to 97.5%, and the increased molar flow rate of water consumption should be 3681 kmol/h.

4.5.6 Determination of Tower Diameter

The diameter of the tower is mainly determined by the treatment capacity of the tower equipment and the allowable gas velocity in the tower. The former refers to the amount of gas mixture treated by the tower equipment per unit of time, i.e., the flow rate of gas mixture; the latter refers to the empty tower velocity of gas in the tower, i.e., the gas flow velocity calculated on the basis of the cross-sectional area of the empty tower. The following relationships exist between them:

$$q_g = \frac{\pi}{4} d_t^2 u_0$$

Thus,

$$d_t = \sqrt{\frac{4q_g}{\pi u_0}} \quad (4.88)$$

where

d_t —Tower diameter, m

q_g —Flow rate of gas mixture, m³/s

u_0 —Gas velocity of empty tower, m/s.

The treatment capacity of the tower equipment is determined by the established production tasks. The gas velocity of empty tower should be selected according to the material properties, tower type and operation mode.

Since the absorbate in the gas phase decreases continuously and the gas pressure decreases gradually during the absorption process, the flow rate of gas mixture in different cross sections of the tower is different, and the maximum volume flow rate of gas mixture in the whole tower is generally taken in the calculation.

In general, about 60 ~ 80% of the gas velocity avoiding liquid overflow u_f are selected to be the gas velocity of empty tower. There are many equations for calculating the gas velocity avoiding liquid overflow u_f . For ring packings, the following equation is often used:

$$\lg \left[\frac{a_{\text{BET}} u_f^2}{g \varepsilon_V^3} \cdot \frac{\rho_G}{\rho_L} \cdot \mu_L^{0.2} \right] = B_m - 1.75 \left(\frac{L}{V} \right)^{1/4} \left(\frac{\rho_G}{\rho_L} \right)^{1/8} \quad (4.89)$$

where

u_f —Gas velocity avoiding liquid overflow, m/s

L/V —Ratio of molar flow rate of liquid to gas

ρ_G/ρ_L —Density ratio of gas over liquid

μ_L —Viscosity of liquid, MPa·s

a_{BET} —Specific surface area of packings, m^2/m^3

ε_V —Packing voidage, m^3/m^3

B_m —Constant; B_m of the Lacy ring is 0.022, and B_m of arc saddle packing is 0.26.

4.5.7 Calculation of the Absorption Process

The calculation of the absorption process can be divided into two types: design calculation and operational calculation. The design calculation refers to calculate the height of the packed bed or the number of actual plates required to complete a given separation requirement. The contents described in the above sections all belong to design calculation. An operational calculation refers to calculate the separation effect of certain mixtures under different operating conditions on the premise that the tower equipment parameters are known. The operational calculation of absorption tower is often encountered in actual production. Its purpose is to guide the operation and adjustment of the absorption tower and to make some measures to ensure separation requirements when operating parameters have changed.

The basic equations used in the operational calculation are the same as those used in the design calculation. They are based on the material balance equations (4.48), (4.49) or (4.50), the phase equilibrium relationship and the packing height Eqs. (4.63), (4.64) or (4.85). The computational process involves solving these basic relations simultaneously.

Generally, the phase equilibrium equation and the height calculation equation of the packing layer are both nonlinear. If the parameters to be obtained in the operational calculation are included in the nonlinear equation, the trial-and-error method must be used to solve them.

When the equilibrium relation conforms to Henry's law, it is more convenient to solve operational problems by using Eqs. (4.81), (4.82), and (4.85), but a trial-and-error method is still needed.

Example 4.13 In the packed absorption tower used in Example 4.9, the overall gas-liquid absorption mass transfer coefficients $K_Y \propto u_V^{0.7}$ (u_V is the gas phase velocity) of the packed material was found in the actual operation, which was not affected by the liquid phase velocity. The height of the packed layer was designed to be 5.00 m, and 99% of gas A could be recovered from the air mixture containing gas A 0.2 (mole fraction). It is known that the working condition of the absorption tower is a gas mass flux of $900 \text{ kg}/(\text{m}^2 \cdot \text{h})$ using inert gas and water as the absorbent with the liquid mass flux of $1000 \text{ kg}/(\text{m}^2 \cdot \text{h})$. The equilibrium equation of gas A is $Y = 0.8X$. On the premise of guaranteeing the recovery efficiency of A, please calculate variety of the height of the packed tower required under the following changed operating conditions: ① doubling the gas velocity ② doubling the liquid velocity.

Known: $K_Y \propto u_V^{0.7}$, $H = 5.00$ m, $y_1 = 0.2$, $\eta_A = 99\%$, $q_G = 900$ kg/(m²·h), $q_L = 1000$ kg/(m²·h), $Y = 0.8X$, $X_2 = 0$, $V = A_T \times q_G/M_{\text{air}}$, $L = A_T \times q_L/M_{\text{water}}$, $M_{\text{water}} = 18$ g/mol, $M_{\text{air}} = 29$ g/mol.

Determine: H' , ΔH .

Solution: The known operating conditions are

$$Y_1 = \frac{y_1}{1 - y_1} = \frac{0.2}{1 - 0.2} = 0.25$$

$$Y_2 = (1 - 99\%)Y_1 = 0.0025$$

$$X_2 = 0$$

$$\frac{L}{V} = \frac{A_T \times q_L/M_{\text{water}}}{A_T \times q_G/M_{\text{air}}} = \frac{\frac{1000}{18}}{\frac{900}{29}} = 1.79$$

According to the material balance, it can be calculated that:

$$X_1 = \frac{V(Y_1 - Y_2)}{L} = \frac{0.25 - 0.0025}{1.79} = 0.138$$

Thus,

$$\Delta Y_1 = Y_1 - 0.8X_1 = 0.25 - 0.8 \times 0.138 = 0.1396$$

$$\Delta Y_2 = Y_2 - Y'_2 = Y_2 - m_e X_2 = Y_2 - 0 = 0.0025$$

i.e.,

$$\Delta Y_m = \frac{\Delta Y_1 - \Delta Y_2}{\ln \frac{\Delta Y_1}{\Delta Y_2}} = \frac{0.1396 - 0.0025}{\ln \frac{0.1396}{0.0025}} = 0.034$$

$$N_{OG} = \frac{Y_1 - Y_2}{\Delta Y_m} = \frac{0.25 - 0.0025}{0.034} = 7.28$$

Finally, determine

$$H_{OG} = \frac{H}{N_{OG}} = \frac{5.00}{7.28} = 0.687 \text{ m}$$

- (1) Since $K_Y \propto u_V^{0.7}$, $K_Y = R \cdot u_V^{0.7}$. Since the gas velocity is doubled, $u'_V = 2u_V$, $V' = 2V$.

$$\frac{K'_Y}{K_Y} = \frac{R(2u_V)^{0.7}}{Ru_V^{0.7}} = 2^{0.7}, K'_Y = 2^{0.7}K_Y$$

$$\begin{aligned}
 H'_{OG} &= \frac{V'}{K'_Y \cdot a A_T} = \frac{2V}{2^{0.7} \times K_Y a A_T} = 2^{0.3} \times \frac{V}{K_Y a A_T} \\
 &= 2^{0.3} \cdot H_{OG} = 1.23 \cdot H_{OG} = 1.23 \times 0.687 = 0.845 \text{ m}
 \end{aligned}$$

In other words, the total height of gas phase transfer unit increases to 1.23 times of original height, and $H'_{OG} = 0.845 \text{ m}$.

Here,

$$\begin{aligned}
 X'_1 &= \frac{V'(Y_1 - Y_2)}{L} = \frac{Y_1 - Y_2}{(L/V)/2} = \frac{0.25 - 0.0025}{\frac{1.79}{2}} = 0.276 \\
 \Delta Y'_1 &= Y_1 - 0.8X'_1 = 0.25 - 0.8 \times 0.276 = 0.0292 \\
 \Delta Y'_2 &= Y_2 - 0.8X'_2 = 0.0025 - 0.8 \times 0 = 0.0025 \\
 \Delta Y'_m &= \frac{\Delta Y'_1 - \Delta Y'_2}{\ln(\Delta Y'_1/\Delta Y'_2)} = \frac{0.0292 - 0.0025}{\ln(0.0292/0.0025)} = 0.0109 \\
 N'_{OG} &= \frac{Y_1 - Y_2}{\Delta Y'_m} = \frac{0.25 - 0.0025}{0.0109} = 22.71 \\
 H' &= H'_{OG} N'_{OG} = 0.845 \times 22.71 = 19.19 \text{ m} \\
 \Delta H &= H' - H = 19.19 - 5 = 14.19 \text{ m}
 \end{aligned}$$

- (2) If the liquid velocity is doubled ($L'' = 2L$), K_Y is basically unchanged because the liquid velocity has little effect on the $K_Y \cdot H_{OG} = V/K_Y a A_T$, V , a , A_T keep unchanged, then H_{OG} keep basically unchanged.

Thus, $H''_{OG} = 0.687 \text{ m}$. Here,

$$X''_1 = \frac{V(Y_1 - Y_2)}{L''} = \frac{Y_1 - Y_2}{2(\frac{L}{V})} = \frac{0.25 - 0.0025}{1.79 \times 2} = 0.0691$$

Then,

$$\begin{aligned}
 \Delta Y''_1 &= Y_1 - 0.8X''_1 = 0.25 - 0.8 \times 0.0691 = 0.195 \\
 \Delta Y''_2 &= Y_2 = 0.0025 \\
 \Delta Y''_m &= \frac{\Delta Y''_1 - \Delta Y''_2}{\ln(\Delta Y''_1/\Delta Y''_2)} = \frac{0.195 - 0.0025}{\ln(0.195/0.0025)} = 0.04418 \\
 N''_{OG} &= \frac{Y_1 - Y_2}{\Delta Y''_m} = \frac{0.25 - 0.0025}{0.04418} = 5.6
 \end{aligned}$$

Finally,

$$\begin{aligned}
 H'' &= H''_{OG} N''_{OG} = 0.687 \times 5.60 = 3.85 \text{ m} \\
 \Delta H &= H'' - H = 3.85 - 5 = -1.15 \text{ m}
 \end{aligned}$$

Answer: When the gas velocity doubles, the packing height increases to 19.19 m, and the increased height is 14.19 m. When the liquid velocity doubles, the packing height decreases to 3.85 m, and the decreased height is 1.15 m.

4.6 Desorption

4.6.1 Overview

Desorption is an inverse process of absorption and a mass transfer process which releases the solute dissolved in the solution from liquid to gas phase. The purpose of desorption is to recover or remove solute dissolved in absorbents. In environmental engineering, NH_3 recovery or removal from industrial wastewater by steam stripping or air blowing is a typical desorption process. The removal of volatile organic acids from sewage by air stripping is also a typical desorption process, and the recovery of polycyclic aromatic hydrocarbons in coking wastewater by steam stripping is also a typical desorption process.

There are many desorption methods, including decompression, heating, steam stripping and gas stripping desorption. Decompression desorption is to reduce the partial pressure of the solute in the gas phase to less than the gas equilibrium partial pressure of the solute in the liquid phase and then release solute from liquid to gas phase. Heating desorption heats the solution and reduces the solubility of the solute in absorbent so that the gas equilibrium partial pressure of the solute in the solution is always higher than the partial pressure of the solute in the gas phase, separating solute from the liquid to gas phase. These two desorption processes (decompression and heating desorption) are simple desorption processes and do not involve complex mass transfer processes between gas and liquid phases. Only gas stripping desorption processes will be discussed in this text book.

4.6.2 Gas Stripping Desorption

Gas stripping desorption is also called carrier gas desorption. Solution sprays from the top of the desorption tower. Carrier gas enters the tower from the bottom of the desorption tower. Under the action of a pressure difference, it flows from bottom to top and contacts the solution countercurrent. There is no or a very small solute concentration in the carrier gas, so the solute transfers from the liquid phase to the gas phase. Finally, the carrier gas with solute flows out from the top of the tower. The function of carrier gas is to provide a gas phase in which the partial pressure of solute is always less than the gas equilibrium partial pressure of solute in solutions. In this way, under the action of the desorption driving force, the solute will be continuously transferred from the liquid phase to the gas phase. Generally,

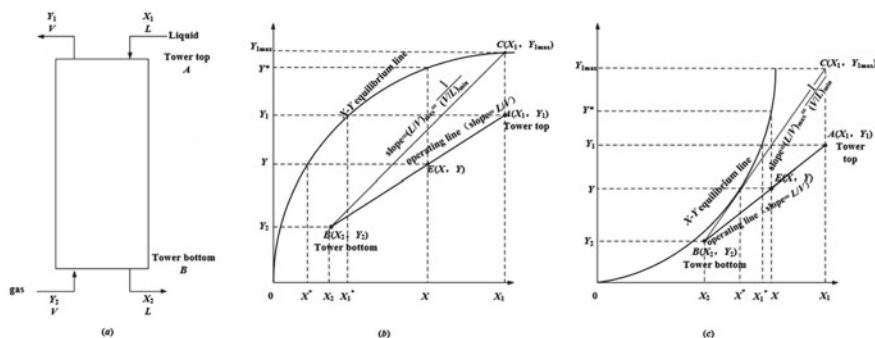


Fig. 4.25 Schematic of countercurrent gas extraction and desorption

gas stripping desorption is a continuous countercurrent operation. Different carrier gases, such as inert gas, air, steam, etc., can be selected for stripping desorption according to the process requirements and the characteristics of the process, similar to the selection of absorbent solvent in an absorption operation.

The desorption principle is the same as absorption, but the driving force sign is opposite; for example, the driving force changes from $p - p^*$ to $p^* - p$. In the graphic calculation, the position of the operating line changes below the equilibrium line. The equation of the desorption operating line is exactly the same as that of the absorption operating line, as shown in Fig. 4.25. Therefore, the calculation equation can be used for desorption only by changing the driving force (concentration difference) in the absorption flux equation. Generally, the molar flow rate of the solvent L (kmol/h) in the liquid with solute the concentration X_1 and X_2 before and after desorption are given by the process conditions, and the solute concentration Y_2 in the carrier gas of the tower is also determined by the process (usually zero). Therefore, in the calculation of desorption process, the molar flow rate of the inert gas V (kmol/h) in gas and the height of packing layer should be determined. Material balance, phase equilibrium and mass transfer flux equation need to be solved simultaneously.

4.6.2.1 Minimum Gas–Liquid Ratio $(V/L)_{\min}$ and Molar Flow Rate of Inert Gas (V)

To be consistent with the absorption process, subscript 1 represents concentrated end A (X_1, Y_1) (top of the tower), and subscript 2 represents low-concentration end B (X_2, Y_2) (bottom of the tower). Those charts are shown in Fig. 4.25. The material balance for the full tower range is:

$$V(Y_1 - Y_2) = L(X_1 - X_2)$$

The equation is the operating line equation of the desorption process. It is a straight line with a slope of L/V on the X - Y coordinate map and passing through the composition points of the top (X_1, Y_1) and bottom (X_2, Y_2) of the desorption tower, as shown in BA in Fig. 4.25. Because of the desorption process requirement, $\Delta X = X - X^* > 0$, $\Delta Y = Y - Y^* < 0$, the operating line BA is below the equilibrium line.

Because point B (X_2, Y_2) has been given by process conditions, when molar flow rate of inert gas (V) decreases, Y_1 of the outlet gas rises, and point A of the operating line approaches the equilibrium line, but the Y_1 limit position is Y_1^* of point C (X_1, Y_1^*), $Y_1^* = Y_{1\max} = f(X_1)$. At this time, the slope L/V of the desorption operating line is the largest $(L/V)_{\max}$, and the gas-liquid ratio V/L is the smallest $(V/L)_{\min}$:

$$\begin{aligned}(L/V)_{\max} &= \frac{Y_{1\max} - Y_2}{X_1 - X_2} \\ (V/L)_{\min} &= \frac{1}{(L/V)_{\max}} = \frac{X_1 - X_2}{Y_{1\max} - Y_2}\end{aligned}\quad (4.90)$$

Usually, $V/L = (1.2 \sim 2)(V/L)_{\min}$ is taken.

When air is used as an inert gas, the V/L value is much greater than the above selection value.

4.6.2.2 Calculation of the Number of Mass Transfer Units and the Number of Equivalents of a Theoretical Plate ($NETP$)

When the phase equilibrium relationship conforms to Henry's law,

$$N_{OL} = \frac{1}{1 - A_a} \ln \left[(1 - A_a) \frac{X_1 - (Y_2/m_e)}{X_2 - (Y_2/m_e)} + A_a \right] \quad (4.91)$$

Accordingly, the equation for calculating the the number of equivalents of a theoretical plate ($NETP$) required for desorption is

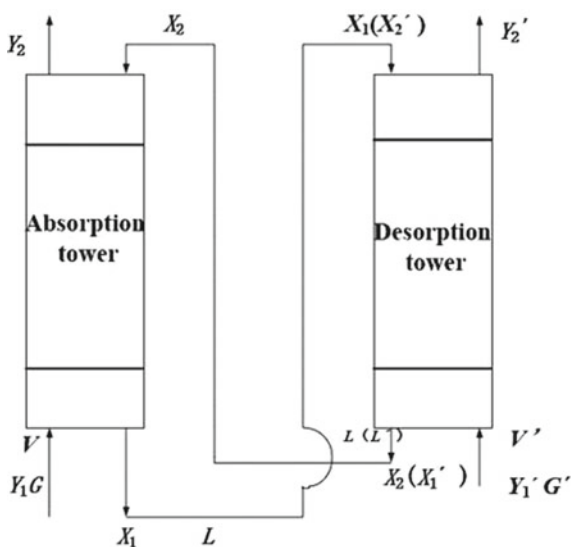
$$N_T = \frac{1}{\ln\left(\frac{1}{A_a}\right)} \ln \left[(1 - A_a) \frac{X_1 - (Y_2/m_e)}{X_2 - (Y_2/m_e)} + A_a \right] \quad (4.92)$$

The range of the desorption factor for desorption operation is $1.2 < \frac{1}{A_a} < 2.0$, and generally $\frac{1}{A_a} = 1.4$. Comparing Eqs. (4.91 ~ 4.92), Eq. (4.93) is obtained.

$$\frac{N_T}{N_{OL}} = (A_a - 1) / \ln A_a \quad (4.93)$$

$A_a = (L/V)/m_e$ is the absorption factor, and $\frac{1}{A_a} = m_e/(L/V)$ is the desorption factor.

Fig. 4.26 Schematic diagram of Example 4.14



Example 4.14 As shown in the Fig. 4.26, an absorption-desorption system is constructed in a factory to reuse raw materials without waste. The packing height of both towers is 10 m. In actual operation, the flow rate of gas treated by the absorption tower is 2000 kmol/h, the circulating absorbent flow rate is 300 kmol/h, and the desorption gas flow rate is 500 kmol/h. The components are $y_1 = 0.01$ (mole fraction same below), $y'_1 = 0$, $y'_2 = 0.03$, $x_2 = 0.001$. In absorption and desorption towers, the phase equilibrium relationship of the gas phase and liquid phase are $y = 0.12x$ and $y' = 0.9x'$, respectively.

Please determine:

- (1) Outlet composition of gas in absorption tower.
- (2) Height of mass transfer unit of desorption tower.
- (3) If the desorption inert gas flow rate changes to 800 kmol/h, what is the outlet composition of the gas in the absorption tower? Assume that L , G , y_1 , y'_1 do not change. The heights of both the absorption and desorption towers keep not change. Namely, H_{OG} and H'_{OG} do not change with the gas flow rate.

Known: $G = 2000$ kmol/h, $L = 300$ kmol/h, $G' = 500$ kmol/h, $y_1 = 0.01$, $y'_1 = 0$, $y'_2 = 0.03$, $x'_1 = x_2 = 0.001$, $y = 0.12x$, $y' = 0.9x'$, $x'_2 = x_1$, $m_{e,1} = 0.12$, $m_{e,2} = 0.9$.

Determine: (1) y_2 , (2) H'_{OG} , (3) y_2 .

Solution: Known operating conditions are

$$V = G(1 - y_1) = 2000 \times (1 - 0.01) = 1980 \text{ kmol/h}$$

$$V' = G'(1 - y'_1) = 500 \times (1 - 0) = 500 \text{ kmol/h}$$

$$Y_1 = \frac{y_1}{1 - y_1} = \frac{0.01}{1 - 0.01} = 0.0101$$

Since $y'_1 = 0$, then $Y'_1 = 0$

$$Y'_2 = y'_2/(1 - y'_2) = 0.03/(1 - 0.03) = 0.0309$$

$$X'_1 = X_2 = x_2/(1 - x_2) = 0.001/(1 - 0.001) = 0.001$$

(1) Total material balance for two towers:

Absorption tower: $V(Y_1 - Y_2) = L(X_1 - X_2)$.

Desorption tower: $V'(Y'_2 - Y_1) = L'(X'_2 - X'_1) = L(X_1 - X_2)$.

Then $V(Y_1 - Y_2) = V'(Y'_2 - Y'_1)$

Substitute data

$$1980 \times (0.0101 - Y_2) = 500 \times (0.0309 - 0)$$

Therefore,

$$Y_2 = 2.29 \times 10^{-3}$$

$$y_2 = \frac{Y_2}{1 + Y_2} = 2.29 \times 10^{-3} / (1 + 2.29 \times 10^{-3}) = 2.28 \times 10^{-3}$$

(2) Material balance for absorption tower

$$L(X_1 - X_2) = V(Y_1 - Y_2)$$

Substitute data

$$300 \times (X_1 - 0.001) = 1980 \times (0.0101 - 0.00229)$$

Solution:

$$X_1 = 0.0525$$

$$X'_2 = X_1 = 0.0525$$

Next, the H'_{OG} of the desorption tower is solved.

$$\Delta Y'_1 = Y_1^* - Y'_1 = m_{e,2}X'_1 - Y'_1 = 0.9 \times 0.001 - 0 = 0.0009$$

$$\Delta Y'_2 = Y_2^* - Y'_2 = m_{e,2}X'_2 - Y'_2 = 0.9 \times 0.0525 - 0.0309 = 0.01635$$

$$\Delta Y'_m = \frac{\Delta Y'_1 - \Delta Y'_2}{\ln \frac{\Delta Y'_1}{\Delta Y'_2}} = \frac{0.0009 - 0.01635}{\ln \frac{0.0009}{0.01635}} = 0.00533$$

Thus,

$$N'_{OG} = \frac{Y'_2 - Y'_1}{\Delta Y'_m} = \frac{0.0309 - 0}{0.00533} = 5.80$$

$$H'_{OG} = \frac{H'}{N'_{OG}} = \frac{10}{5.80} = 1.72 \text{ m}$$

Similarly, for absorption towers

$$\Delta Y_1 = Y_1 - Y_1^* = Y_1 - m_{e,1}X_1 = 0.0101 - 0.12 \times 0.0525 = 0.0038$$

$$\Delta Y_2 = Y_2 - Y_2^* = Y_2 - m_{e,1}X_2 = 2.29 \times 10^{-3} - 0.12 \times 0.001 = 0.00217$$

$$\Delta Y_m = \frac{\Delta Y_1 - \Delta Y_2}{\ln \frac{\Delta Y_1}{\Delta Y_2}} = \frac{0.0038 - 0.00217}{\ln \frac{0.0038}{0.00217}} = 2.91 \times 10^{-3}$$

Thus,

$$N_{OG} = \frac{Y_1 - Y_2}{\Delta Y_m} = \frac{0.0101 - 2.29 \times 10^{-3}}{2.91 \times 10^{-3}} = 2.68$$

- (3) After the desorption inert gas flow rate V' changes to 800 kmol/h, the material balance of the absorption tower is determined:

$$V(Y_1 - Y_2) = L(X_1 - X_2)$$

$$1980 \times (0.0101 - Y_2) = 300 \times (X_1 - X_2) \quad (①)$$

The material balance of the desorption tower is:

$$V'(Y'_2 - Y'_1) = L'(X'_2 - X'_1) = L(X_1 - X_2)$$

$$800 \times (Y'_2 - 0) = 300 \times (X_1 - X_2)$$

$$(X_1 - X_2) = \frac{800}{300}(Y'_2 - 0)$$

$$(X_1 - X_2) = 2.67Y'_2 \quad (②)$$

For the absorption tower, because H and H_{OG} are unchanged, N_{OG} is unchanged.

$$N_{OG} = \frac{1}{1 - \frac{m_{e,1}V}{L}} \ln \left[\left(1 - \frac{m_{e,1}V}{L} \right) \frac{Y_1 - m_{e,1}X_2}{Y_2 - m_{e,1}X_2} + \frac{m_{e,1}V}{L} \right]$$

$$= \frac{1}{1 - \frac{0.12 \times 1980}{300}} \ln \left[\left(1 - \frac{0.12 \times 1980}{300} \right) \frac{0.0101 - 0.12X_2}{Y_2 - 0.12X_2} + \frac{0.12 \times 1980}{300} \right]$$

$$= 2.68$$

That is,

$$\frac{0.0101 - 0.12X_2}{Y_2 - 0.12X_2} = 4.587 \quad (3)$$

For the desorption tower, according to the assumption that H'_{OG} is unchanged, $H = H_{OG}N_{OG} = H'_{OG}N'_{OG}$, it is known that N'_{OG} remains unchanged after the change in desorption inert gas flow rate.

$$\begin{aligned} \Delta Y'_m &= \frac{\Delta Y'_1 - \Delta Y'_2}{\ln \frac{\Delta Y'_1}{\Delta Y'_2}} = \frac{(m_{e,2}X'_1 - Y'_1) - (m_{e,2}X'_2 - Y'_2)}{\ln \frac{m_{e,2}X'_1 - Y'_1}{m_{e,2}X'_2 - Y'_2}} \\ &= \frac{m_{e,2}(X_2 - X_1) - (Y'_1 - Y'_2)}{\ln \frac{m_{e,2}X_2}{m_{e,2}X_1 - Y'_2}} \end{aligned}$$

According to Eq. ②, $(X_1 - X_2) = 2.67 Y'_2$, then $(X_2 - X_1) = -2.67 Y'_2$; In addition $Y'_1 = 0$,

$$\begin{aligned} \Delta Y'_m &= \frac{0.9(-2.67)Y'_2 - (0 - Y'_2)}{\ln \frac{0.9X_2}{0.9X_1 - Y'_2}} \\ \Delta Y'_m &= \frac{1.4Y'_2}{\ln \frac{0.9X_1 - Y'_2}{0.9X_2}} \end{aligned}$$

Therefore,

$$N'_{OG} = \frac{Y'_2 - Y'_1}{\Delta Y'_m} = \frac{Y'_2 - 0}{\frac{1.4Y'_2}{\ln \frac{0.9X_1 - Y'_2}{0.9X_2}}} = \frac{\ln \frac{0.9X_1 - Y'_2}{0.9X_2}}{1.4} = 5.80$$

It can be concluded that:

$$\frac{0.9X_1 - Y'_2}{0.9X_2} = 3361 \quad (4)$$

In conjunction with ① ~ ④:

$$Y_2 = 2.207 \times 10^{-3}, y_2 = \frac{Y_2}{1 + Y_2} = \frac{2.207 \times 10^{-3}}{1 + 2.207 \times 10^{-3}} = 2.20 \times 10^{-3}$$

Answer: The outlet composition of the gas in absorption tower y_2 is 2.28×10^{-3} , and the height of mass transfer unit of desorption tower H'_{OG} is 1.72 m. If the desorption inert gas flow rate is 800 kmol/h, y_2 is 2.20×10^{-3} .

$$Y_j^* = m_{ej}X_j \quad (4.94)$$

where

X_j —Molar ratio of j component in liquid phase, dimensionless

Y_j^* —Molar ratio of the j component in the gas phase, which is in equilibrium with the liquid phase X_j , dimensionless

m_{ej} —Phase equilibrium constant of absorbate component j .

The phase equilibrium constants m_e of each solute component are different from each other, so each solute component has its own equilibrium line, as shown in Fig. 4.24 through the coordinate origin of the three straight lines OA, OB and OC. From the relative position of the equilibrium line, it can be seen that the solubility of component A is the smallest (insoluble) in $m_{eA} > m_{eB} > m_{eC}$, which is called the light component, the solubility of component C is the largest (soluble), which is called the heavy component, and the solubility of component B is between A and C.

In the gas and liquid in and out of the tower, the concentration of each component is different, and their material balance equations are different. In view of the absorption of low concentration gas, the gas and liquid flow rate (V and L) as well as the liquid–gas ratio L/V along the tower can be regarded as constants. Thus, the operating lines of each component are a set of parallel lines, such as DE , FG and HI in Fig. 4.24. Referencing Eq. (4.49), the operating line equation of any component j is:

$$Y_j = \frac{L}{V}(X_j - X_{j2}) + Y_{j2} \quad (4.95)$$

where

Y_j, X_j —Molar ratio of j component in gas and liquid phases in contact with a cross section of the tower

Y_{j2}, X_{j2} —Molar ratio of the j component in the gas and liquid phases at the top cross section of the tower.

The slope of the operating line of each component is the same, but the slope of the equilibrium line is not the same. There is always one component (or more) whose slope of the equilibrium line is close to the slope of the operating line, and the two lines are nearly parallel, that is, $(L/V)/m_e$ is close to 1. As shown in Fig. 4.27, component B, which is generally a medium solubility component, is called the key component.

The more insoluble component A compared to the key component B, the slope of the equilibrium line is larger than that of operating line $\frac{(L/V)}{m_e} < 1$, the two lines converge at the bottom of the tower. The solution is discharged from the bottom of the tower, where the concentration of A is close to the equilibrium concentration with the component A in gas mixture of inlet. However, the concentration of A in the gas, which discharged from the top of the tower, is still high, indicating that the absorption is not complete; that is, the recovery efficiency or removal efficiency are low.

It can also be seen from the distribution of the theoretical plates that the concentration of the theoretical plates near the top of the tower varies greatly, while the concentration of the theoretical plates near the bottom of the tower varies very little,

so the absorption of insoluble components is mainly carried out near the top of the tower.

In contrast with the soluble component of the key component (C in figure), the situation is exactly the opposite. The two lines tend to converge at the top of the tower. When the gas is sent out from the top of the tower, the concentration of C is very low, indicating that C is completely absorbed; that is, the recovery efficiency is very high. The absorption of soluble components is mainly near the bottom of the tower, and the plate near the top of the tower does not play much of a role in separation.

From the above analysis, it is obviously impossible to require a tower to absorb all components equally well. The design principle of a multicomponent absorption tower is to determine the number of theoretical plates (or number of transfer units) needed according to the absorption requirements of one component and then calculate how many theoretical plates (or number of transfer units) can absorb other components at the same time to calculate the gas and liquid composition of the tower according to the gas and liquid composition of the intake tower.

4.7.2 Chemical Absorption

In the process of absorption, chemical reactions can occur between absorbates in gases and other components in absorbent. This process of absorption accompanied by chemical reactions is called chemical absorption. For example, the absorption of flue gas containing SO_2 , NO_2 and CO_2 or odorous gas containing H_2S and mercaptan by NaOH or Na_2CO_3 aqueous solutions is typical chemical absorption. In waste gas treatment and purification projects, physical absorption often cannot completely remove gaseous pollutants; at most, it can only reduce the pollutants in gas to their equilibrium partial pressure with the concentration of pollutants in absorbent, while chemical absorption can completely remove gaseous pollutants in theory. Therefore, chemical absorption is more important in the control of waste gas pollution.

1) Characteristics of chemical absorption

Chemical absorption is a process of simultaneous mass transfer and chemical reaction, which is more complex than physical absorption. Compared with physical absorption, chemical absorption has the following characteristics:

- (1) Mass transfer driving force is great. When the absorbate enters the absorbent, it is consumed by chemical reaction, and there are few or no free absorbate in the absorption solution, so the equilibrium partial pressure of the absorbate is very low or zero, and the absorption driving force is improved greatly.
- (2) The overall gas-liquid mass transfer coefficient is great. If the concentration of the active component in the absorbent solution is great enough and the fast irreversible reaction occurs, the absorbate will be completely consumed near the gas-liquid interface; then, the diffusion resistance of the absorbate in the liquid film will be greatly reduced, or even reduced to zero, thus increasing the mass transfer coefficient of the liquid phase.

- (3) The effective mass transfer area is large. In the absorption equipment, there are always some liquids that flow slowly or stagnate. In physical absorption, these liquids are often saturated by solutes and lose their absorption capacity. However, chemical absorption needs to absorb more solutes, and then to achieve saturation. Therefore, some mass transfer area is not effective for physical absorption in some time, but it may still be effective for chemical absorption. Therefore, under the same gas–liquid flow conditions, the effective mass transfer area of chemical absorption is larger than that of physical absorption.

Based on the above characteristics, mass transfer rate of chemical absorption is high, which can reduce the size of equipment, save equipment investment, absorbent capacity (the absorbed amount per unit volume of absorbent) is large, absorbent dosage can be reduced, operational cost can be reduced, and the gas purification effect is good. Chemical absorption also has the following shortcomings: the price of absorbent is more expensive, the corrosiveness of absorbent to equipment is stronger, the crystallization of reaction products can foul equipment, the regeneration of absorbent is more difficult, and the cost of regeneration is higher. Therefore, the selection of absorption schemes should be considered comprehensively.

2) Chemical absorption mechanism

The mechanism of chemical absorption is much more complicated than that of physical absorption. Its absorption rate depends not only on the diffusion rate of the absorbate in gas and liquid phases but also on the diffusion rate of active components, the chemical reaction rate and the diffusion rate of reaction products in liquid phase. Chemical absorption can be roughly divided into five consecutive steps, of which any one may play a decisive role, as shown in Fig. 4.28. The five steps are as follows:

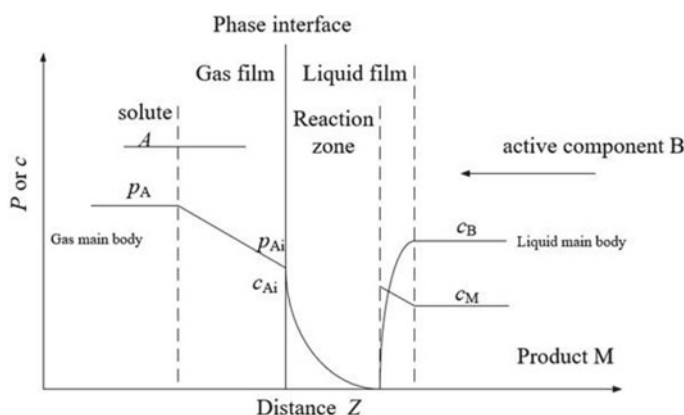


Fig. 4.28 Schematic diagram of the chemical absorption process

- (1) Absorbate A diffuses from the main body of the gas phase to the gas–liquid interface, which is similar to physical absorption.
- (2) Absorbate A diffuses into the reaction film in the liquid phase.
- (3) Active component B, which can react with A in absorbent, diffuses into the reaction film in the liquid phase.
- (4) Chemical reactions between A and B occur in the reaction film.
- (5) The reaction products M diffuse from the reaction zone to the main body of the liquid phase.

The location of the reaction film in the liquid phase is close to the gas–liquid interface, depended on the relative size of the reaction rate and diffusion rate. The faster the reaction proceeds, the closer the reaction zone is to the interface, and the narrower the reaction zone. In contrast, if the reaction proceeds very slowly, absorbate A may also diffuse into the main body of the liquid phase and still fail to react, and the reaction zone will widen. Therefore, the factors affecting the chemical absorption rate include not only the same factors as physical absorption (system properties and gas–liquid flow conditions, etc.) but also the factors related to the chemical reaction rate, such as the chemical reaction rate constant and the concentration of materials involved in the reaction.

Considering the above factors, if the liquid mass transfer coefficient k'_L of chemical absorption can be found to be several times that of pure physical absorption k_L , then the flux relationship of chemical absorption can be written.

$$N'_A = k'_L(c_{Ai} - c_{AL}) = \beta_a k_L(c_{Ai} - c_{AL}) \quad (4.96)$$

where

β_a —Multiple of the absorption flux increased by chemical reaction is called the enhancement factor or reaction factor

c_{Ai} —Concentration of absorbate gas A at the gas–liquid interface

c_{AL} —Concentration of absorbate gas A in the main body of the liquid phase

N'_A —Chemical absorption flux, $\text{kmol}/(\text{m}^2 \cdot \text{s})$.

By combining mass transfer theory with the chemical reaction principle, the relationship between β_a and various influencing factors in some cases can be derived. Because the reaction mechanism cannot be accurately known, the k'_L values obtained by calculation of the enhancing factor are sometimes not very reliable, and some necessary data are not easily obtained, so k'_L or correlation equations are often directly measured by experiments in design. Of course, these k'_L or correlation equation are determined for a specific reaction system, so they can only be used in this system. There are more variables to be considered for chemical absorption than physical absorption, such as the concentration of reactants in absorbent and the temperature and pressure of the absorption system.

Due to the reaction of the absorbate with active components in the liquid phase, chemical absorption can accelerate the mass transfer flux of the absorbate and increase the absorption capacity of the absorbent. On the one hand, the gas partial pressure of the absorbate is only in equilibrium with the absorbate which is physically dissolved in the solution. The concentration of absorbate in liquid phase is very

low, even reaches zero, since the absorbate has reacted with the active components in liquid. Therefore, the absorbate in liquid phase no longer affects the gas-liquid equilibrium, the chemical absorption can make the absorbent absorb more absorbate. On the other hand, the absorbate is consumed by chemical reaction during liquid phase diffusion, which makes the mass transfer resistance of the liquid phase decrease with reduced effective film thickness. The mass transfer driving force increases with decreased interfacial liquid concentration of absorbate, which increases the mass transfer flux of chemical absorption.

3) Classification of chemical absorption

According to the rate of chemical reaction in the chemical absorption process, it can be divided into the following five categories, as shown in Table 4.9, in which M = the amount of reaction of absorbate in the liquid film/the amount of absorbate which does not react and diffuses into the main body of the liquid through the liquid film.

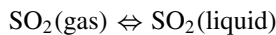
Example 4.15 There is a pool beside Warehouse B. The equilibrium partial pressure of SO_2 is 5.05 kPa when the equilibrium absorption point of SO_2 is reached at a room temperature of 20 °C. It is known that the solubility coefficient of SO_2 is $H_{pc} = 1.56 \times 10^{-2} \text{ kmol}/(\text{kPa} \cdot \text{m}^3)$ and that the first dissociation constant of SO_2 in water is $K_1 = 1.7 \times 10^{-2} \text{ kmol}/\text{m}^3$. Please calculate the solubility of SO_2 under this condition by considering only the first hydrolysis of SO_2 in water without considering other reactions.

Known: $p_A^* = 5.05 \text{ kPa}$, $H_{pc} = 1.56 \times 10^{-2} \text{ kmol}/(\text{kPa} \cdot \text{m}^3)$, $K_1 = 1.7 \times 10^{-2} \text{ kmol}/\text{m}^3$.

Determine: c_A .

Solution: In the case of dissociation, SO_2 absorption involves two processes.

Mass transfer process of SO_2 from the gas phase to the liquid phase:



The dissociation reaction of SO_2 occurs in water:



The concentration of SO_2 in water which does not react can be calculated by Henry's law.

$$[\text{SO}_2] = H_{pc} p_A^* = 1.56 \times 10^{-2} \times 5.05 = 0.0788 \text{ kmol}/\text{m}^3$$

According to the dissociation equilibrium constant equation, the concentration of HSO_3^- can be obtained as follows:

$$K_1 = \frac{[\text{H}^+][\text{HSO}_3^-]}{[\text{SO}_2]}$$

Table 4.9 Chemical absorption types

Chemical reaction rate	M	Relative magnitude of reaction quantity and diffusion quantity	Main location of chemical reaction	Graphical representation
Very slow reaction	$\rightarrow 0$	The total amount of absorbate reaction in liquid phase is much less than the total amount of diffusion transfer	The chemical reaction is neglected and treated by physical absorption	
Slow reaction	$\ll 1$	The amount of absorbate in liquid film is much less than that of diffusion through liquid film	The reaction is carried out in the main liquid phase	

(continued)

Table 4.9 (continued)

Chemical reaction rate	M	Relative magnitude of reaction quantity and diffusion quantity	Main location of chemical reaction	Graphical representation
Medium-rate reaction	≈ 1	The amount of absorbate reaction in liquid film is equal to the amount of diffusion transfer through liquid film	The reaction takes place not only in liquid film but also in the main body of liquid phase	
Rapid reaction	$\gg 1$	Absorbate completes all the chemical reactions and is consumed to zero during the diffusion of absorbate in liquid film	All reactions take place in the liquid film	

(continued)

Table 4.9 (continued)

Chemical reaction rate	M	Relative magnitude of reaction quantity and diffusion quantity	Main location of chemical reaction	Graphical representation
Instantaneous reaction	$\rightarrow \infty$	The absorbate completes all the reactions at the gas-liquid interface	The reaction takes place at the gas-liquid interface	

Without considering other reactions,

$$[\text{H}^+] = [\text{HSO}_3^-]$$

The following substitution is available:

$$[\text{HSO}_3^-] = \sqrt{K_1[\text{SO}_2]} = \sqrt{1.7 \times 10^{-2} \times 0.0788} = 0.0366 \text{ kmol/m}^3$$

Therefore, the total concentration of absorbate in aqueous solution measured by SO_2 is as follows:

$$\begin{aligned} c_A &= [\text{HSO}_3^-] + [\text{SO}_2] = 0.0366 + 0.0788 \\ &= 0.1154 \text{ kmol/m}^3 = 7.4 \text{ kg/m}^3 \end{aligned}$$

Note: The equilibrium problem of the chemically reversible reaction of $\text{SO}_2 + \text{H}_2\text{O} \rightleftharpoons \text{H}_2\text{SO}_3$ is neglected here. It is considered that all SO_2 is converted to H_2SO_3 and then dissociated.

Answer: The solubility of SO_2 is 7.4 kg/m^3 under this condition.

Example 4.16 Warehouse B prepares an absorption device to prevent SO_2 gas leakage. SO_2 in the air can be absorbed by water or alkali liquor in emergencies. It is known that the ambient air pressure is 101.3 kPa, the temperature is 25°C and the mole fraction of SO_2 in the air is 0.05 during the leak. The gas film absorption mass transfer coefficient is $1 \times 10^{-6} \text{ kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$, the liquid film absorption mass transfer coefficient is $8 \times 10^{-6} \text{ m/s}$, the Henry's coefficient of SO_2 is $H_{pm} = 3.55 \times 10^3 \text{ kPa}$, and the density of the absorbent is 1000 kg/m^3 .

Determine: Mass transfer flux of absorbate when clean water and alkali solution are used as absorbent separately (assuming that alkali absorption is an instantaneous irreversible reaction).

Known: $y = 0.05$, $k_G = 1 \times 10^{-6} \text{ kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$, $k_L = 8 \times 10^{-6} \text{ m/s}$, $H_{pm} = 3.55 \times 10^3 \text{ kPa}$, $\rho_0 = 1000 \text{ kg/m}^3$, $M_0 = 18 \text{ kg/kmol}$.

Determine: N_A or N'_A in clean water and alkali solution.

Solution:

- (1) When SO_2 is absorbed by clean water
Solubility coefficient:

$$H_{pc} = \frac{1}{H_{pm}} \frac{\rho_0}{M_0} = \frac{1}{3.55 \times 10^3} \times \frac{1000}{18} = 1.56 \times 10^{-2} \text{ kmol}/(\text{kPa} \cdot \text{m}^3)$$

The overall gas-liquid two phases mass transfer coefficients is:

$$\begin{aligned} \frac{1}{K_G} &= \frac{1}{k_G} + \frac{1}{H_{pc}k_L} = \left(\frac{1}{1 \times 10^{-6}} + \frac{1}{1.56 \times 10^{-2} \times 8 \times 10^{-6}} \right) \\ &= 9.012 \times 10^6 \text{ (m}^2 \cdot \text{s} \cdot \text{kPa)/kmol} \end{aligned}$$

Then:

$$K_G = 1.11 \times 10^{-7} \text{ kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$$

The driving force of mass transfer is:

$$\Delta p_A = p_A - p_A^* = 0.05 \times 101.3 - 0 = 5.065 \text{ kPa}$$

The mass transfer flux is: $N_A = K_G \Delta p_A = 1.11 \times 10^{-7} \times 5.065 = 5.62 \times 10^{-7} \text{ kmol}/(\text{m}^2 \cdot \text{s})$.

- (2) Under the condition of alkali solution absorption, because of the instantaneous irreversible reaction, SO_2 reacts when it reaches the liquid film at the phase interface. There is no accumulation or mass transfer process to the main body of the liquid phase. It can be considered that the concentration of SO_2 in the solution is 0, and there is no resistance to mass transfer in the liquid phase. Therefore, the total mass transfer coefficient of the gas phase is as follows:

$$K_G = k_G = 1 \times 10^{-6} \text{ kmol}/(\text{m}^2 \cdot \text{s} \cdot \text{kPa})$$

The total mass transfer driving force is:

$$\Delta p_A = 5.065 \text{ kPa}$$

The mass transfer flux is:

$$N_A = k_G \Delta p_A = 1 \times 10^{-6} \times 5.065 = 5.065 \times 10^{-6} \text{ kmol}/(\text{m}^2 \cdot \text{s})$$

Answer: The mass transfer flux of SO_2 in clean water is $5.62 \times 10^{-7} \text{ kmol}/(\text{m}^2 \cdot \text{s})$ and $5.065 \times 10^{-6} \text{ kmol}/(\text{m}^2 \cdot \text{s})$ in alkali solution.

Exercises

Part I: Multiple Choice and Fill in the Blanks

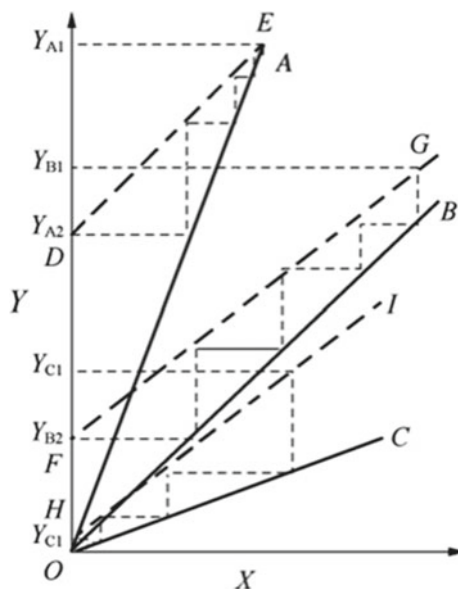
- At a certain temperature and a low total gas pressure, the Henry's coefficient of a gas in water is very small, indicating that the gas ().
 - is soluble in water
 - is hardly soluble in water
 - is insoluble in water
 - solubility in water has nothing to do with the Henry's coefficient
- Gas-liquid two-phase equilibrium depends on the following two conditions: if $p_G > p_L^*$ or $c_G^* > c_L$, it belongs to the () process; if $p_L^* > p_G$ or $c_L > c_G^*$, it belongs to the () process.

- A. absorption
 - B. desorption
 - C. diffusion
 - D. mass transfer
3. Which of the following options is not a factor affecting the diffusion coefficient ().
- A. temperature
 - B. pressure
 - C. properties of diffusion components
 - D. reaction area
4. A larger Henry's coefficient H_{pm} for a gas means the gas ().
- A. easier to dissolve
 - B. more difficult to dissolve
 - C. have moderate solubility
 - D. insoluble
5. The favorable conditions for absorption operation are ().
- A. high temperature, high gas partial pressure
 - B. high temperature, low gas partial pressure
 - C. low temperature, high gas partial pressure
 - D. low temperature, low gas partial pressure
6. On the $Y-X$ diagram, the absorption operating line is always located on the equilibrium line ().
- A. upper
 - B. lower
 - C. two lines coincide
 - D. uncertainty
7. Simple steady-state molecular diffusion can be divided into () and ().
- A. equimolecular counterdiffusion
 - B. equimolecular codiffusion
 - C. unidirectional diffusion
 - D. bidirectional diffusion
8. Which of the following options is not a equilibrium separation process ().
- A. absorption
 - B. adsorption
 - C. heat transfer
 - D. extraction

9. The basic principle of absorption is to separate the components of the gas mixture according to the difference of () in the liquid absorbent.
 - A. density
 - B. specific heat capacity
 - C. volatility
 - D. solubility
10. In the design of the desorption tower, the relationship between the operating line of the desorption tower and the X - Y equilibrium curve is ().
 - A. intersection
 - B. above
 - C. below
 - D. overlap
11. In the absorption operation, the change in the following values does not affect the absorption mass transfer coefficient ().
 - A. changes in the number of mass transfer units
 - B. changes in the height of mass transfer units
 - C. change in the structure size of the absorption tower
 - D. change in packing type and size of the absorption tower
12. When the absorption factor A_a is less than 1 and the height of the packed tower is unrestricted, gas-liquid two-phase equilibrium will be achieved at () of the absorption tower performing counter-current operation.
 - A. the top
 - B. the bottom
 - C. the middle
 - D. any cross section
13. The favorable conditions for a desorption operation are ().
 - A. low temperature and high gas partial pressure
 - B. low temperature and low gas partial pressure
 - C. high temperature and high gas partial pressure
 - D. high temperature and low gas partial pressure
14. The absorption rate depends mainly on the diffusion rate through the two-film. To increase the absorption rate, which () should be done.
 - A. increasing the thickness of the gas film and reducing the thickness of the liquid film
 - B. reducing the thickness of both gas and liquid film
 - C. increasing the thickness of both gas and liquid film
 - D. reducing the thickness of the gas film and increasing the thickness of the liquid film

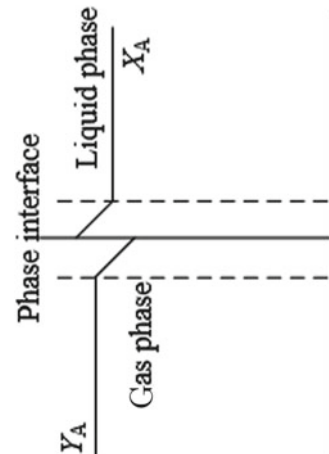
15. The water is used to absorb the CO_2 in air by a countercurrent absorption tower. If the water flow rate increases, the partial pressure of CO_2 in outlet gas will ().
- A. increase
 - B. decrease
 - C. keep unchanged
 - D. be unable to be judged
16. For a desorption operation, the actual concentration of the solute in the liquid is () the concentration of the liquid equilibrium of solute in gas.
- A. less than
 - B. greater than
 - C. equal to
 - D. unable to be judged for
17. In desorption operation, total pressure p () and temperature T () will be beneficial to desorption.
- A. increase
 - B. reduce
 - C. constant
 - D. uncertainty
18. In the countercurrent absorption tower, the absorption of soluble components mainly occurs near (), while the absorption of insoluble components mainly occurs near ().
- A. the top
 - B. the bottom
 - C. the middle
 - D. any cross section
19. Multicomponent absorption tower for countercurrent absorption of gas mixture is shown in Fig. 4.29. Lines OA , OB and OC usually represent the gas () line, while DE , FG and HI usually represent the gas () line. The solubility of component A is (), and it is absorbed mainly in the () of the tower. The solubility of component C is (), and it is absorbed mainly in the () of the tower.
- A. operation
 - B. equilibrium
 - C. maximum
 - D. minimum
 - E. tower bottom
 - F. tower top

Fig. 4.29 Equilibrium line and operating line of each component in three-component absorption



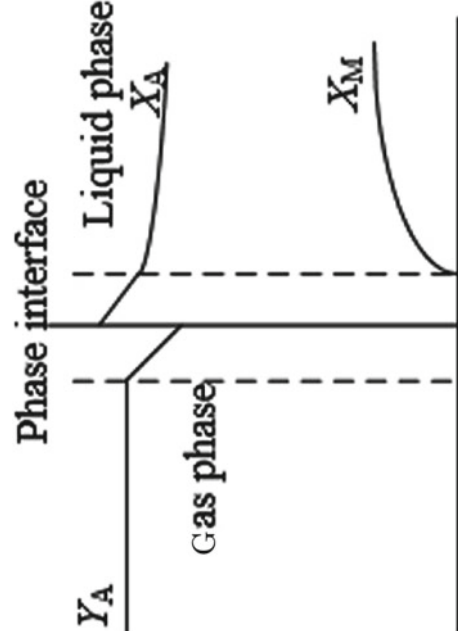
20. According to the rate of chemical reaction in the process of chemical absorption, it can be divided into the following five categories, as listed in Table 4.10, where M' = the amount of reaction of absorbate gas in the liquid film/the amount of absorbate which does not react and diffuses into the main body of the liquid through the liquid film. Please fill in the appropriate selection in the form.
 - A. very slow reaction
 - B. slow reaction
 - C. medium reaction
 - D. rapid reaction
 - E. instantaneous reaction
 - F. chemical reactions are neglected and treated by physical absorption.
 - G. reaction occurs in the main body of liquid phase
 - H. reaction occurs not only in the liquid film but also in the main body of the liquid phase
 - I. the reaction is completed in the liquid film
 - J. reaction occurs at the gas-liquid interface
21. Chemical absorption makes the resistance of liquid film become () and that of gas film become () compared with the corresponding resistance of original physical absorption.
 - A. decrease
 - B. increase
 - C. invariant
 - D. uncertain

Table 4.10 Chemical absorption types

Chemical reaction rate	M	Relative magnitude of reaction quantity and diffusion quantity	Main location of chemical reaction	Graphical representation
()	$\rightarrow 0$	The total amount of solute reaction in liquid phase is much less than the total amount of diffusion transfer	()	

(continued)

Table 4.10 (continued)

Chemical reaction rate	M	Relative magnitude of reaction quantity and diffusion quantity	Main location of chemical reaction	Graphical representation
()	$\ll 1$	The amount of reaction of solute in liquid film is much less than that of diffusion through liquid film	()	

(continued)

Table 4.10 (continued)

Chemical reaction rate	M	Relative magnitude of reaction quantity and diffusion quantity	Main location of chemical reaction	Graphical representation
()	$\rightarrow \infty$	The solute can complete all the reactions at the gas-liquid interface	()	
()	$\gg 1$	All reactions of solutes in the diffusion of liquid films are completed in the liquid films	()	

(continued)

Table 4.10 (continued)

Chemical reaction rate	M	Relative magnitude of reaction quantity and diffusion quantity	Main location of chemical reaction	Graphical representation
()	≈ 1	The reaction amount of solute in liquid film is equivalent to the diffusion transfer amount through liquid film	()	

22. Which of the following options is not characteristic of chemical absorption ().
- large mass transfer driving force
 - large liquid mass transfer coefficient
 - large effective mass transfer area
 - large absorbent dosage
23. The absorption operation is based on _____ to achieve the purpose of separating gas mixtures.
24. The expression equation of Henry's law in $p^* = H_{pm}x$. If the Henry's coefficient of a gas in a solvent is very small, this gas belongs to _____ gas for the solvent.
25. The Henry's coefficient H_{pm} _____, the phase equilibrium constant m_e _____ and the solubility coefficient _____ when the total pressure decreases in the low concentration gas-liquid equilibrium system.
26. In the desorption process, the partial pressure of the solute in the gas phase is always _____ than the equilibrium partial pressure of the solute, so the desorption operating line is always _____ the equilibrium line.
27. The two-film theory simplifies the whole mass transfer process to _____.
28. When the gas film resistance is much greater than the liquid film resistance and the solubility coefficient H_{pc} _____, i.e., $1/k_G \gg 1/(H_{pc}k_L)$, the interphase mass transfer process is controlled by _____, and then $K_G \approx k_G$.
29. When the liquid film resistance is much greater than the gas film resistance and the solubility coefficient H_{pc} _____, i.e., $1/k_L \gg H_{pc}/k_G$, the interphase mass transfer process is controlled by _____, and then $K_L \approx k_L$.
30. In a low-concentration gas-liquid equilibrium system with not too high total pressure, the Henry's coefficient _____ when the solute concentration in solution changes, _____ when the total pressure decreases, and _____ when the temperature rises.
31. In a single-phase system, the rate of material transferred by molecular diffusion, i.e. the rate of molecular diffusion follows the law of _____, which can be expressed as _____. The rate of molecular diffusion per unit mass transfer area perpendicular to the direction of mass transfer is _____ to the concentration gradient, and the direction of mass transfer moves along the direction of concentration _____.
32. According to the two-film theory, when the solubility of the absorbate is very small in the liquid phase, the total mass transfer coefficient expressed by the liquid phase concentration _____ mass transfer coefficient in the liquid phase.
33. The expression of Henry's law is $p^* = H_{pm}x$. If the Henry's coefficient H_{pm} of a gas in water is very large, then the gas belongs to _____ gas.
34. The relationship between the several mass transfer coefficients can be expressed as $\frac{1}{K_G} = \frac{1}{k_G} + \frac{1}{H_{pc}k_L}$, in which $\frac{1}{k_G}$ represents _____, and which indicates that the absorption process is controlled by _____ when the item _____ can be ignored.

35. For the countercurrent absorption tower, when the ratio of the slope of the operating line to the slope of the equilibrium line is _____, gas–liquid equilibrium may be achieved at the _____ of the tower; when the ratio of the slope of the operating line to the slope of the equilibrium line is _____, gas–liquid equilibrium may be achieved at the _____ of the tower.
36. For the countercurrent absorption tower, when the absorption factor $A_a < 1$ and the height of packed tower is infinite, gas–liquid two-phase equilibrium will be reached at _____.
37. The desorption operation is based on the principle: _____ in order to separate the liquid mixture.
38. In the process of multicomponent absorption by an absorption tower, the absorption of soluble components mainly occurs in _____, while the absorption of insoluble components mainly occurs in _____.

Part II: Computational Questions + Answers

1. It is known that at a total pressure of 101.3 kPa and a temperature of 20 °C, the oxygen content in 1000 g water is 0.03 g. At this time, the equilibrium partial pressure of oxygen above the solution is 70 kPa, and the density of the solution is 1000 kg/m³. Determine the following:

- (1) Solubility coefficient H_{pc} [kmol/(m³ · kPa)],
- (2) Henry's coefficient H_{pm} (kPa),
- (3) Phase equilibrium coefficient m_e ,

Answer: Solubility coefficient H_{pc} is 1.34×10^{-5} kmol/(m³ · kPa). Henry's coefficient H_{pm} is 4.142×10^6 kPa. Phase equilibrium coefficient m_e is 4.09×10^4 .

2. In the wet method of removing atmospheric particulate matter, the dust particles in the gas mixture containing SO₂ are washed by clear water in a Venturi pipe, and the gas flow and washing water are separated in a gas–liquid separator. The outlet gas contains 0.05 (mole fraction) of SO₂, and the operating pressure is atmospheric.

In the following two cases:

- (1) The operating temperature was 10 °C;
- (2) The operating temperature was 50 °C.

Please determine the removal amount (g) of SO₂ with per 1 kg of discharge solution. (Looking up the table, it can be seen that at 10 °C, $H_{pm} = 2.45 \times 10^3$ kPa, and at 50 °C, $H_{pm} = 8.71 \times 10^3$ kPa.)

Answer: The loss of SO₂ is 7.35 g at 10 °C and 2.07 g at 50 °C.

3. Please determine the following process to be absorption or desorption and the driving force. Then please show them on the X - Y diagram.
 - (1) The water solution containing 0.001 (mole fraction) of SO_2 contacts the gas mixture containing 0.03 (mole fraction) of SO_2 , the total pressure is 101.3 kPa, and $T = 35^\circ\text{C}$. (Looking up the table, it can be seen that when $T = 35^\circ\text{C}$, $H_{pm} = 0.567 \times 10^4$ kPa for SO_2).
 - (2) The gas-liquid composition and total pressure are the same as (1), $T = 15^\circ\text{C}$ ($H_{pm} = 0.294 \times 10^4$ kPa for SO_2 at $T = 15^\circ\text{C}$).
 - (3) The gas-liquid composition and temperature are the same as (1), and the total pressure is 300 kPa (absolute pressure).

Answer: (1) Desorption $\Delta y = 0.026$, (2) Absorption $\Delta y = 0.001$, (3) Absorption $\Delta y = 0.0111$.

4. At 101.3 kPa and 20°C , 1 m^3 polluted gas containing 0.1 (mole fraction) of SO_2 and 0.9 (mole fraction) of air was dissolved and absorbed by 1 m^3 of clean water in a 2 m^3 container. Please calculate the total mass transfer driving force at the moment of initial contact between the polluted gas and clean water (respectively expressed by partial pressure difference, mole fraction in gas and molar concentration difference of liquid phase). Then please find out the final composition of SO_2 in water and the total pressure of residual gas.
Known: $H_{pm} = 3.55 \times 10^3$ kPa for SO_2 at $T = 20^\circ\text{C}$.

Answer: Overall mass transfer driving force: partial pressure difference: $\Delta p = 10.13$ kPa, mole fraction: $\Delta y = 0.1$, molar concentration difference in liquid phase: $\Delta c = 0.16$ kmol/ m^3 , $c_A = 0.004$ kmol/ m^3 , $p = 91.429$ kPa.

5. In the process of treating polluted gas containing ammonia, water is used to absorb ammonia in the gas mixture in the packed tower. The gas and water enter from the top of the tower and contact with each other from top to bottom. The gas flow rate is $1000\text{ m}^3/\text{h}$ (standard), the ammonia content is 0.01 (mole fraction), the average temperature in the tower is 25°C , and the total pressure is atmospheric. Under this condition, the equilibrium relationship of ammonia between the two phases is $Y = 0.93X$. Please determine the following:
 - (1) If the water flow rate is $5\text{ m}^3/\text{h}$ and there is no ammonia in the water, please calculate the maximum absorption rate of ammonia.
 - (2) If the water flow rate is $10\text{ m}^3/\text{h}$ and there is no ammonia in the water, please calculate the maximum absorption rate of ammonia.
 - (3) If the water flow rate is $5\text{ m}^3/\text{h}$ and the ammonia content in the water is 5 kg/1000 kg, please calculate the maximum absorption rate of ammonia.

Answers: 87%, 92.5%, 44%.

6. In the process of acetone absorption with water, the operating pressure of the absorption tower is 101.32 kPa and the temperature is 293 K. The acetone content in the gas entering the absorption tower is 0.026 (mole fraction), and the required absorption efficiency is 80%. Under operating conditions, the equilibrium relationship of acetone between the two phases is $Y = 1.18X$

(molar ratio). Please calculate the minimum liquid-gas ratio $(L/V)_{\min}$. If the absorption efficiency is 90%, then what is the minimum liquid-gas ratio?

Answer: 0.944, 1.062.

7. In a countercurrent absorption tower, the soluble components in the mixed gas are absorbed by pure absorbent and the equipment height is infinite. The concentration of gas entering the tower is $Y_1 = 8\%$, and the gas-liquid equilibrium relationship is $Y = 2X$. Please determine the following:

- (1) If the liquid-gas ratio L/V is 2.5, what is the absorption efficiency?
- (2) If the liquid-gas ratio L/V is 1.5, what is the absorption efficiency?

Answers: 100%, 75%

8. A device uses air to cool water at 50 °C directly. It is known that $p_{\text{H}_2\text{O}} = 2.7$ kPa absolute pressure, the total pressure is 101.3 kPa (absolute pressure), and $K_G = 0.037$ kmol/(m²·h·kPa). Determine: K_Y

Answer: 8.9×10^{-4} kmol/(m²·s).

9. At 101.3 kPa (absolute pressure) and 25 °C, water absorbs B vapor in the air. Assume the phase equilibrium relationship conform to Henry's law and the solubility coefficient $H_{pc} = 2.11 \times 10^{-3}$ kmol/(m³·Pa). It is known that the gas phase mass transfer coefficient $k_G = 7.63 \times 10^{-5}$ kmol/(m²·h·Pa) and the liquid phase mass transfer coefficient $k_L = 0.09$ m/h. Please determine: the overall mass transfer coefficients K_G , K_L and the proportion of gas phase mass transfer resistance in total resistance.

Answer: $K_G = 5.44 \times 10^{-5}$ kmol/(m²·h·Pa), $K_L = 0.026$ m/h, 71.3%.

10. On a cross section of an ammonia absorption tower operated at 110 kPa, the gas containing 0.03 (mole fraction) of ammonia meets the water with an ammonia concentration of 1 kmol/m³. It is known that the gas film mass transfer coefficient $k_G = 5 \times 10^{-9}$ kmol/(m²·s·Pa), and the liquid film mass transfer coefficient $k_L = 1.5 \times 10^{-4}$ m/s. The equilibrium relationship between ammonia and water can be expressed by Henry's law. The solubility coefficient H_{pc} is 7.3×10^{-4} kmol/(m³·Pa). Please determine the following:

- (1) The two-phase composition at the gas-liquid interface;
- (2) Total driving force, overall mass transfer coefficient and total mass transfer flux expressed by partial pressure difference and molar concentration difference;
- (3) Gas film resistance, liquid film resistance and its ratio over total resistance.

Answer: (1) The composition of gas-liquid interface is $p_i = 1.45$ kPa, $c_i = 1.06$ kmol/m³.

(2) The total driving force Δp is 1.93 kPa, overall mass transfer coefficient K_G is 4.78×10^{-9} kmol/(m²·s·Pa) and total mass transfer flux is 9.22×10^{-6} kmol/(m²·s) when they are expressed by partial pressure difference. The total driving force Δc is 1.409 kmol/m³, overall mass transfer coefficient K_L is 6.55×10^{-6} m/s, total mass transfer flux is 9.26×10^{-6} kmol/(m²·s).

(3) Gas film resistance is $2 \times 10^8 \text{ (m}^2 \cdot \text{s} \cdot \text{Pa)/kmol}$. Liquid film resistance is $9.1 \times 10^6 \text{ (m}^2 \cdot \text{s} \cdot \text{Pa)/kmol}$. The ratio of gas film resistance over total resistance is 95.6%. The ratio of liquid film resistance over total resistance is 4.4%.

11. In a countercurrent packed absorption tower, a pure solvent is used to absorb the target organic pollutant in the gas mixture. If both the equilibrium line and the operating line are straight under the operating conditions and the ratio of the equilibrium line to the operating line is 0.8, the total number of gas phase mass transfer units (N_{OG}) is 10. Please try to find out the absorption efficiency η_A of the absorption tower.

Answer: 97%

12. Design a packed absorption tower for the absorption of acetone using water. The cross-sectional area of the tower is 1 m^2 , and the inlet flow rate of the gas mixture is 70 kmol/h , in which the composition of acetone Y_1 is 0.02. The absorption efficiency is required to be 90% using water without acetone. The operating pressure of the tower is 101.3 kPa and the temperature is 293 K . Under these conditions, the equilibrium relationship of acetone between the two phases is $Y = 1.18X$. Take the liquid-gas ratio to be 1.4 times the minimum liquid-gas ratio and the total gas phase volumetric mass transfer coefficient K_{Ya} is $2.2 \times 10^{-2} \text{ kmol/(s} \cdot \text{m}^3)$. Please use six methods to calculate the height of the required packing layer.

Answer: 4.41 m

13. In a packed tower with a height of 8 m , a pure absorbent is used to absorb the soluble components in a gas mixture. Under operating conditions, the phase equilibrium constant $m_e = 0.6$. When $L/V = 0.8$, the absorbate recovery efficiency can reach 85%. Now, another packings with better performance is used to increase the absorption efficiency to 95% under the same operating conditions. How many times is the volumetric mass transfer coefficient of this packed tower than the original one?

Answer: 1.98 times

14. Aromatics in coke oven gas are absorbed by wash oil. The temperature in the absorption tower is $27 \text{ }^\circ\text{C}$ and the pressure is 106.7 kPa . The flow rate of inert gas in coke oven gas is $850 \text{ m}^3/\text{h}$, the inlet gas contains 0.02 (molar ratio, the same below) of aromatics, and the recovery efficiency of aromatics is required to be higher than 95%. The aromatics in the wash oil entering the absorption tower is 0.005. The specific absorbent dosage (L/V) is 1.5 times of the theoretical minimum specific absorbent dosage $(L/V)_{\min}$. The absorption phase equilibrium relationship is: $Y^* = 0.125X$. The total gas phase mass transfer unit height is 0.875 m . Please try to find out the height equivalent to a theoretical plate *HETP* of this packed absorption tower by analytic method.

Answer: 1.05 m

15. An absorption tower with a certain height absorbs acetone from an acetone-air mixture with clear water counter-currently at 101.3 kPa , 293 K . The recovery efficiency of acetone can reach up to 95% at an operating liquid-gas ratio of 2.1. The equilibrium relationship of acetone between the two phases is known

to be $Y = 1.18X$. The absorption process is controlled by gas film and the overall volume mass transfer coefficient K_{Ya} is proportional to the 0.8 power of the gas flow rate.

- (1) If the gas flow rate increases by 20%, while the liquid flow rate and the composition of the intake gas and liquid keep unchanged, please calculate the following:
 - ① What is the recovery efficiency of acetone?
 - ② What is the increased amount of acetone absorbed per unit time?
- (2) If the gas flow rate, the composition of the intake gas and liquid, operating temperature and pressure of the absorption tower remain unchanged, to increase the recovery efficiency of acetone from 95% to 98%, the amount of absorbent should be increased to how many times the original amount?

Answers: (1) 92.3%, 16.6%, (2) 1.87 times

16. A desorption tower is used to treat water to reduce the content of CO_2 from 200 ppm to 5 ppm (1 ppm = $1/10^6$, in terms of mass). The operating temperature of the tower is 25°C and the total pressure is 0.1 MPa. The air fed to the bottom of the tower contains 0.1% CO_2 (volume percentage). The Henry's constant $H_{pm} = 1.64 \times 10^2$ MPa at the operating temperature. The spray density of water fed into the tower is $10,000 \text{ kg}/(\text{m}^2 \cdot \text{h})$. Under these conditions, K_{xa} is estimated to be $1111 \text{ kmol}/(\text{m}^3 \cdot \text{h})$. The amount of water treated per hour is 50 t, and the actual air flow rate used is 50 times the theoretical minimum. Please calculate the volume of air entering the tower per hour (at 25°C) and the height of the packing layer.

Answer: (2040 m^3/h , 2.05 m)

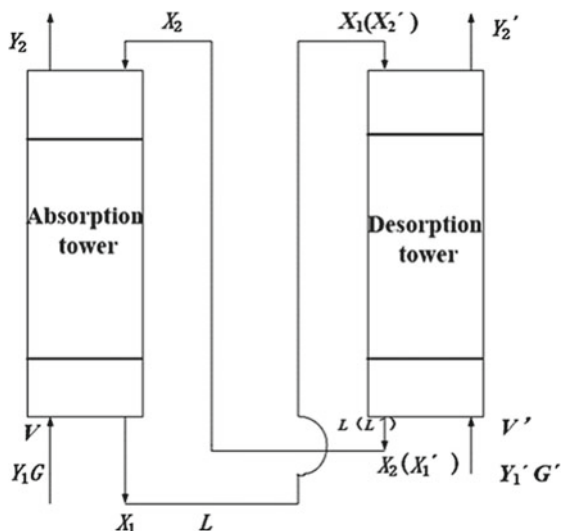
17. An absorption-desorption system is constructed in factory A, as shown in the Fig. 4.30. In actual operation, the amount of gas treated by the absorption system is 4000 kmol/h, the circulating absorbent is 600 kmol/h, and the desorption gas flow rate is 1000 kmol/h. The packing layer heights of both two systems are 15 m, and compositions are $y_1 = 0.012$, $y'_1 = 0$, $y'_2 = 0.04$, $x_2 = 0.0015$. In the absorption and desorption systems, the phase equilibrium relationship of the gas phase and liquid phase are $y = 0.15x$ and $y' = x'$. Assume that L , G , y_1 , y'_1 are constant. The heights of both the absorption and desorption towers do not change with the gas flow rate. Please calculate the following:

- (1) Gas composition at the outlet of the absorption tower and the mass transfer unit height of the desorption tower;
- (2) If the gas flow rate of desorption tower changes to 750 kmol/h, please determine the gas composition at the outlet of the absorption tower.

Answer: (1) $y_2 = 1.976 \times 10^{-3}$; $H'_{OG} = 3.48 \text{ m}$; (2) $y_2 = 2.657 \times 10^{-3}$.

18. Design a packed absorption tower to absorb gas A with water. The inner diameter of the tower is 1 m, the flow rate of gas containing A is 2000 m^3 per hour, the operating pressure is $1.013 \times 10^5 \text{ Pa}$, and the temperature is 293 K. The composition of the gas entering the tower is 6% (volume percentage of A), the absorption efficiency is 76%, and the specific absorbent dosage is 1.5

Fig. 4.30 Schematic diagram of question 17



times the minimum. Under these conditions, the equilibrium relationship of A between the gas and liquid phases is $Y_A^* = 0.13X_A/(1 + 0.87X_A)$. The total volume mass transfer coefficient of the gas phase K_{Ya} is $20 \text{ mol}/(\text{s} \cdot \text{m}^3)$. Please calculate the required packing layer height using three methods.

Answer: 1.91 m.

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Chapter 5

Chemical and Biological Reaction Kinetics and Reactors



Yan Liu, Liang Li, Dong An, and Chen-Xi Li

5.1 Chemical and Biological Reaction Kinetics

5.1.1 Classification of Reactions

1) Classification by chemical reaction characteristics

The chemical activity of a material can be described by a chemical reaction or by a combination of chemical reactions. There are many kinds of chemical reactions, and it is difficult to master them one by one in the learning process with the continuous expansion of human cognition. Generally, chemical reactions can be classified according to specific criteria based on their internal laws and people's cognitive habits. Common methods are as follows:

- (1) Chemical reactions can be divided into simple reactions and complex reactions according to the differences in reaction process and mechanisms. A simple reaction is a chemical reaction in which only one elementary reaction is involved, and the reactants go through one step to form the final products. However, complex reactions usually have complex process, including two or more elementary reactions. Reactants need to go through multiple steps to form the final products. According to the relationship between elementary steps, complex reactions can be subdivided into parallel reactions, consecutive reactions, simultaneous reactions and lumped reactions.
- (2) Chemical reactions are directional. There is a class of reactions that can be either forward or backward, which are known as reversible reactions. However, other reactions can only go in one direction, and the result is that the reactant becomes the product completely, which is called an irreversible reaction. It is

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363

important to note that irreversible reactions are relative and can be reversed when conditions change.

- (3) Chemical reactions can be divided into inorganic reactions, organic reactions and biochemical reactions according to the differences in the properties of the materials involved in the reactions.
- (4) Chemical reactions can be divided into zero-order reactions, first-order reactions, second-order reactions and multistage reactions according to the macrodynamic characteristics of the reactions. In addition, from a microscopic point of view, they can be divided into single-molecule reactions, bi-molecule reactions and three-molecule reactions.
- (5) Chemical reactions are generally accompanied by certain thermal effects, so all chemical reactions can be divided into exothermic reactions and endothermic reactions.

2) **Classification according to reaction process conditions**

- (1) Chemical reactions can be divided into gas phase reactions, liquid phase reactions, solid phase reactions and heterogeneous reactions according to the physical states of all the materials involved in the reaction (including reactants, products, catalysts, etc.).
- (2) According to temperature conditions, chemical reactions can be divided into isothermal reactions, adiabatic reactions and nonadiabatic variable temperature reactions.
- (3) According to pressure conditions, chemical reactions can be divided into atmospheric, pressurized and depressurized reactions.
- (4) According to reaction flow conditions, chemical reactions can be divided into ideal flow models (horizontal push flow, full mixed flow) and nonideal flow models.
- (5) According to the specific operational mode of a chemical reaction kettle, chemical reactions can be divided into three categories: intermittent reactions, continuous reactions and semi-intermittent reactions.

3) **Classification according to the characteristics of elements involved in chemical reactions**

- (1) According to the electron gain and loss in reactions, they can be divided into oxidation reactions and reduction reactions.
- (2) According to the characteristics of chemical particles in reactions, they can be divided into molecular reactions, ionic reactions and atomic reactions.

5.1.2 Reaction rate and Equation

Experimental results have shown that the factors that influence the chemical reaction rate are the intrinsic properties of reactants and the variable external factors.

Variable external factors mainly include concentration (reactant, product, catalyst concentration, etc.), temperature, pH, pressure, solvent characteristics, and ionic strength, etc..

1) Reaction rate

Chemical kinetics is the study of the process and rate of chemical reactions. The reaction process refers to the process by which reactants are transformed into products, that is, the reaction mechanism. In a chemical reaction, the amount of products increases gradually with the decrease in the total amount of reactants. The rate of chemical reaction can be expressed by the rate at which any reactant decreases or any product increases. For example, the volume of an aqueous solution is V , and the number of materials that reactant A reduces in $d\tau$ is dn_A . The reaction rate can be expressed as:

$$r_A = \frac{1}{V} \frac{dn_A}{d\tau} = \frac{\text{Variation in number of moles of material A}}{\text{unit volume} \times \text{time}} = \frac{dc_A}{d\tau} = \frac{\text{Changes in A concentrations}}{\text{unit time}} \quad (5.1)$$

where

r_A —Rate of reaction of A, $\text{mol}/(\text{m}^3 \cdot \text{s})$ or $\text{kg}/(\text{m}^3 \cdot \text{s})$

V —Reaction volume, m^3

τ —Reaction time, s

n_A —Moles or kilograms of reactant A, mol or kg

c_A —Concentration of reactant A, mol/m^3 or kg/m^3 .

Symbol selection for r_A :

When A represents the reactant, the concentration of the reactant decreases with time, and thus r_A takes a negative value. When A is the product, r_A takes a positive value. As shown in Fig. 5.1:

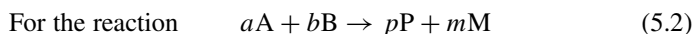
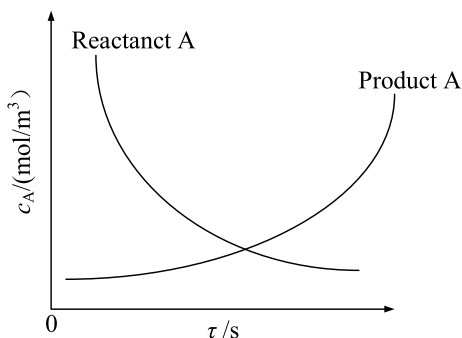


Fig. 5.1 Variety of reactants and products with time



Equation (5.2) represents the stoichiometric relationship between reactants A & B and products P & M. As shown above, the a mol reactant A molecule reacts with the b mol reactant B molecule to generate p mol product P molecule and m mol product M molecule, and the total mass of reactants is equal to that of products. Therefore, this equation is called a stoichiometric equation. Stoichiometric equations do not provide any information about how molecules A and B react to form product molecules P and M and how fast they react. These problems fall within the range of chemical dynamics.

In Eq. (5.2), the following relationship exists between the reaction rate of each component and stoichiometry: n_A , n_B , n_P and n_M are the molar quantities of materials A, B, P and M at time τ , respectively. Then, the following can be obtained from Eq. (5.2):

$$-\frac{1}{a} \frac{dn_A}{d\tau} = -\frac{1}{b} \frac{dn_B}{d\tau} = \frac{1}{p} \frac{dn_P}{d\tau} = \frac{1}{m} \frac{dn_M}{d\tau} \quad (5.3)$$

The above equation can be rewritten as

$$d\ddot{U} = -\frac{dn_A}{a} = -\frac{dn_B}{b} = \frac{dn_P}{p} = \frac{dn_M}{m} = \frac{dn_I}{i} \quad (5.4)$$

$d\ddot{U}$ is defined as the reaction progress, i is the stoichiometric equation coefficient of species I, and when component I is the reactant, i takes its negative value. When component I is the product, its positive value is taken as follows:

$$pP + mM - aA - bB = 0$$

Equation (5.5) can be obtained from Eqs. (5.3) and (5.4).

$$\frac{r_I}{i} = \frac{1}{i} \frac{dc_I}{d\tau} = \frac{1}{V} \frac{d\ddot{U}}{d\tau} \quad (5.5)$$

In the equation, r_I and c_I are the reaction rate and concentration of species I at time τ respectively, and V is the volume of the reaction solution. According to Eq. (5.4), when the initial mass of A is n_{A_0} , then the reaction progress is $\ddot{U} = (n_{A_0} - n_A)/a$. If η_A represents the conversion efficiency of A. $\eta_A = (n_{A_0} - n_A)/n_{A_0}$, Eq. (5.6) can be obtained:

$$\eta_A = \frac{a\ddot{U}}{n_{A_0}} \quad (5.6)$$

2) Reaction Order

The chemical experimental results show that the reaction rate of product M can be expressed as follows:

$$r_M = \frac{d[M]}{d\tau} = \frac{dc_M}{d\tau} = kc_A^a c_B^b \quad (5.7)$$

For reactant A, this is an a -order reaction. For the reactant B, this is a b -order reaction. The total reaction order is $(a + b)$. Typically, a and b are the order of reactions of reactants A and B, dimensionless. k is called the rate constant of the reaction in units of $[\text{concentration}]^{1-(a+b)}/\tau$, which has no relationship with the concentration of reactants or products but only relates to temperature, pressure, pH and other environmental factors. c_A is the concentration of A in mol/m^3 or kg/m^3 , and c_B is the concentration of B in mol/m^3 or kg/m^3 .

The reaction rate of product P can be expressed in another equation:

$$r_P = \frac{d[P]}{d\tau} = \frac{dc_P}{d\tau} = k'c_A^{a'} c_B^{b'} \quad (5.8)$$

Similarly, the reaction rates of reactants A and B can be expressed as:

$$-r_A = -\frac{d[A]}{d\tau} = -\frac{dc_A}{d\tau} = k''c_A^{a''} c_B^{b''} \quad (5.9)$$

$$-r_B = -\frac{d[B]}{d\tau} = -\frac{dc_B}{d\tau} = k'''c_A^{a'''} c_B^{b'''} \quad (5.10)$$

Equations (5.9) and (5.10) not only represent different reaction order and rate constants, respectively, but also introduce a negative sign on the left side because the reaction rates of reactants r_A and r_B are always negative. $-r_A$ and $-r_B$ are positive values, and the rate constants k'' and k''' are positive.

In addition, it is also worth noting that the reaction order also has the following characteristics:

- (1) The rate equation of both reactants and products is expressed in terms of the concentration of reactants or products.
- (2) The exponential numbers (a' and b' , a'' and b'' , a''' and b''') for the concentrations of reactants A and B (c_A and c_B) in Eqs. (5.8), (5.9) and (5.10) are not necessarily the same as the coefficients of stoichiometric Eqs. 5.2 (a and b) and may not be integers either.
- (3) If the rate equation cannot be expressed in the similar form of Eqs. (5.7) to (5.10), the concept of “reaction order” cannot be applied to the chemical reaction. Take the following reaction as an example:



Its rate equation is

$$r_{\text{HBr}} = \frac{k'[\text{H}_2][\text{Br}]^{1/2}}{1+k''[\text{HBr}]/[\text{Br}_2]} \quad (5.12)$$

If the kinetic behavior is considered at the beginning of the reaction, the following rate equation can be obtained:

$$r = k[\text{H}_2][\text{Br}_2]^{1/2} \quad (5.13)$$

As mentioned above, k is the kinetic constant of the reaction. If the concentration of reactants are controlled to $[\text{H}_2] \ll [\text{Br}_2]$, then only the effect of H_2 concentration can be observed in the kinetic experiment. Then, the rate equation is

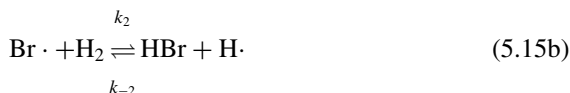
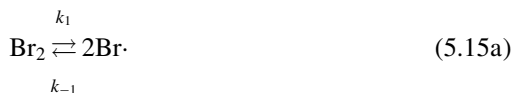
$$r = k_{\text{obs}}[\text{H}_2] \quad (5.14)$$

The k_{obs} is called the pseudo rate constant. The form of a reaction rate equation is directly related to the corresponding kinetic experimental conditions. The rate equation can only be applied within the corresponding experimental conditions; otherwise, incorrect results and inferences will occur.

Kinetic experimental results show that the reaction for HBr follows the reaction rate equation shown in Eq. (5.12). Since the limited form of Eq. (5.7) is not available, the concept of reaction order is not applicable; that is, the reaction does not have a simple reaction order. When the reaction has the reaction rate equation shown in Eq. (5.13), the reaction order is 1.5, where H_2 and Br_2 show reaction orders of 1 and 0.5, respectively. When the reaction has a reaction rate equation, as shown in Eq. (5.14), the reaction is defined to be a pseudo-first-order reaction or a kinetic first-order reaction. From an experimental point of view, the fitting results of the reaction order may be a positive integer, negative integer, zero or even a fraction.

3) Elementary reaction

In elementary reactions, reactants undergo a step reaction to directly generate products; that is, reactants (molecules, atoms, ions or free radicals) are directly converted to products by a collision (or chemical behavior), also known as a simple reaction. Under this condition, the exponent of the concentration corresponding to the kinetic equation is equal to its stoichiometric coefficient and its reaction order. In complex reactions, the initial reactant needs to go through a series of simple reaction steps before the final product can be generated. However, the stoichiometric equation gives the relationship between the initial reactant and the final product without considering the intermediate steps and products. Therefore, the reaction order is generally not equal to the coefficient of the stoichiometric equation. In general, all the reaction sequences constituting a stoichiometric equation can be called the reaction mechanism of the original reaction. Equation (5.11) can be explained as follows:



These elementary reactions constitute the mechanism of the reaction in Eq. (5.11). $\text{Br}\cdot$ and $\text{H}\cdot$ are called free radicals (atoms, ions, or molecules with unpaired electrons are called free radicals). Unpaired electrons make free radicals very active. As shown in Eq. (5.15a ~ c), the elementary reaction is actually composed of many reactions between H_2 & Br_2 and H & Br with identical chemical compositions and properties but different microcosmic states. From this perspective, the discussion of elementary reactions still belongs to the macroscopic category.

In Eq. (5.15a ~ c), k_1 , k_2 and k_3 are the rate constants of the forward reaction, while k_{-1} and k_{-2} are the rate constants of the reverse reaction. Therefore, this set of equations actually consists of five elementary reactions whose order is exactly the same as the stoichiometric coefficients. Using this characteristic, the reaction rate expression of each reaction species can be written directly. For example, the rate of formation for HBr is:

$$r_{\text{HBr}} = k_2[\text{Br}\cdot][\text{H}_2] - k_{-2}[\text{HBr}][\text{H}\cdot] + k_3[\text{H}\cdot][\text{Br}_2] \quad (5.16)$$

Equation (5.15a ~ c) forms a chain reaction. The reaction of k_1 is called the initiation step, while the reactions of k_2 and k_3 are called the transfer step. The $\text{Br}\cdot$ generated in reaction k_3 repeats the reaction of k_2 to produce $\text{H}\cdot$, which leads to reaction k_3 producing HBr . The reaction proceeds, and thus, it is called a chain reaction. $\text{Br}\cdot$ and $\text{H}\cdot$ are called chain transport materials. The reaction k_{-1} is called the termination step, while k_{-2} is called the suppression step.

5.1.3 Homogeneous Reaction Kinetics

This section mainly introduces the reaction kinetics equations and related influencing factors of a single reaction, parallel reaction, reversible reaction and consecutive reaction. Through learning these contents, the basic concepts of chemical reaction kinetics can be gradually understood.

5.1.3.1 Simple reaction

1) Zero-order reaction

The single component reaction that conforms to the kinetics of the zero-order reaction is the simplest elementary reaction. In a single component zero-order reaction $A \xrightarrow{k} P$, the initial concentration of reactant A is c_{A0} , where the kinetic constant of the zero-order reaction is k ; then, the concentration of reactant A (c_A) at time τ can be calculated according to the following equation. According to the definition of a zero-order reaction, it can be obtained that:

$$-\frac{dc_A}{d\tau} = kc_A^0 = k$$

The above equation can be integrated from time 0 to time τ to obtain:

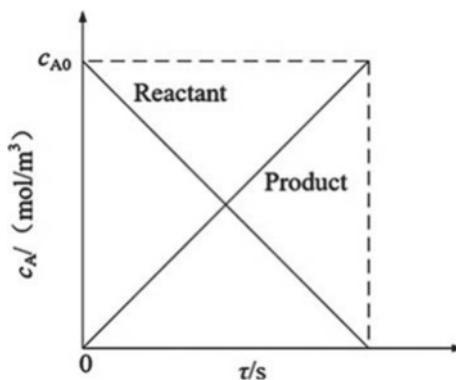
$$\int_{c_{A0}}^{c_A} -dc_A = \int_0^{\tau} k d\tau$$

Finally, the expression of c_A is shown in Eq. (5.17), whose zero-order reaction c_A - τ curve is shown in Fig. 5.2. When the time is c_{A0}/k , the concentration of reactant A reaches 0, and the reaction is finished.

$$c_A = c_{A0} - k\tau \quad (5.17)$$

If the units of concentration c_A and time τ are mol/L and s, respectively, then the unit for reaction kinetic constant k can be obtained from Eq. (5.17) in mol/(L·s). A typical characteristic of zero-order reactions is that the reaction rate is independent of the concentration of the reactants. In enzymatic reactions in the field of biochemistry, when the substrate concentration is high, the reaction rate has a low correlation with the substrate concentration, which can be regarded as a pseudo-zero-order reaction.

Fig. 5.2 Curve c_A - τ of zero-order reaction



2) First-order reaction

If the reaction of the above single component conforms to first-order reaction kinetics, c_{A0} and k are still used as the initial concentration and the first-order reaction kinetic constant, and then the concentration of reactant A (c_A) at time τ can be obtained by a similar method. The basic differential equation is written in terms of a first-order reaction, which can be integrated at time intervals $(0, \tau)$:

$$-\frac{dc_A}{d\tau} = kc_A$$

$$\int_{c_{A0}}^{c_A} \frac{dc_A}{c_A} = -\int_0^\tau k d\tau$$

$$\ln c_A - \ln c_{A0} = \ln \frac{c_A}{c_{A0}} = -k\tau \quad (5.18)$$

The following can be obtained from the above equation:

$$c_A = c_{A0} \exp(-k\tau) \quad (5.19)$$

By the reaction $A \rightarrow S$, $c_A + c_S = c_{A0}$, in which c_S is the concentration of S at time τ . From Eq. (5.19):

$$c_S = c_{A0} - c_{A0} \exp(-k\tau)$$

From Eqs. (5.18) and (5.19), the concentration of reactant A (c_A) and the concentration of product S (c_S) are both exponentially related to time (τ). As shown in Fig. 5.3, when $\tau \rightarrow \infty$, the concentration of reactant A (c_A) approaches zero, the concentration of product S (c_S) tends to be c_{A0} , and the reaction is almost complete.

Fig. 5.3 Concentration-time curve of first-order reaction

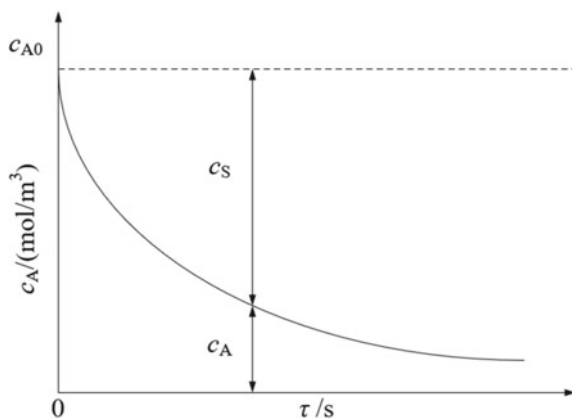
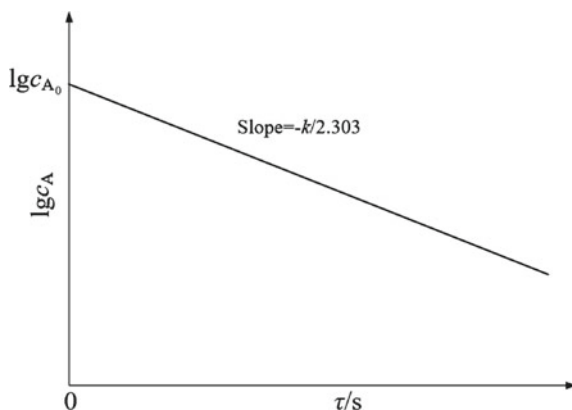


Fig. 5.4 Calculating the rate constant of the first-order reaction



Take the logarithm of both sides of Eq. (5.19) (base 10), which can be written as a linear form of Eq. (5.20). Taking $\lg c_A$ as the Y-axis and time τ as the X-axis, linear fitting using the least squares method can be used to obtain a line, as shown in Fig. 5.4. The first-order reaction kinetic constant k can be obtained from the slope of line.

$$\lg c_A = -\frac{k}{2.303}\tau + \lg c_{A_0} \quad (5.20)$$

The rate of first-order reactions is sometimes described by the concept of half-life. The half-life ($\tau_{1/2}$) is the time needed for the initial concentration of a material c_{A_0} to be reduced to $\frac{1}{2}c_{A_0}$, as shown in Eq. (5.21).

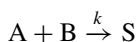
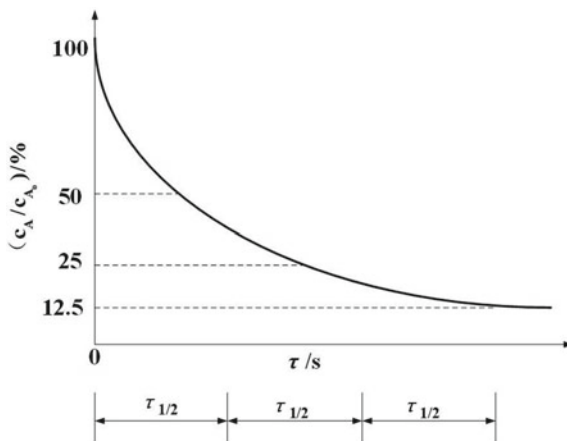
$$\begin{aligned} \frac{c_A}{c_{A_0}} &= e^{-k\tau_{1/2}} = \frac{1}{2} \\ \tau_{1/2} &= \frac{\ln 2}{k} = \frac{0.693}{k} \end{aligned} \quad (5.21)$$

The decay of radioactive elements is a first-order reaction, so the half-life is often used to indicate their decay rate.

From Eq. (5.21), it can be seen that the half-life of the first-order reaction is a constant independent of the initial concentration of the reactant, which can be represented by the concentration-time curve in Fig. 5.5. The time needed for material A to decrease from 100 to 50% of c_{A_0} is the same as the time needed for A to decrease from 50 to 25% of c_{A_0} , that is, $\tau_{1/2}$. Similarly, the time needed to reduce from 0.25 c_{A_0} to 0.125 c_{A_0} is also the half-life $\tau_{1/2}$.

3) Second-order reaction

If the following reaction involve two reactants A and B, and conforms to second-order reaction kinetics, the concentration of the product S (c_S) can be derived separately under the conditions of $c_{A_0} \neq c_{B_0}$ and $c_{A_0} = c_{B_0}$, respectively, in which c_{A_0} and c_{B_0} are the initial concentrations of A & B.

Fig. 5.5 Half-life of first-order reaction

When $c_{A_0} \neq c_{B_0}$, c_S represents the concentration of product S at time τ . As seen from the above equation, the molar ratio of reactants A and B and product S is 1:1:1. Therefore, when the concentration of product S increases from 0 to c_S , the concentrations of reactants A and B decrease from c_{A_0} and c_{B_0} to $(c_{A_0} - c_S)$ and $(c_{B_0} - c_S)$, respectively. According to the definition of a second-order reaction:

$$\frac{dc_S}{d\tau} = kc_A c_B = k(c_{A_0} - c_S)(c_{B_0} - c_S)$$

When integrated over the time interval $(0, \tau)$:

$$\int_0^{c_S} \frac{dc_S}{(c_{A_0} - c_S)(c_{B_0} - c_S)} = \int_0^\tau k d\tau$$

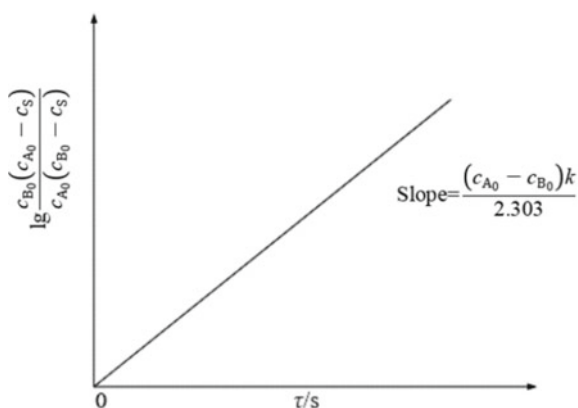
$$\frac{1}{c_{B_0} - c_{A_0}} \left(\ln \frac{c_{B_0} - c_S}{c_{A_0} - c_S} - \ln \frac{c_{B_0}}{c_{A_0}} \right) = k\tau$$

The above equation can be written as

$$\frac{2.303}{c_{A_0} - c_{B_0}} \lg \frac{c_{B_0}(c_{A_0} - c_S)}{c_{A_0}(c_{B_0} - c_S)} = k\tau \quad (5.22)$$

Taking $\lg \frac{c_{B_0}(c_{A_0} - c_S)}{c_{A_0}(c_{B_0} - c_S)}$ as axis y and time τ as axis x and using the least squares method for linear fitting, a line as shown in Fig. 5.6 can be obtained, the slope of which is the second-order reaction kinetic constant k .

Fig. 5.6 Calculating the rate constant of the second-order reaction



When the initial concentration $c_{A_0} = c_{B_0}$, the reaction rate equation is simplified to:

$$\frac{dc_s}{d\tau} = k(c_{A_0} - c_s)^2$$

When integrated over the same interval $(0, \tau)$:

$$\frac{c_s}{c_{A_0}(c_{A_0} - c_s)} = k\tau \quad (5.23)$$

Substituting $\frac{1}{2}c_{A_0}$ into Eq. (5.23), it can be concluded that when the initial concentration of the two reactants is equal, the half-life of the second-order reaction is

$$\tau_{1/2} = \frac{1}{kc_{A_0}} \quad (5.24)$$

The Eq. (5.24) shows that the half-life of the second-order reaction is inversely proportional to the initial concentration of the reactant. If $(\tau_{1/2})_{100}$ and $(\tau_{1/2})_{50}$ represent the half-life for $100\%c_{A_0}$ and $50\%c_{A_0}$, respectively, it can be obtained that:

$$(\tau_{1/2})_{100} : (\tau_{1/2})_{50} = \frac{1}{kc_{A_0}} : \frac{1}{k(0.5c_{A_0})} = 1 : 2$$

In other words, in the reaction process, the time required to reduce from $0.5c_{A_0}$ to $0.25c_{A_0}$ is twice the time required to reduce from c_{A_0} to $0.5c_{A_0}$. In a similar way, the time required to reduce from $0.25c_{A_0}$ to $0.125c_{A_0}$ is four times as long as the time required to reduce from c_{A_0} to $0.5c_{A_0}$.

4) The solution of reaction order

In the process of chemical reaction dynamics research, under the condition that the specific form of the kinetic equation is unknown, the general form can be written as $-\frac{dc}{d\tau} = kc_A^a c_B^b \dots$; then, the reaction order of each reactant can be found and the kinetic equation of the reaction can be determined finally. In this process, the concentration of reactants at different times is determined by experiments, and the reaction order is determined by mathematically fitting experimental data, which is a crucial step to establish the kinetic equation. The calculation methods of reaction order mainly include the differential method, half-life method and integral method.

(1) Graphical differentiation

The so-called differential method is a method used to determine the reaction order with the differential form of the rate equation.

Assuming a first-order reaction $A \rightarrow S$, the general form of the rate equation can be written as $r_A = kc^{n'}$, and the curve of the $c - \tau$ curve is measured experimentally, as shown in Fig. 5.7. Then, the slope of the tangent line at any point on the curve is the instantaneous rate corresponding to the concentration.

When the reactant concentration is c_1 , $r_1 = kc_1^{n'}$. When the reactant concentration is c_2 , $r_2 = kc_2^{n'}$. By taking logarithm of two equations:

$$\lg r_1 = \lg k + n' \lg c_1$$

$$\lg r_2 = \lg k + n' \lg c_2$$

Fig. 5.7 Relationship between reactant concentration and time

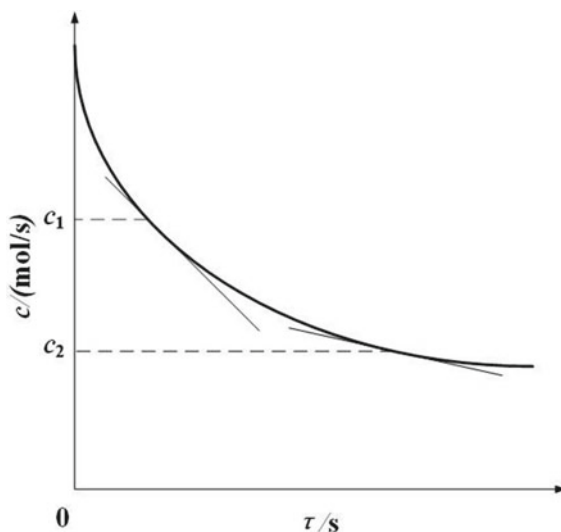
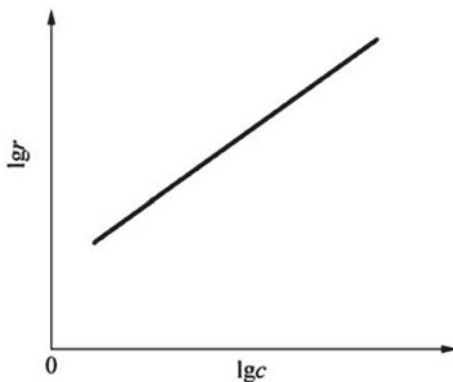


Fig. 5.8 The relationship between $\lg r$ and $\lg c$



$$n' = \frac{\lg r_1 - \lg r_2}{\lg c_1 - \lg c_2}$$

By determining the reaction rate for any two concentrations on the curve, the corresponding reaction order n' will be obtained.

The logarithm of the general equation of the rate equation is conducted to obtain its linear form $\lg r = \lg k + n' \lg c$. Plot $\lg r$ with $\lg c$ (see Fig. 5.8), and the slope of the line is n' .

This method needs the determination of the slope at different times and obtains the reaction rate at the corresponding moment from the $c - \tau$ graph. Each data point on the line corresponds to a different time, so the reaction order is called the order based on time.

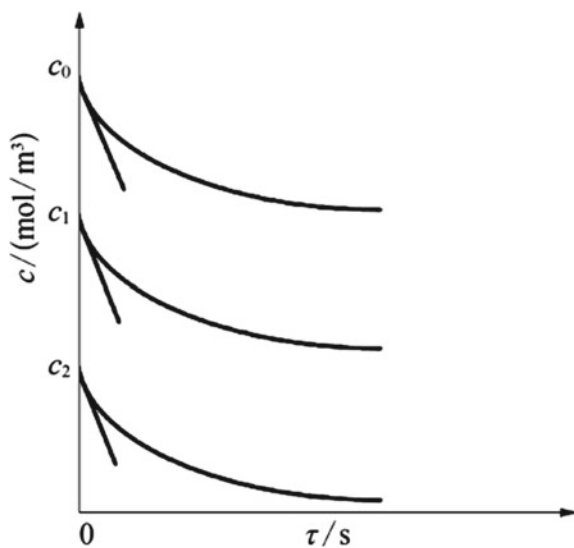
(2) Differentiation

This method sets the concentration of various starting reactants and measures the corresponding initial rate at different starting concentrations, i.e., the slope of the tangent line of the curves at $\tau = 0$ shown in Fig. 5.9, and then plots $\lg r_0$ with $\lg c$ to get a straight line. The order of reaction n' can be obtained from the slope, which is called the order based on the concentration. The initial rate is not affected by the product and other factors, which is equivalent to the order without interference factors, so the reaction order obtained by this method is relatively reliable.

If two or more materials are involved in a chemical reaction and the initial concentration of each reactant is different, the rate equation can be written as a general equation: $r = k c_A^a c_B^b$.

At this time, the order of reactions can also be obtained by the differential method, and the reaction order of a and b for different reactants can be obtained. In the specific experimental process, excessive addition of B and other reactants with only changed the initial concentration of reactant A, the instantaneous rate corresponding to the different initial concentrations of A was obtained. Another condition is that all reactants keep the same concentration except for changing initial concentration of reactant A corresponding rate also can be got.

Fig. 5.9 Curve of reactant concentration with time



$$r = k' c_A^a$$

Thus, $k' = k c_B^b$, $\lg r = \lg k' + a \lg c_A$, and then a can be determined.

Then, by excessive adding other reactants except B or keeping the other reactants at the same concentration, but only changing the initial concentration of the reactant B, the instantaneous rates will be obtained corresponding to different initial concentrations of reactant B.

$$r' = k'' c_B^b$$

$$k'' = k c_A^a$$

$$\lg r' = \lg k'' + b \lg c_B$$

Then, b will be determined.

The whole reaction order is $n' = a + b + \dots$

(3) Half-life

For different order reactions, the relation between the half-life and initial concentration is:

$$0 \text{ order} : \tau_{1/2} = \frac{c_{A0}}{2k_0}$$

$$1 \text{ order} : \tau_{1/2} = \frac{\ln 2}{k_1}$$

$$2 \text{ order} : \tau_{1/2} = \frac{1}{k_2 c_{A_0}}$$

$$3 \text{ order} : \tau_{1/2} = \frac{3}{2k_3 c_{A_0}^2}$$

$$n' \text{ order} : \tau_{1/2} = \overset{\vee}{G} \frac{1}{c_{A_0}^{n'-1}}$$

Two different initial concentrations of c_{A_0} and c'_{A_0} are set for simultaneous experiments with the reactant A.

$$\frac{\tau_{1/2}}{\tau'_{1/2}} = \left(\frac{c'_{A_0}}{c_{A_0}} \right)^{n'-1}$$

$$\lg \frac{\tau_{1/2}}{\tau'_{1/2}} = (n' - 1) \lg \frac{c'_{A_0}}{c_{A_0}}$$

$$n' = 1 + \frac{\lg \frac{\tau_{1/2}}{\tau'_{1/2}}}{\lg \frac{c'_{A_0}}{c_{A_0}}}$$

n' can be determined from two sets of data. More sets of data are considered and then a line is plotted according to $\lg \tau^{1/2} = (1 - n') \lg c_{A_0} + \lg \overset{\vee}{G}$; the slope of this line will be $1 - n'$.

Example 5.1 For the reaction of 1,2-dichloropropanol with sodium hydroxide to form epichlorohydrin, the relationship between $\tau_{1/2}$ and c_{A_0} for 1,2-dichloropropanol is measured experimentally as follows:

The serial number	Reaction temperature/K	The initial concentration of reactants/(mol/L) 1, 2-dichloropropanol, NaOH	$\tau_{1/2}$
1	303.2	0.475, 0.475	4.80
2	303.3	0.166, 0.166	12.9

Determine the order of this reaction.

Solution:

$$n' = 1 + \frac{\lg \frac{\tau_{1/2}}{\tau'_{1/2}}}{\lg \frac{c'_{A_0}}{c_{A_0}}} = 1 + \frac{\lg 4.80 - \lg 12.90}{\lg 0.166 - \lg 0.475} = 1.94 \approx 2$$

Thus, it is a second-order reaction.

(4) Integration

The integral method is also called the trial method. After a series of corresponding concentration-time kinetic data are measured experimentally, the following two attempts can be generally made:

- ① The concentration-time values of each group are substituted into the definite integral equation of the rate with a simple series reaction, and then the k value is calculated. If the calculated k value is basically a constant, the reaction is the order of equations substituted. If k is not constant, the assumption must be made again.
- ② Plot the following values:

$$\ln c_A \sim \tau$$

$$\text{or } \frac{1}{c_A} \sim \tau$$

$$\text{or } \frac{1}{c_A^2} \sim \tau$$

If all data points can be fitted to a straight line, the reaction is the corresponding order. The integral method is suitable for reactions with simple order and is not suitable for serious deviations from integers.

5.1.3.2 Parallel Reaction

When reactant A reacts to form product S, the rate constant is k_1 , and it can also form another product P with a rate constant of k_2 . Such a reaction is called a parallel reaction or a competitive reaction. Such reactions can be expressed by the following equation:



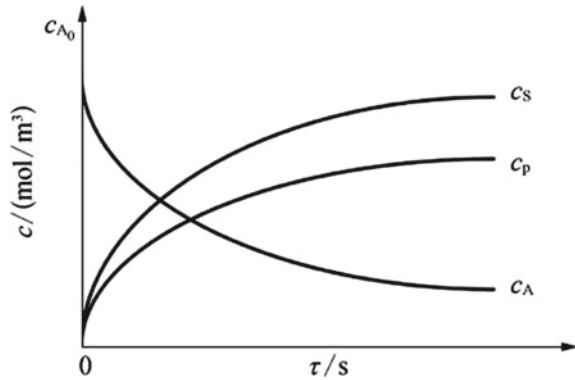
Assuming the initial concentrations of A, S, and P are c_{A0} , c_{S0} and c_{P0} , respectively. The reaction rates of c_A , c_S and c_P can be expressed as:

$$-\frac{dc_A}{d\tau} = k_1 c_A + k_2 c_A = (k_1 + k_2) c_A \quad (5.25)$$

$$\frac{dc_S}{d\tau} = k_1 c_A \quad (5.26)$$

Fig. 5.10

Concentration-time curve of first-order parallel reaction



$$\frac{dc_p}{d\tau} = k_2 c_A \quad (5.27)$$

By integrating Eq. (5.25):

$$\int_{c_{A0}}^{c_A} \frac{dc_A}{c_A} = - \int_0^{\tau} (k_1 + k_2) d\tau$$

The following can be obtained:

$$c_A = c_{A0} e^{-(k_1 + k_2)\tau} \quad (5.28)$$

In addition,

$$c_s = \frac{k_1 c_{A0}}{k_1 + k_2} [1 - e^{-(k_1 + k_2)\tau}] \quad (5.29)$$

$$c_p = \frac{k_2 c_{A0}}{k_1 + k_2} [1 - e^{-(k_1 + k_2)\tau}] \quad (5.30)$$

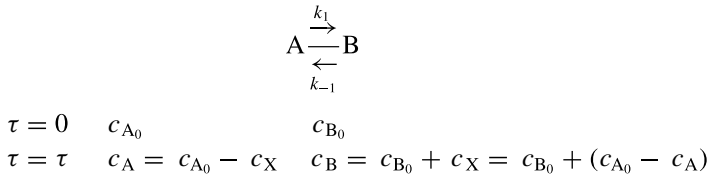
The variety of the concentration of each component with time in the first-order parallel reaction is shown in Fig. 5.10.

5.1.3.3 Reversible Reaction

The reversible reaction is generally expressed as:



k_{-1} is often used to represent the reverse reaction rate constant as follows:



If the initial concentrations of A and B are c_{A_0} and c_{B_0} , the concentrations of A and B (c_A and c_B) at time τ can be derived by the following methods.

At time τ , assuming c_X is the molar concentration of B generated through the chemical reaction of reactant A, the following equation can be written:

$$-\frac{d(c_{A_0} - c_X)}{d\tau} = k_1(c_{A_0} - c_X) - k_{-1}(c_{B_0} + c_X)$$

This can be simplified and written as an integral:

$$\begin{aligned} \frac{dc_X}{d\tau} &= k_1(c_{A_0} - c_X) - k_{-1}(c_{B_0} + c_X) \\ \int_0^{c_X} \frac{dc_X}{(k_1 + k_{-1})c_X - (k_1c_{A_0} - k_{-1}c_{B_0})} &= \int_0^\tau -d\tau \end{aligned}$$

Which is integrated to obtain

$$\frac{1}{k_1 + k_{-1}} \ln \frac{(k_1 + k_{-1})c_X - (k_1c_{A_0} - k_{-1}c_{B_0})}{-(k_1c_{A_0} - k_{-1}c_{B_0})} = -\tau$$

The above equation can be written as

$$\ln \left(\frac{N}{N - c_X} \right) = (k_1 + k_{-1})\tau$$

In the equation,

$$N = \frac{k_1c_{A_0} - k_{-1}c_{B_0}}{k_1 + k_{-1}} \quad (5.31)$$

Thus,

$$\frac{N}{N - c_X} = e^{(k_1 + k_{-1})\tau}$$

namely,

$$\begin{aligned}
c_X &= N[1 - e^{-(k_1+k_{-1})\tau}] \\
c_A &= \frac{k_{-1}(c_{A_0} + c_{B_0})}{k_1 + k_{-1}} \\
&\quad + \frac{(k_1c_{A_0} - k_{-1}c_{B_0})}{k_1 + k_{-1}} e^{-(k_1+k_{-1})\tau} \\
c_B &= \frac{k_1(c_{A_0} + c_{B_0})}{k_1 + k_{-1}} \\
&\quad - \frac{(k_1c_{A_0} - k_{-1}c_{B_0})}{k_1 + k_{-1}} e^{-(k_1+k_{-1})\tau}
\end{aligned} \tag{5.32}$$

When $k_{-1} \rightarrow 0$, that is, when the reverse reaction does not exist or can be ignored, $N = c_{A_0}$, and Eq. (5.32) becomes

$$c_X = c_{A_0}(1 - e^{-k_1\tau})$$

Therefore, the concentrations of A & B are:

$$\begin{aligned}
c_A &= c_{A_0} - c_X = c_{A_0}e^{-k_1\tau} \\
c_B &= c_{B_0} + c_X = c_{B_0} + c_{A_0}(1 - e^{-k_1\tau})
\end{aligned}$$

5.1.3.4 Consecutive Reaction

There is a class of chemical reactions that are completed by successive related steps. Some or all of the products of the previous step are used as part or all of the reactants of the next step. Such reactions are called consecutive reactions.

The mathematical processing of consecutive reaction kinetics is relatively complex, and the following only describes the dynamic derivation process of consecutive reactions composed of two one-way first-order reactions.



$$\begin{array}{cccc}
\tau = 0 & c_{A_0} & 0 & 0 \\
\tau = \tau & c_A & c_B & c_C
\end{array}$$

$$c_A + c_B + c_C = c_{A_0}$$

For component A, the reaction rate is:

$$-\frac{dc_A}{d\tau} = k_1c_A$$

If integrated on both sides:

$$\int_{c_{A_0}}^{c_A} -\frac{dc_A}{c_A} = \int_0^\tau k_1 d\tau$$

Then,

$$c_A = c_{A_0} e^{-k_1 \tau}$$

For component B, the reaction rate is

$$\frac{dc_B}{d\tau} = k_1 c_A - k_2 c_B = k_1 c_{A_0} e^{-k_1 \tau} - k_2 c_B$$

Solve this linear differential equation,

$$c_B = \frac{k_1 c_{A_0}}{k_2 - k_1} (e^{-k_1 \tau} - e^{-k_2 \tau})$$

For component C, the reaction rate is

$$\frac{dc_C}{d\tau} = k_2 c_B$$

Thus,

$$c_C = c_{A_0} - c_A - c_B = c_{A_0} \left(1 - \frac{k_2}{k_2 - k_1} e^{-k_1 \tau} + \frac{k_1}{k_2 - k_1} e^{-k_2 \tau} \right)$$

Due to the complexity of the mathematical processing of consecutive reaction kinetics, it can be approximately solved. When the rate of one of the reactions is very slow, the rate is approximately taken as the rate of the whole reaction, and the slow step is called the rate control step of the consecutive reaction.

(1) When $k_1 \gg k_2$, the second step is the rate control step, then:

$$c_C = c_{A_0} (1 - e^{-k_2 \tau})$$

(2) When $k_2 \gg k_1$, the first step is the rate control step, then:

$$c_C = c_{A_0} (1 - e^{-k_1 \tau})$$

The intermediate product of the consecutive reaction is both the product of the previous step reaction and the reactant of the latter step reaction, so its concentration has a process of increasing first and then decreasing, and there will be a maximum value at some point in the reaction process. The time of occurrence of this value and its value depend on the relative size of the two rate coefficients, as shown in Fig. 5.11. The intermediate concentration, c_B , reaches its maximum value at the specific time τ_m and its first derivative is zero. The τ_m can be calculated by the following method.

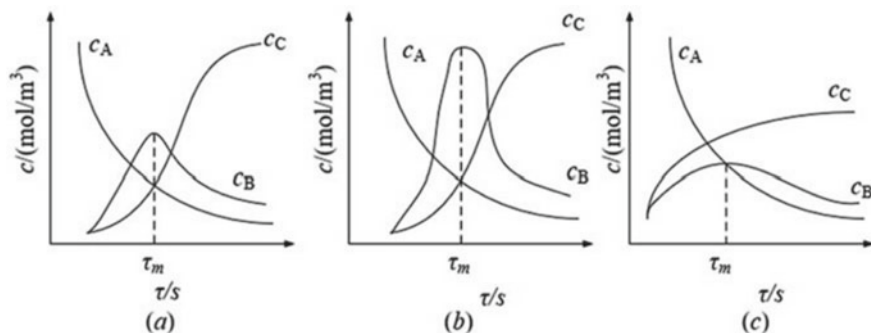


Fig. 5.11 Variety of intermediates under different conditions

$$c_B = \frac{k_1 c_{A_0}}{k_2 - k_1} (e^{-k_1 \tau} - e^{-k_2 \tau})$$

$$c_{Bm} = \frac{k_1 c_{A_0}}{k_2 - k_1} (e^{-k_1 \tau_m} - e^{-k_2 \tau_m})$$

$$\frac{dc_B}{d\tau} = \frac{k_1 c_{A_0}}{k_2 - k_1} (k_2 e^{-k_2 \tau} - k_1 e^{-k_1 \tau}) = 0$$

Because $c_{A_0} \neq 0$, $k_1 \neq 0$, so $k_2 e^{-k_2 \tau} - k_1 e^{-k_1 \tau} = 0$, and $\tau = \tau_m$ at this time, the solution is:

$$\tau_m = \frac{\ln k_2 - \ln k_1}{k_2 - k_1}$$

5.1.3.5 Influence of Temperature on the Reaction Rate Constant and Reaction Rate

The essence of the effect of temperature on the reaction rate is that temperature has an effect on the reaction rate constant; that is, temperature changes the reaction rate constant and then affects the reaction rate.

1) Van't Hoff approximation rule

A large number of experimental data show that for every 10 K increase in temperature, the reaction rate constant increases by approximately 2 ~ 4 times, resulting in a 2 ~ 4 times increase in the reaction rate. This empirical rule can be used to estimate the effect of temperature on the reaction rate constant. This empirical rule was first summarized by the Dutch chemist van't Hoff, so it is generally called the van't Hoff approximation rule.

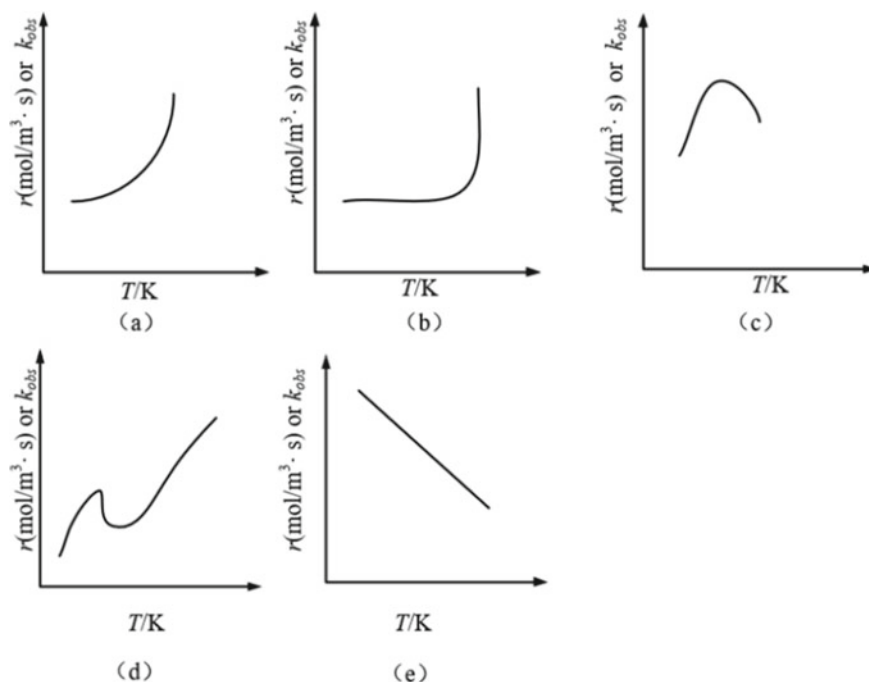


Fig. 5.12 The curves of the effect of temperature on the reaction rate constants

For example, a chemical reaction is known to take 10 min to complete at 390 K. If the reaction temperature is lowered to 290 K, how long will the reaction take to complete?

Solution: For every 10 K increase, the lower limit of the rate increase is 2 times.

$$\frac{k(390 \text{ K})}{k(290 \text{ K})} = \frac{\tau(290 \text{ K})}{\tau(390 \text{ K})} = 2^{10} = 1024$$

$$\tau(290 \text{ K}) = 1024 \times 10 \text{ min} \approx 7\text{d}$$

2) Five types of influence of temperature on the reaction rate constant and reaction rate

- (1) With increasing temperature, the reaction rate or pseudo rate constant gradually increases, and the reaction rate is exponentially correlated with temperature, which is the most common type of reaction, as shown in Fig. 5.12a.
- (2) At the beginning, the temperature has little influence. When the temperature exceeds a certain limit, the reaction rate or pseudo rate constant suddenly increases, and the reaction is conducted rapidly in the form of an explosion, as shown in Fig. 5.12b.

- (3) At a low temperature, the rate or pseudo rate constant increases with temperature, and when the temperature exceeds a certain limit, the reaction rate significantly decreases, such as in multiphase catalytic reactions and enzymatic catalytic reactions, as shown in Fig. 5.12c.
- (4) With increasing temperature, the rate increases to a certain value and then decreases, and then increases rapidly. One or more side reactions may occur, as shown in Fig. 5.12d.
- (5) The reaction rate or pseudo reaction rate constant decreases with increasing temperature. This type is rare, such as when nitric oxide is oxidized to nitrogen dioxide, as shown in Fig. 5.12e.

3) Arrhenius equation

(1) Exponential equation

$$k = A' \exp\left(-\frac{E_a}{RT}\right)$$

The equation describes the exponential correlation between the reaction rate constant and temperature.

where

A' —Pre-exponential factor, a constant independent of temperature

k —Rate constant

E_a —Arrhenius activation energy, a constant independent of temperature, J/mol

R —Molar gas constant, 8.314 J/(mol·K)

T —Temperature, K.

(2) Logarithmic equation

$$\ln k = -\frac{E_a}{RT} + B$$

The logarithmic equation describes the linear relationship between the logarithm of the kinetic constant and the reciprocal temperature. The activation energy E_a can be obtained by plotting $1/T$ with $\ln k$ according to k values measured at different temperatures.

(3) Definite integral equality

$$\ln \frac{k_2}{k_1} = \frac{E_a}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

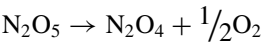
The definite integral equality assumes that the activation energy is independent of the temperature and can be calculated according to the k value at two different temperatures.

(4) Differential equation

$$\frac{d\ln k}{dT} = \frac{E_a}{RT^2}$$

The rate of change of k with respect to T in the differential equation is determined by the value of E_a .

Example 5.2 Take the decomposition reaction of N_2O_5 in tetrachloromethane liquid as an example to explain the relationship between k and T and find the activation energy E_a .



The rate constants measured by experiments at different temperatures are listed in Table 5.1.

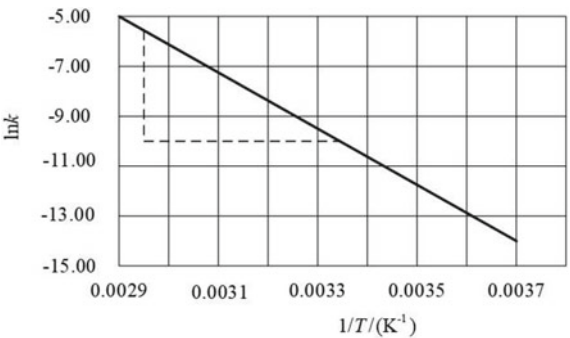
Solution:
The data in Table 5.1 are plotted, with $\ln k$ as the ordinate and $1/T$ as the abscissa, and a line can be obtained, as shown in Fig. 5.13. To find two points on the line, take two points in the ordinate spacing as $\Delta \ln k$, in the abscissa spacing as $\Delta \frac{1}{T}$, and slope $\alpha = \Delta \ln k / \Delta \frac{1}{T}$. For example, two points at 338 K and 298 K are exactly on the line.

Table 5.1 The decomposition rate constant of N_2O_5 at different temperatures

Temperature T/K	$\frac{1}{T}/(\text{K}^{-1})$	$k/(\text{s}^{-1})$	$\ln k$
338	0.002959	487×10^{-5}	-5.325
328	0.003049	150×10^{-5}	-6.502
318	0.003145	49.8×10^{-5}	-7.605
308	0.003247	13.5×10^{-5}	-8.910
298	0.003356	3.46×10^{-5}	-10.272
273	0.003663	0.0787×10^{-5}	-14.055

* The decomposition reaction of N_2O_5 is a first-order reaction, and the unit of the rate constant k is s^{-1}

Fig. 5.13 Diagram of decomposition reaction



According to the data of these two points in Table 5.1, the following equation can be obtained.

$$\alpha = \frac{\Delta \ln k}{\Delta \frac{1}{T}} = \frac{(-10.272) - (-5.325)}{0.003356 - 0.002959} = \frac{-4.947}{0.000397} = -12460$$

Substituting $\alpha = -12,460$, $T = 298$ K, and $\ln k = -10.272$ into the linear equation $\ln k = \alpha/T + B$ yields

$$\begin{aligned} -10.272 &= -12460/298 + B \\ B &= -10.272 + 41.81 = 31.54 \end{aligned}$$

Because

$$\alpha = \frac{-E_{\alpha}}{R}$$

thus,

$$\begin{aligned} E_{\alpha} &= -R \times \alpha = (-8.314) \times (-12460) \\ &= 103600 \text{ J/mol} = 103.6 \text{ kJ/mol} \end{aligned}$$

$$\ln k = -12460 \frac{1}{T} + 31.54$$

Answer: The relationship between $\ln k$ and $1/T$ is shown in Fig. 5.13. The mathematical expression is $\ln k = -12460 \frac{1}{T} + 31.54$. The activation energy of the N_2O_5 decomposition reaction in tetrachloromethane liquid is 103.6 kJ/mol.

5.1.4 Biochemical Reaction Kinetics

5.1.4.1 Kinetics of Enzyme-Catalyzed Reaction

The kinetics of enzyme-catalyzed reactions are a science that systematically studies the reactions and their influencing factors. These factors mainly include enzyme concentrations, substrate concentrations, pH, temperature, inhibitors and activators. The results of kinetics of enzyme-catalyzed reaction are helpful for optimizing the enzymatic reaction conditions and improving the reaction efficiency. They are also helpful for elucidating the correlation between the structure and function of enzymes and providing basic data for the study of the microaction mechanism of enzymes. They also aid in understanding the role of enzymes in metabolism. In environmental engineering, both the activated sludge and biofilm treatment technologies are used to convert organic pollutants into CO_2 , N_2 , CH_4 , and H_2 under the action of enzymes produced by microorganisms.

1) Concept and characteristics of enzymes

Enzymes are special organic compounds synthesized by living cells that can catalyze specific reactions inside or outside cells. An enzyme is a kind of biocatalyst with high selectivity and catalytic activity. Except for a few ribonucleic acids (RNA), the essence of enzymes are proteins, which have the physical and chemical characteristics of general proteins. However, enzymes are not common proteins, they are the proteins with catalytic activity. Compared with common chemical catalysts, enzymes have the following characteristics:

(1) High catalytic efficiency

A large number of experimental data have shown that the catalytic efficiency of enzymes is $10^7 \sim 10^{13}$ times higher than that of common catalysts. Similar to general catalysts, enzymes can accelerate the rate of chemical reactions but do not change the equilibrium point of chemical reactions; that is, the reaction equilibrium constant remains unchanged. Enzymes promote both forward and reverse reaction rates at the same proportion in the catalytic process, which can shorten the time required to achieve equilibrium from a few hours to a few seconds. The accelerated reaction mechanism is that enzymes reduce the activation energy required for chemical reactions.

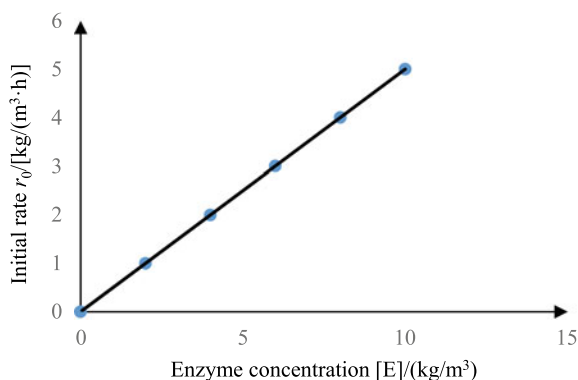
(2) Catalytic specificity

Unlike common catalysts, enzymes are highly selective to substrates, and their catalytic reactions are highly specific. Except for few spontaneous reactions, most reactions in organisms are catalyzed by specific enzymes. An enzyme can find a specific substrate from thousands of reactants. This is the specificity of an enzyme, which can be classified as absolute specificity, relative specificity and stereospecificity, depending on the specificity of enzyme catalysis. If an enzyme can only catalyze a specific substrate, it can be considered as having absolute catalytic specificity. For example, urease can only catalyze the hydrolysis of urea to decompose it into carbon dioxide and ammonia. If an enzyme can catalyze the reaction of a class of compounds or a class of chemical bonds, it can be considered as having relative catalytic specificity. For example, esterase can catalyze the hydrolysis of triglycerides and other ester bonds. If an enzyme has strict requirements for the three-dimensional configuration of substrate molecules, it can be categorized as having stereoisomeric catalytic specificity. For example, *L*-lactate dehydrogenase only catalyzes *L*-lactate dehydrogenation and has no effect on *D*-lactate.

(3) Catalytic activity can be adjusted

The catalytic activity of enzymes is directly related to a variety of factors. For example, some enzymes can be reversibly bound to some chemical groups through covalent bonds to regulate their catalytic activities. This process is called the covalent modification of enzymes. Inducers or inhibitors can change the synthesis and decomposition rate of enzymes, regulate the content of enzymes in cells, and thus change the catalytic reaction rate. In addition, there are some other regulatory pathways of enzyme activity, such as allosteric agents which can regulate the catalytic activity of allosteric enzymes, as well as hormones which can regulate enzyme activity through a second messenger.

Fig. 5.14 Effect of enzyme concentration on the initial rate of reaction



2) Kinetics of enzyme-catalyzed reaction

To study the effect of a certain factor on the enzymatic catalytic reaction rate, it is necessary to keep other related factors of the system constant and maintain strict initial reaction rate conditions. For example, when studying the correlation between catalytic reaction rate and enzyme concentration, the substrates used in the system must be sufficient to saturate all enzymes, the products generated are not sufficient to affect the catalytic efficiency of enzymes, and other relevant conditions of the reaction system (such as pH and temperature) have not changed significantly. The effects of enzyme concentration, substrate concentration, temperature, pH and other factors on the enzyme catalytic reaction rate will be discussed.

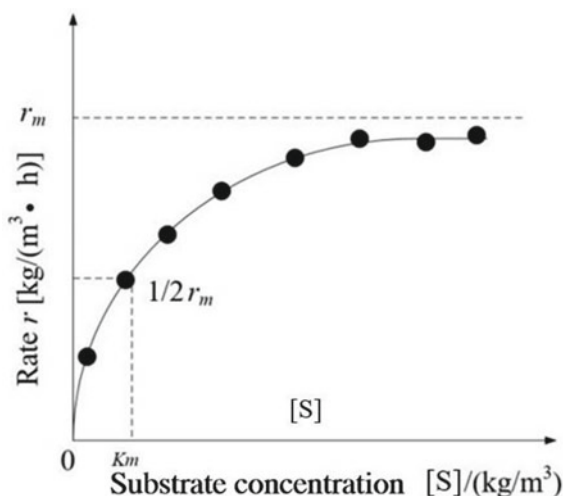
(1) Enzyme concentration

As shown in Fig. 5.14, when the temperature and pH are constant, all enzymes are in a saturated state if the concentration of a specific substrate far exceeds the enzyme concentration. The initial rate of the enzymatic catalytic reaction is proportional to the enzyme concentration $[E]$ in the system, and this method can be used to determine the activity of a specific enzyme.

(2) Substrate concentration

Figure 5.15 shows the relationship between substrate concentration and the initial rate of enzymatic catalytic reaction under the condition of constant enzyme concentration. When the substrate concentration is low, the reaction rate increases sharply with substrate concentration and presents a positive correlation. However, when the substrate reaches a certain limit, the reaction rate tends to be constant and basically does not change with increasing substrate concentration. At this point, the limit is called the saturation concentration of the substrate. When the substrate concentration is equal to or greater than the saturation concentration, all the active centers of the catalyst enzyme are occupied by the substrate, and each enzyme molecule has given full play to its catalytic capacity. Therefore, the catalytic reaction rate is limited by the enzyme concentration and no longer increases with substrate concentration.

Fig. 5.15 Effect of substrate concentration on the initial rate of reaction



① Michaelis–Menten equation

The most reasonable theory to explain the relationship between substrate concentration and reaction rate in enzymatic catalysis is intermediate product theory. Enzymes first bind to substrates to form complexes of enzymes and substrates, which are then decomposed into products and free enzymes. This procedure can be expressed as follows

$$r = \frac{r_m [S]}{K_m + [S]} \quad (5.33)$$

where

r —Rate of enzyme catalyzed reaction, $\text{kg}/(\text{m}^3 \cdot \text{h})$

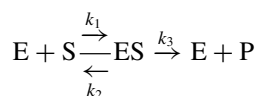
r_m —Maximum reaction rate, $\text{kg}/(\text{m}^3 \cdot \text{h})$

$[S]$ —Substrate concentration, kg/m^3

K_m —Michaelis constant, kg/m^3 .

This equation is a basic equation for studying the kinetics of enzyme reactions, and it conforms to the experimental curve obtained in Fig. 5.15.

② Derivation of Michaelis–Menten equation



where E, S, ES and P represent the free enzyme, substrate, enzyme-substrate complex and reaction product, respectively. k_1 is the reaction rate constant for the generation reaction of ES, and k_2 and k_3 represent the reaction rate constants of the decomposition reaction of ES into $\text{E} + \text{S}$ and $\text{E} + \text{P}$.

$$\text{Rate of formation of ES} = k_1[E][S]$$

$$\text{Rate of decomposition of ES} = (k_2 + k_3)[ES]$$

When the reaction reaches equilibrium,

$$\frac{[E][S]}{[ES]} = \frac{k_2 + k_3}{k_1} \quad (5.34)$$

By assuming $\frac{k_2 + k_3}{k_1} = K_m$, then

$$\frac{[E][S]}{[ES]} = K_m \quad (5.35)$$

Assuming that the total concentration of the enzyme is $[E_0]$, then

$$[E] = [E_0] - [ES] \quad (5.36)$$

Substitute Eq. (5.36) into Eq. (5.35) and then transfer the term to obtain

$$[ES] = \frac{[E_0][S]}{K_m + [S]} \quad (5.37)$$

Because the enzyme-catalyzed reaction rate is determined by the effective enzyme concentration, namely, the concentration of the intermediate ES, thus

$$r = k_3[ES] \quad (5.38)$$

Substituting Eq. (5.38) into Eq. (5.37) and transferring the term obtains

$$r = \frac{k_3[E_0][S]}{K_m + [S]} \quad (5.39)$$

If the substrate concentration in the reaction system is so high that the enzyme is completely saturated, i.e., $[E_0] = [ES]$, the maximum reaction rate r_m is reached, so

$$r_m = k_3[E_0] \quad (5.40)$$

Substitute Eq. (5.40) into Eq. (5.39) to obtain

$$r = \frac{r_m[S]}{K_m + [S]} \quad (5.41)$$

Michaelis–Menten equation is obtained.

③ Discussion of the Michaelis–Menten equation

When the substrate concentration is much lower than the Michaelis constant, $K_m \gg [S]$, then

$$r = \frac{r_m}{K_m}[S] \quad (5.42)$$

That is, the enzymatic catalytic reaction rate is proportional to the concentration of the corresponding substrate, which conforms to the first-order reaction kinetics.

When the substrate concentration is much higher than the Michaelis constant $[S] \gg K_m$, then

$$r = r_m \quad (5.43)$$

That is, the enzymatic catalytic reaction rate is independent of the substrate concentration and conforms to zero-order reaction kinetics.

④ The meaning of Michaelis constant K_m and the maximum reaction rate r_m

Assuming that the substrate concentration is $[S]$ and the reaction rate is half of the maximum reaction rate, the following equation can be established:

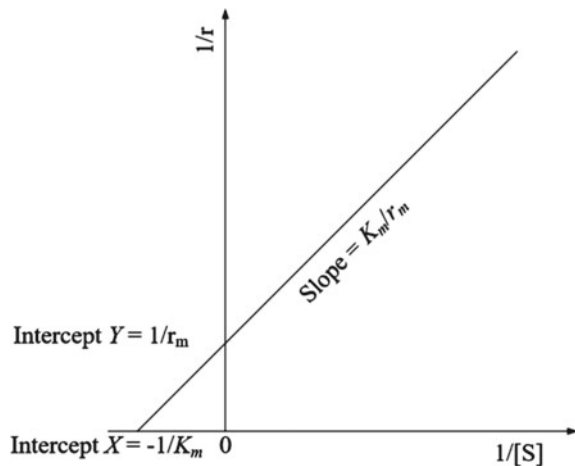
$$r = \frac{1}{2}r_m = \frac{r_m[S]}{K_m + [S]} \quad (5.44)$$

Further sorting of the the equation results in $K_m = [S]$. Therefore, K_m represents the substrate concentration when the enzymatic catalytic reaction rate reaches half of the maximum rate, also known as the half rate constant K_m . The higher this value is, the lower the affinity between the enzyme and substrate is, and the maximum reaction rate can be reached only when the substrate concentration is high. In contrast, the smaller K_m is, the greater the affinity between the enzyme and the substrate is, and the maximum reaction rate can be reached when the substrate concentration is low.

K_m is one of the characteristic constants of the enzyme, the value of which is related to the structure of the enzyme, the corresponding substrate and environmental factors (such as temperature, pH, ionic strength) and has nothing to do with the concentration of the enzyme itself. The values of K_m range widely for a variety of enzymes. When several enzymes can catalyze the same substrate, the values of each enzyme's K_m vary widely. When an enzyme can catalyze several substrates simultaneously, the values corresponding to every substrates K_m are different.

The reaction rate r_m reaches the maximum when all the active centers of the enzyme are occupied by substrates, whose value is directly proportional to the concentration of enzyme. If the total concentration of the enzyme is known, then the conversion number of enzymes can be calculated from r_m .

Fig. 5.16 Drawing of Lineweaver–Burk method (Zhou & Wang, 2015)



⑤ Determination of K_m and r_m

The Michaelis–Menten equation can be transformed for the convenience of measuring K_m and r_m . For example, the reciprocal form of Michaelis–Menten equation if one inverts both sides of the equation shown below. At this point, the curve turns into a straight line, and the least squares method can be used to fit the data to obtain K_m and r_m , which is called the Lineweave-Burk method.

$$\frac{1}{r} = \frac{1}{r_m} + \frac{K_m}{r_m} \frac{1}{[S]}$$

A straight line can be obtained, as shown in Fig. 5.16, by plotting $\frac{1}{r}$ versus $\frac{1}{[S]}$. The slope of the line is $\frac{K_m}{r_m}$, the intercept of the line intersecting the Y-axis is $\frac{1}{r_m}$, the point where it intersects the X-axis is $-\frac{1}{K_m}$, and the obtained equation is:

$$\frac{1}{r} = \frac{K_m}{r_m} \cdot \frac{1}{[S]} + \frac{1}{r_m}$$

Source of Fig. 5.16: Reprinted from book “*Microbiology of Environmental Engineering, Fourth ed.*” with kind permission from Prof. Shifen Wang.

(3) Temperature

The catalytic efficiency of enzymes is highly sensitive to temperature changes, which is one of the important characteristics of enzymes. When other conditions are constant, each enzyme exhibits its maximum activity only at a certain temperature, which is called the optimal temperature. All enzymes have their optimal temperature under certain conditions. Generally, the optimal temperature of enzymes in animals is 37 ~ 50 °C, while the optimal temperature of most enzymes in microbial cells is within the range of 25 ~ 60 °C. In the appropriate temperature range, the rate of

enzymatic chemical reaction can be increased by 1 ~ 2 times when the temperature increases by 10 °C.

Temperature has a double effect on enzyme activity: first, the enzyme reaction temperature is within a certain range (0 ~ 40 °C), which accelerates with increasing temperature, and second, enzymes are proteins. As the temperature increases, the denaturation rate of enzymes increases. Therefore, the effect of temperature on the enzymatic catalytic reaction rate is the comprehensive result of the two opposite effects.

During biological treatment process in environmental engineering, the activity of enzymes is expressed as the metabolic activity of microbial communities. The effect of temperature on the aerobic biodegradation reaction rate can be expressed as follows:

$$\frac{r_T}{r_{20}} = \theta_t^{T-20} \quad (5.45)$$

where

r_T —Reaction rate at temperature of T °C, kg/(m³·h)

r_{20} —Reaction rate at 20 °C, kg/(m³·h)

θ_t —Temperature coefficient, 1.02 ~ 1.25 in general

T —Temperature, °C.

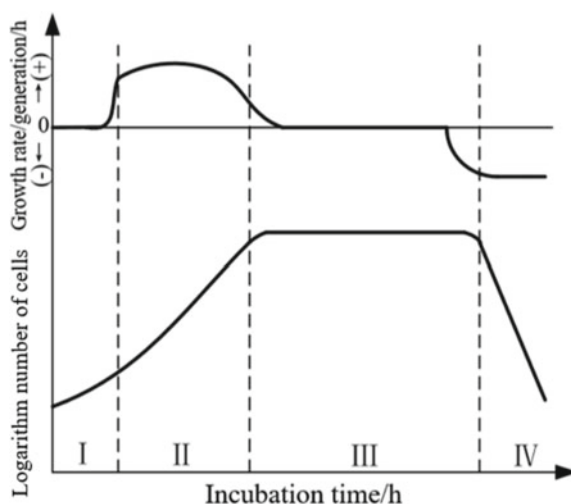
(4) pH

The pH of the system has an important influence on the enzymatic reaction rate. The activity of an enzyme at different pH can be tested in the same medium, which shows that the maximum catalytic efficiency of enzymes can be achieved at a certain pH, which is called the optimal pH for this enzyme. The existence of an optimal pH for enzyme catalysis shows that the reactive group of the enzyme and substrate molecule, coenzyme and prosthetic group as well as their ionization states are directly related to enzyme catalytic efficiency. However, the optimal pH of the enzyme does not belong to the characteristic constant, and it can be changed by the concentrations of both buffer and substrates, as well as the kinds of buffer. It is generally believed that when the system is at an appropriate pH, the dissociation state of the active group on the enzyme molecule is the most suitable to bond with the substrate. When the pH of the system changes, the binding ability of the enzyme to the substrate will be significantly reduced. The pH of the microbial wastewater treatment process is generally maintained between 6 ~ 9, and the optimal pH of most processes is within the range of 6 ~ 8.

5.1.4.2 Growth Curve of Cell

To investigate the growth and reproduction principles in the microbial community, a certain species of bacteria can be inoculated in the appropriate liquid culture medium, and the bacteria can reproduce by asexual binary fission. After fission, each daughter

Fig. 5.17 Typical growth curve of bacteria (Zhou & Wang, 2015). I Lag phase II Exponential growth phase III Stationary phase IV Decline phase. *Source of Fig. 5.17:* Reprinted from book “*Microbiology of Environmental Engineering, Fourth ed.*” with kind permission from Prof. Shifen Wang



cell produced has independent living ability. Under the condition that no material was added or removed and the volume of the culture medium remained unchanged, the time was taken as the abscissa, and the number of bacteria was taken as the ordinate. According to the change in the number of bacteria in the experimental period, the growth curve is drawn, which reflects the change in the number of bacteria. As shown in Fig. 5.17, the typical growth curve can be divided into four stages according to the change law of the number of bacteria: a lag phase, exponential growth phase, stationary phase and decline phase.

1) The lag phase

The process of bacterial growth and reproduction does not usually begin immediately in the medium after inoculation. Especially in the initial period of time, there is no significant change in the number of cells, and this stage is called the lag phase. This is the period when bacteria adapt to the new environment. There are two main reasons resulting in the lag. First is that bacteria must be induced to synthesize enzymes related to the utilization of new nutrients in the new environment, because of the difference in the composition of pre- and post-culture medium. Second is that some specific coenzymes or activators needed for the enzymatic catalytic reactions within bacteria have a high ability to transfer through the cell film, which can be lost from the cells through diffusion during the transfer process to the new medium.

2) The exponential growth phase

At the end of the lag phase, the bacteria begin to adapt to the new environment and undergo division and reproduction. The number of bacteria in the culture medium increases significantly. At this time, the growth of bacteria enters the exponential growth phase. In this period, a straight line can be approximately obtained by plotting with the natural or common logarithm of the number of bacteria and the corresponding time. The number of bacteria in the exponential growth phase increase

almost exponentially to $2^0 \rightarrow 2^1 \rightarrow 2^2 \rightarrow 2^3 \rightarrow \dots 2^{n_g}$. If a division is divided into “one generation”, the number of bacterial colonies doubles with each generation. Because the population growth of bacteria is carried out at an exponential rate, this stage is called the exponential growth phase. This law can be expressed by Eq. (5.46):

$$X'_{C2} = X'_{C1} \cdot 2^{n_g} \quad (5.46)$$

where

X'_{C1} , X'_{C2} —Number of cells at times τ_1 and τ_2 , respectively

n_g —Generation number

R_C —Growth rate, which is the generation number per unit time, s^{-1} or h^{-1}

G_τ —Generation time, which is the time needed for completing once of cell division, s or h

τ —Time, s or h

τ_1 , τ_2 —Time at τ_1 , τ_2 , s or h.

From the above equation, it can be deduced that:

$$n_g = 3.3 \lg \frac{X'_{C2}}{X'_{C1}} \quad (5.47)$$

Assume R_C is the growth rate, that is, generation numbers per unit time, namely,

$$R_C = \frac{n_g}{\tau_2 - \tau_1} \quad (5.48)$$

By substituting Equation (5.47) into Eq. (5.48),

$$R_C = \frac{3.3 \lg \frac{X'_{C2}}{X'_{C1}}}{\tau_2 - \tau_1} \quad (5.49)$$

Assume G_τ is the generation time, that is, the time required for the cell to divide once, so G_τ is the reciprocal of R_C .

$$G_\tau = \frac{1}{R_C} = \frac{\tau_2 - \tau_1}{3.3 \lg \frac{X'_{C2}}{X'_{C1}}} \quad (5.50)$$

It can be concluded that the more generation numbers (n_g) of bacterial cells in a certain period of time, the shorter the generation time (G_τ) and the faster the growth rate (R_C).

3) The stationary phase

With the growth and reproduction of bacteria in the culture medium, nutrients such as nitrogen and phosphorus are gradually consumed, and the concentration of harmful metabolites also increases gradually. The growth and reproduction rate of bacteria begins to decline gradually. Finally, the bacteria reproduction and apoptosis rates are basically equal. The cell concentrations in the medium reaches the maximum and keeps unchanged during this period.

4) The decline phase

With the depletion of nutrients and the accumulation of harmful metabolic wastes, bacteria death rate gradually accelerates. When the bacteria death rate exceeded the growth rate, the cell concentration in the culture medium began to decline gradually, and the bacteria growth entered the decline phase. The shape and size of cells are very inconsistent during the decline phase, abnormal cells will sometimes be produced, the life activities of bacteria mainly rely on endogenous respiration, and a large number of bacteria die.

5.1.4.3 Kinetics of Microorganisms

1) Microbial growth rate

The growth rate of microorganisms is proportional to the cell concentration, which is expressed as follows:

$$r_X = \frac{dX_C}{d\tau} = \mu_B X_C \quad (5.51)$$

where,

r_X —Microbial growth rate, kg(cells)/(m³·h)

X_C —Cell concentration, kg(cells)/m³

μ_B —Specific growth rate, h⁻¹

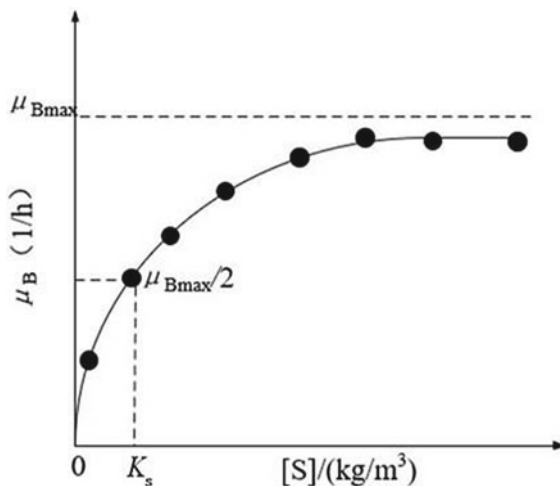
τ —Time, h.

According to the above equation, the specific growth rate is equivalent to the rate constant of the first-order reaction, which can be expressed as:

$$\mu_B = \frac{1}{X_C} \frac{dX_C}{d\tau} \quad (5.52)$$

The specific growth rate is related to the cell type, culture medium composition, substrate concentration, culture temperature, environmental pH, dissolved oxygen concentration and other factors. The relationship between the specific growth rate and concentration of restricted substrates is shown in Fig. 5.18. At a low limiting substrate concentration, μ_B increases with the concentration of restricted substrates and finally

Fig. 5.18 Effect of restricted substrate concentration on specific growth rate of cells



reaches its maximum value μ_{Bmax} . The curve is expressed by a mathematical equation such as Monod equation, i.e.,

$$\mu_B = \frac{\mu_{Bmax}[S]}{K_S + [S]} \quad (5.53)$$

where

μ_B —Specific growth rate, h^{-1}

μ_{Bmax} —Maximum specific growth rate, h^{-1}

$[S]$ —Substrate concentration, kg/m^3

K_S —Saturation coefficient, kg/m^3 , equals the value of $[S]$ when $\mu_B = \frac{\mu_{Bmax}}{2}$.

2) Growth rate equation in the presence of inhibitory factors

Certain substrates or products (e.g. glucose, methanol, ethanol, etc.) inhibit bacteria growth. The reasons for their growth inhibition may include changes in film fluidity and permeability, changes in the chemical potential of substrates, intermediate metabolites or products, changes in enzyme activity (inhibition), effects on enzyme synthesis (repression), depolymerization of polysubunits, and effects on cell function. In addition, elevated concentrations of certain substrates can activate one metabolic pathway but inhibit another. For example, during glucose metabolism, an increase in oxygen concentration inhibits the fermentation pathway and activates the oxidative metabolic pathway, this phenomenon known as the 'Pasteur effect'. When the glucose concentration reaches a high level, even under aerobic conditions, the metabolism of oxygen consumption will be inhibited, and metabolism will turn to the fermentation pathway, which is called the glucose effect.

(1) Substrate inhibition

The inhibition of bacteria growth by high concentrations of substrates can be divided into competitive inhibition and noncompetitive inhibition. If growth inhibition is caused by a rate-limiting step in a catalytic substrate degradation reaction, the type of inhibition on growth is the same as that of enzyme activity. The main substrate inhibition equation is as follows:

① Noncompetitive inhibition

$$\mu_B = \frac{\mu_{Bmax}}{\left(1 + \frac{K_S}{[S]}\right)\left(1 + \frac{[S]}{K_I}\right)} \quad (5.54)$$

In the equation, K_I —substrate inhibition coefficient, kg/m^3 .
If $K_I \gg K_S$, then

$$\mu_B = \frac{\mu_{Bmax}[S]}{K_S + [S] + \frac{[S]^2}{K_I}} \quad (5.55)$$

② Competitive inhibition

$$\mu_B = \frac{\mu_{Bmax}[S]}{K_S\left(1 + \frac{[S]}{K_I}\right) + [S]} \quad (5.56)$$

For inhibition constant K_I , there is a difference between competitive inhibition and noncompetitive inhibition.

(2) Product inhibition

The kinetics of microbial product inhibition can be divided into competitive inhibition and noncompetitive inhibition. Sometimes, under the condition that the specific inhibition mechanism is not clear, the specific growth rate can be approximately written as exponential or linear attenuation. Commonly used growth inhibition equations are:

① Noncompetitive inhibition

$$\mu_B = \frac{\mu_{Bmax}}{\left(1 + \frac{K_S}{[S]}\right)\left(1 + \frac{[P_C]}{K_P}\right)} \quad (5.57)$$

② Competitive inhibition

$$\mu_B = \frac{\mu_{Bmax}[S]}{K_S\left(1 + \frac{[P_C]}{K_P}\right) + [S]} \quad (5.58)$$

where

$[P_C]$ —Metabolite concentration, kg/m^3

K_P —Metabolite inhibition coefficient, kg/m^3 .

3) Substrate consumption rate

There are a large number of cells in a microbial reaction system, and there are certain differences between each cell. In the actual application process, it is impossible to know the substrate consumption rate of each cell. Therefore, the macroscopic consumption rate of the system is often analyzed and discussed, regardless of the intracellular differences, and the cell is considered a stable chemical component.

The relationship between the substrate consumption rate and microbial growth rate can be expressed as follows:

$$-r_s = \frac{d[S]}{d\tau} = \frac{r_X}{Y_{X/S}} \quad (5.59)$$

where

r_s —Substrate consumption rate, $\text{kg}(\text{substrate})/(\text{m}^3 \cdot \text{h})$

r_X —Microbial growth rate, $\text{kg}(\text{cells})/(\text{m}^3 \cdot \text{h})$

$[S]$ —Substrate concentration, kg/m^3

τ —Reaction time, h

$Y_{X/S}$ —Apparent cell yield coefficient, $\text{kg}(\text{cells})/\text{kg}(\text{substrate})$.

The substrate consumption rate divided by the amount of bacteria is called the specific substrate consumption rate, in terms of γ , that is,

$$\gamma = \frac{r_s}{X_C} \quad (5.60)$$

where

X_C —Cell concentration, kg/m^3

γ —Specific substrate consumption rate, $\text{kg}(\text{substrate})/[(\text{kg}(\text{cell}) \cdot \text{h})]$.

According to Eqs. (5.51), (5.59) and (5.60), the following can be concluded:

$$-\gamma = \frac{\mu_B}{Y_{X/S}} \quad (5.61)$$

If μ_B is expressed by the Monod, Eq. (5.61) can be deformed into:

$$-\gamma = \frac{\mu_{B\max}}{Y_{X/S}} \frac{[S]}{K_S + [S]} = (-\gamma_{\max}) \frac{[S]}{K_S + [S]} \quad (5.62)$$

where

- γ —Specific substrate consumption rate, kg(substrate)/(kg(cell) · h)
 γ_{\max} —Maximum specific substrate consumption rate, kg(substrate)/(kg(cell) · h)
 $[S]$ —Substrate concentration, kg/m³
 K_S —Saturation coefficient, kg/m³, equals the value of $[S]$ when $\mu_B = \frac{\mu_{B\max}}{2}$
 $Y_{X/S}$ —Apparent cell yield coefficient, kg(cell)/kg(substrate).

When cells take nitrogen sources, inorganic salts, vitamins and other materials as the consumptive substrate, $Y_{X/S}$ is a constant. Because those materials are only components of the cell not energy sources, Eq. (5.62) can be true. However, when the consumptive substrate is both a cell component (carbon source) and an energy source, the energy consumed to maintain normal cell metabolism should be considered. In this case:

$$\begin{aligned} (\text{Total carbon source consumption rate}) &= (\text{Consumption rate for growth}) \\ &+ (\text{Consumption rate for maintaining metabolism}) \end{aligned}$$

$$-r_S = \frac{r_X}{Y_{X/S \max}} + m_k X_C \quad (5.63)$$

where

- $Y_{X/S \max}$ —Maximum cell yield coefficient without maintenance metabolism, kg(cell)/kg(substrate)
 m_k —Maintenance coefficient, kg(substrate)/(kg(cell)·h)
 X_C —Cell concentration, kg/m³
 r_S —Substrate consumption rate, kg(substrate)/(m³·h).

Equation (5.63) is divided by the cell concentration X_c on both sides, and Eq. (5.64) can be obtained:

$$-\gamma = \frac{\mu_B}{Y_{X/S \max}} + m_k \quad (5.64)$$

Equation (5.64) shows the correlation between the specific consumption rate of substrate γ and the specific rate of cell growth μ_B , which can also be considered as a linear model with two parameters. After further analysis of Eq. (5.64), γ with μ_B can be further simplified to the following Eq. (5.65). Since μ_B is a function of $[S]$, γ is also a function of $[S]$.

$$-\gamma = f(\mu_B) = f[S] \quad (5.65)$$

5.2 Reactor

A reactor is a container for a biological or chemical reaction. Through a series of engineering measures, certain reaction conditions are provided in the container to make the reaction proceed in the desired direction faster and more efficiently. For example, in the treatment of municipal wastewater by an activated sludge method, a series of engineering methods, such as supplying air (oxygen) in the aeration tank, stirring and reflux, are adopted to make microorganisms decompose organic pollutants in wastewater into CO_2 and H_2O . The biochemical reaction can be conducted much faster and more efficiently than the natural biochemical process of a simulated river.

For example, when pollutants undergo biological or chemical reactions in the reactor, they are mainly biological or chemical processes but are also accompanied by physical processes such as diffusion and backmixing. Biological or chemical processes and physical processes are interrelated and interact with each other, which are complex. There are many engineering problems in the large-scale practical engineering application process, which is quite different from the conditions of biological or chemical reactions in small laboratory reactors. Therefore, reaction engineering is the study of reactor design amplification and process optimization by integrating biological or chemical reactions with engineering problems. This includes the macroscopic kinetics of biological/chemical reaction, that is, the quantitative relationship between the rate of biological/chemical reaction and various parameters, the back-mixing and distribution of residence time for various materials in continuous flow reactor, the characteristics of the reactor and the optimization of the reaction process.

In general, reaction engineering is the study of the process of large-scale biological/chemical reactions, the basic laws of equipment characteristics and the relationship between various parameters. Its specific tasks include:

- (1) Proper selection and reasonable design of a reactor;
- (2) The use of laboratory research data to enlarge effectively and solve the problems that may appear in practical engineering applications;
- (3) Optimize the design and control of the reaction process;
- (4) Improve and strengthen the existing technology and equipment, reduce energy consumption, and improve economic efficiency.

Through bench- or pilot-scale tests, appropriate process conditions should be found, and more importantly, a kinetic model of the biological/chemical reaction should be established. In addition, it is necessary to study the transfer process of the reactor, establish the flow model of materials in the reactor, and obtain the parameters of the transfer process. In this way, the reactor amplification time can be greatly shortened, and the optimal process conditions and operating conditions can be found in a wide range.

5.2.1 Flow Model of Material in Reactor

5.2.1.1 Basic Concepts

Material particle: The material particle refers to the micro element or micro group that represents the characteristics of the material. Macroscopic material is composed of numerous particles.

Residence time/mean residence time: The residence time of a material particle from the inlet to the outlet during the continuous operation of the reactor is called the residence time of the material particle. In an actual reactor, the residence time of each material particle is not the same, and there is a distribution, that is, a residence time distribution. The mean residence time of each particle is called the mean residence time.

5.2.1.2 Flow Model

In a flow reactor, the flow of material particles is different, resulting in uneven material concentration. The reaction time difference experienced by material particles has a very direct impact on the reaction results. Therefore, for a flow reactor, the material flow in the reactor must be considered. It is difficult to directly observe the flow of material particles in the reactor, so a flow model is generally used to describe it. Generally, the flow model can be divided into an ideal flow model and a nonideal flow model, and the ideal flow model can be subdivided into a plug flow model and a continuous stirred tank model.

1) Plug flow model

The plug flow model considers that the material entering the reactor moves forward along the flow direction, such as a piston in an engine cylinder, and does not mix with each other. Specifically, the model includes the plug flow model, piston flow model, ideal displacement model, ideal extrusion model, pipe flow model and other types.

(1) Model characteristics

- ① Material parameters such as temperature, concentration and pressure change continuously along the flow direction.
- ② The material parameters on any cross section perpendicular to the flow direction are all the same, and there is no obvious boundary layer in the reactor.
- ③ Materials are not mixed between cross sections along the flow direction inside the reactor.
- ④ The residence time of material particles on any cross section of the reactor is the same.

- ⑤ With no backmixing, material particles with different residence times in the reactor are not mixed.

(2) Scope of Application

The tubular reactors with great ratio of height over diameter.

2) **Continuous stirred tank model**

In a continuous stirred tank model, when the material enters the reactor, the new material and the original material inside the reactor are completely mixed. This model can be divided into a complete mixed model, ideal mixed model, stirred kettle model and other types.

(1) Model characteristics

- ① The material particles in the reactor are completely mixed, and the material parameters are the same everywhere and equal to the parameters at the outlet.
- ② The materials entering the reactor at the same time are dispersed instantaneously and uniformly mixed with the original materials in the reactor.
- ③ The residence time of material particles in the reactor varies. The residence time of particles in the material leaving the reactor at the same time is also different.
- ④ Backmixing approaches infinity.

(2) Scope of application

A fully mixed reactor with a strong agitation.

3) **Axial backmixing model**

In most practical continuously running reactors, the flow of materials is neither a full mixed flow nor a plug flow but somewhere in between. A new flow model needs to be used for mathematical description, and the axial backmixing model is one of them.

In this model, the degree of material flow deviation from the plug flow in the flow system can be described by superimposing axial diffusion or axial backmixing on the main body of the plug flow, and the direction of axial backmixing is opposite to that of the main body flow, as shown in Fig. 5.19.

Assuming that axial backmixing can be represented by Fick's law and that the flow of materials is stable, the axial backmixing model can be described by the following ordinary differential equation:

$$D_a \frac{d^2 c_A}{dl^2} - u \frac{dc_A}{dl} - r_A = 0 \quad (5.66)$$

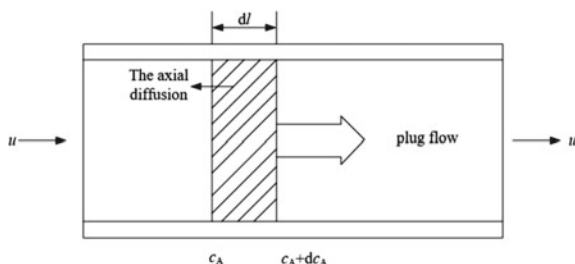


Fig. 5.19 Schematic diagram of axial diffusion flow model

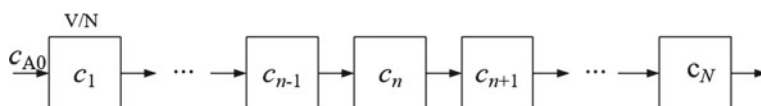


Fig. 5.20 Multi-reactor series model of flow system. V —Volume of flow system N —The equivalent number of series reactors in the flow system

where

D_a —Axial diffusion coefficient, m^2/s

$\frac{dc_A}{dl}$ —Axial concentration gradient, kg/m^4

u —Flow velocity, m/s

r_A —Chemical reaction rate, $\text{kg}/(\text{m}^3 \cdot \text{s})$.

When the initial and boundary conditions are determined, a definite solution to this equation can be obtained, that is, a mathematical description of the chemical reaction results under a specific flow condition.

4) Multiple continuous stirred tank model

As shown in Fig. 5.20, this model considers a continuous flow system to be the result of multiple continuous stirred tank reactors. This method uses the number of reactors in series (N) to characterize the degree to which the actual flow deviates from the ideal plug flow or the continuous stirred tank reactor. This model is more intuitive, and the parameter (N), which represents the flow characteristics, can be determined by experiments.

5.2.2 Reactor and Operational Mode

5.2.2.1 Classification by Operational Mode

The reactor can be divided into batch reactors, continuous flow reactors, semibatch or semicontinuous reactors and sequencing batch reactors according to the method of adding materials.

1) **Batch reactor**

Reactants are added into the reactor “pot by pot” for reaction. After each addition of reaction materials, all reaction products are removed after a certain time of reaction to meet the requirements.

The main characteristics of such reactors are as follows:

(1) Operational characteristics

There is no input or output of materials during the whole reaction process. It is generally used for homogeneous liquid phase reactions, especially for small amounts of treatment.

(2) Typical characteristics

The concentration, temperature and other parameters of the reaction system are kept uniform within the reactor and only change with the reaction time, and all material particles experience the same reaction time.

(3) Main advantages

Flexible operation, low equipment cost, suitability for small batch production or small-scale wastewater treatment.

(4) Main disadvantages

There are low utilization rate of equipment and high labor intensity. Different batches of operating conditions are not easily made the same, product quality (such as effluent water quality) is not easy to control.

2) **Continuous reactor**

During the reaction process, while feeding continuously, the reactor continuously discharges. The temperature, reactant concentration and other parameters of the reactor only change with the internal space position and have nothing to do with the running time of the reactor. There are two different types of continuous reactors: plug flow reactor and continuous stirred tank reactor.

3) **Half Continuous/batch reactor**

In this operation mode, one or several reactants are added into the reactor initially, and then products are removed or new reactants are added over time. The parameters of materials in the reactor change over time, and the operation mode shifts between continuous and intermittent flow, such as in traditional anaerobic sludge digester.

4) **Sequencing batch reactor (SBR)**

This is an intermittent reactor with continuous circulation in time. It is widely used in wastewater treatment, as well as the volume of the reactor and its treatment flow can be very large and highly automated. For example, in the sequencing batch reactor of an activated sludge process, the following five steps are repeated: ① water filling, ② reaction (aeration), ③ settlement (precipitation), ④ drainage, and ⑤ idle.

5.2.2.2 Classification by the Shape of the Reactor

According to the geometry, the reactors can be summarized as tubular, tank (kettle) and tower reactors as well as fixed bed, expansion bed and fluidized bed reactors.

Tubular reactors generally adopt a continuous operation mode, which is typically characterized by a large height-diameter ratio or length-width ratio and a small mixing effect of materials in the reaction process. The traditional plug-flow activated sludge process belongs to this type. Flue gas absorption towers also belong to this type.

A tank (kettle) reactor can be operated continuously or intermittently. Its characteristic is that the ratio of height to diameter or length to width is small, generally close to 1. Agitators are used to mix materials, such as mechanical aeration sedimentation tanks.

5.2.2.3 Classification by the Phase of Reactants

A reactor can be divided into a homogeneous reactor and a multiphase reactor according to the phases of the reactants. Internal reactions in homogeneous reactors usually occur in a single gas or liquid phase, such as a pH-regulating tank. If the reaction in a reactor involves two-phase or even multiphase media, it is called a multiphase reactor, such as a flue gas absorption tower, aeration tank in an activated sludge process, biological filter, etc.

5.2.2.4 Classification by Flow Mode

According to the flow and mixing state of reactants, reactors can be classified as ideal flow reactors and nonideal flow reactors. Ideal flow reactors can be further categorized into continuous stirred tank reactor (CSTR) and plug flow reactor.

5.2.3 Calculation of Ideal Homogeneous Reactor

The material balance equation based on conservation laws of both matter and energy is the basic equation of reactor calculation. All reactors are capable of carrying out material balance for one certain component of the system.

The input amount of materials in unit of time = the output amount of materials in unit of time + the accumulation amount of materials in unit of time + the amount of materials that disappear due to reaction in unit of time.

This equation is a general one. For different types of reactors, the items should be listed according to the specific situation to obtain the specific calculation equation. After obtaining the specific balance equation of the reactor, the corresponding reaction rate equation is substituted, and the boundary conditions are determined according to the characteristics of the reactor. The time required to reach a certain

concentration or conversion efficiency under different biological/chemical reaction rate equations is obtained, and then the corresponding reactor volume can be obtained according to the amount of treatment.

5.2.3.1 Batch Reactor

The volume of a batch reactor is determined by the daily processing capacity of the material and the time required for the reaction. Since the batch operation is intermittent, the time required for each batch operation includes the reaction time (τ) and auxiliary time (τ'), such as feeding, discharging and cleaning, in each batch operation. The effective volume of the reactor is the volume occupied by the reactants (V_R). The total volume (V_T) of the reactor can be calculated from the following equation:

$$V_R = V_T \varphi_{R/T} \quad (5.67)$$

In the equation, $\varphi_{R/T}$ is the loading coefficient, which refers to the ratio of the effective volume (V_R) of the reactor over the total volume (V_T) of the reactor. For liquid without foam and boiling, $\varphi_{R/T}$ (0.7 ~ 0.85) can be taken. For other material systems, $\varphi_{R/T}$ (0.4 ~ 0.6) can be taken. For wastewater treatment reactors in environmental engineering, the total volume of reaction V_T is obtained after the determination of the effective volume V_R by the following method. After the determination of length, width and height according to V_R , the original length and width, add 0.3 ~ 0.5 m super high on the original height, and then obtain the total volume V_T .

To calculate the total volume of the reactor, the effective volume should be calculated first. The effective volume depends on the reaction time τ .

In the batch reactor, the composition and temperature of each point in the reactor are all the same due to agitation. There is no material input or output in the reactor, and thus, the balance equation of component A in the whole reactor is as follows:

The amount of materials disappearing due to reaction in unit of time = the amount of materials accumulated in unit of time

$$(-r_A)V = -\frac{dn_A}{d\tau} \quad (5.68)$$

where

$(-r_A)$ —Reaction rate of component A in the reaction system, $\text{mol}/(\text{m}^3 \cdot \text{s})$

V —Volume of reactants at time τ , m^3

n_A —Number of moles of reactant A, mol , $n_A = n_{A0}(1 - x'_A)$

n_{A0} —Number of moles of initial reactant A, mol

x'_A —Conversion efficiency of reactant A, the fraction of the reactant A consumed over initial reactant A, $x'_A = \frac{n_{A0} - n_A}{n_{A0}}$.

$$(-r_A)V = \frac{-d[n_{A0}(1 - x'_A)]}{d\tau} = n_{A0} \frac{dx'_A}{d\tau} \quad (5.69)$$

According to the above equation, the following will be obtained:

$$\tau = n_{A0} \int_0^{x'_A} \frac{dx'_A}{(-r_A)V} \quad (5.70)$$

The above equation is the basic equation for calculating the reaction time necessary for reactants to reach a certain conversion efficiency in a batch reactor. This equation is applicable to cases where V varies with the reaction.

Special computational cases.

- (1) For the constant volume process, the volume of materials before and after the reaction does not change much, which can be regarded as a constant; then, Eq. (5.70) becomes

$$\tau = \frac{n_{A0}}{V} \int_0^{x'_A} \frac{dx'_A}{-r_A} = c_{A0} \int_0^{x'_A} \frac{dx'_A}{(-r_A)} \quad (5.71)$$

Again,

$$\begin{aligned} c_A &= c_{A0}(1 - x'_A) \\ dc_A &= -c_{A0}dx'_A \end{aligned}$$

Therefore,

$$\tau = - \int_{c_{A0}}^{c_A} \frac{dc_A}{(-r_A)} \quad (5.72)$$

- (2) For an irreversible first-order constant volume reaction

$$-r_A = c_{A0} \frac{dx'_A}{d\tau} = kc_A = kc_{A0}(1 - x'_A) \quad (5.73)$$

where k is the reaction rate constant, and the units vary with the reaction order. For the first-order reaction, the k unit is s^{-1} .

Equation (5.73) can be sorted into

$$\frac{dx'_A}{d\tau} = k(1 - x'_A)$$

$$\int_0^{x'_A} \frac{dx'_A}{1-x'_A} = \int_0^\tau k d\tau$$

$$\tau = \frac{1}{k} \ln \frac{1}{1-x'_A} \quad (5.74)$$

- (3) For different reactions, the time τ required to achieve a certain conversion efficiency can be obtained from the integral of Eq. (5.71) according to the kinetic differential equation. The boundary condition of the integral is:

$$\begin{cases} \tau = 0, & x'_A = 0 \\ \tau = \tau, & x'_A = x'_A \end{cases}$$

In the batch stirred reactor, the reaction time τ required for zero-order, first-order and second-order reactions to reach a specific conversion efficiency and outflow concentration is listed in Table 5.2.

- (4) As shown in the above discussion, in a batch reactor, as long as c_{A0} is the same, the reaction time required for each batch of reaction materials to achieve a certain conversion efficiency only depends on the reaction rate ($-r_A$). For the same reaction, no matter how much reactant is processed, as long as the same conversion efficiency is achieved, each batch of reaction times must be the same. Therefore, in the amplification design, as long as the factors affecting the reaction rate in mass production are the same as those in the small test, the reaction time can be guaranteed to be the same.
- (5) For the various results of Eqs. (5.70), (5.71) and (5.72), the relationship between various parameters can also be intuitively expressed in graphical form, as shown in Fig. 5.21.

Based on the above discussion, the reaction time τ required for different reactions at a certain conversion efficiency can be calculated so that the effective volume V_R of the reactor can be determined to further confirm the volume of the reactor.

$$V_R = \frac{\text{Daily treatment volume}}{24} \times (\tau + \tau') = q_h(\tau + \tau') \quad (5.75)$$

Table 5.2 Reaction time of different chemical reactions in the batch stirred reactor

Reaction order	Dynamic differential equation	Time to reach a certain conversion efficiency	Time to obtain a certain concentration
Zero-order reaction	$-r_A = k$	$\tau = \frac{x'_A c_{A0}}{k}$	$\tau = \frac{c_{A0} - c_A}{k}$
First-order reaction	$-r_A = kc_A$	$\tau = \frac{1}{k} \ln \frac{1}{1-x'_A}$	$\tau = \frac{1}{k} \ln \frac{c_{A0}}{c_A}$
Second-order reaction	$-r_A = kc_A^2$	$\tau = \frac{1}{k} \frac{x'_A}{c_{A0}(1-x'_A)}$	$\tau = \frac{1}{k} \left(\frac{1}{c_A} - \frac{1}{c_{A0}} \right)$

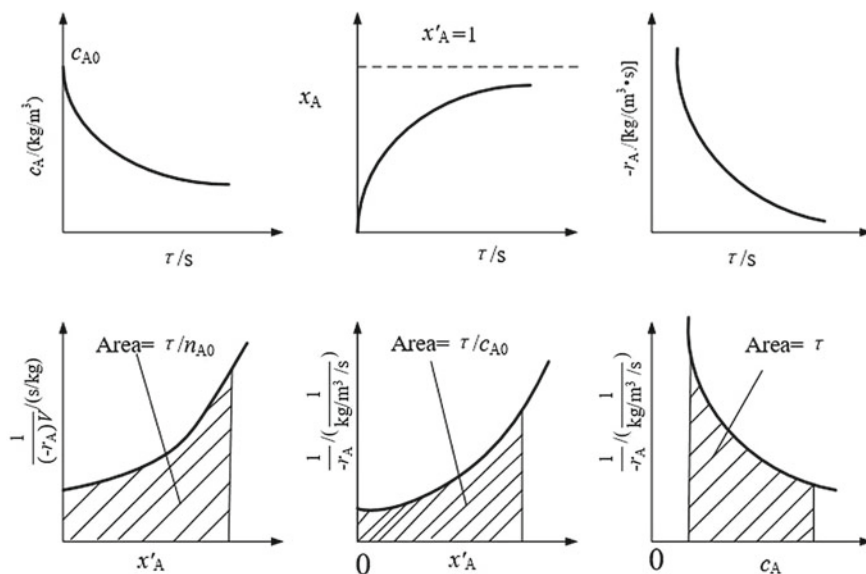


Fig. 5.21 Performance of batch reactor and graphical representation of its equation

where

q_h —Average amount of materials processed per hour, m^3/h

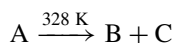
τ' —Auxiliary time of feeding, discharging and cleaning in each batch of operation, h

V_R —Effective volume of reactor, m^3

τ —Reaction time, h.

For a certain volume of reaction material, a reactor can be selected. If the volume of one reactor is too large, several small volume reactors can be used in parallel. When the reactor volume is too large, the mixing effect is difficult to achieve the uniformity of the small reactor, which will result in an uneven mixing of materials, resulting in uneven concentration and temperature distribution, thus reducing the conversion efficiency and failing to meet the design requirements. However, if several smaller reactors are selected, the corresponding auxiliary equipment will increase, and the operating cost of the equipment will increase. Therefore, the volume and number of reactors should be determined by weighing the advantages and disadvantages of reactants, product quality, operation and economic benefits.

Example 5.3 The following decomposition reaction is carried out in a batch reactor:



By experimental measurement, the reaction is a first-order reaction, and its kinetic constant is 0.231 h^{-1} at 328 K. The daily treatment capacity and initial concentration

of reactant A are 189 m^3 and 1.00 kg/dm^3 , respectively. The height-diameter ratio of the reactor is 8, and the additional height is 0.3 m. The auxiliary time required for each operation is $\tau' = 4 \text{ h}$. Try to find the effective volume and total volume of the reactor required when the conversion efficiency of reactant A is 90%.

Solution: (1) Calculate the time (τ) required to achieve the required conversion efficiency.

According to Eq. (5.74):

$$\tau = \frac{1}{k} \ln \frac{1}{1 - x'_A} = \frac{1}{0.231} \ln \frac{1}{1 - 0.9} = 9.97 \text{ h}$$

(2) Calculate the volume of the reactor

According to the Eq. (5.75)

$$\begin{aligned} V_R &= \frac{\text{Daily processing capacity}}{24} \times (\tau + \tau') = q_h(\tau + \tau') \\ &= \frac{189}{24} \times (9.97 + 4) \\ &= 110.01 \text{ m}^3 \end{aligned}$$

$$V = \frac{\pi}{4} d^2 h = \frac{\pi}{4} d^2 \times 8d = 2\pi d^3 = 110.01 \text{ m}^3$$

$$d^3 = 17.51$$

$$d = 2.60 \text{ m}$$

$$h = 8d = 20.80 \text{ m}$$

$$h_{\text{total}} = 20.80 + 0.30 = 21.10 \text{ m}$$

$$V_{\text{total}} = \frac{\pi}{4} d^2 h_{\text{total}} = \frac{\pi}{4} \times 2.60^2 \times 21.10 = 111.97 \text{ m}^3$$

Answer: The effective volume and total volume of the reactor are 110.01 m^3 and 111.97 m^3 , respectively.

5.2.3.2 Plug Flow Reactor

The plug flow reactor usually operates in continuous mode, and its remarkable feature is that there is no backmixing of material particles in the reactor, and the residence time is equal. In the cross section perpendicular to the material flow direction, the flow rate, concentration, conversion efficiency and temperature are all equal, but along the material flow direction, they are changed. In the axial direction of the reactor, a micro element volume dV_R is taken for the material balance of component A.

Under stable flow, there is no material accumulation. In this situation, the material balance equation is:

The input amount of materials in unit of time = the output amount of materials in unit of time + the amount of materials that disappeared due to reaction in unit of time

Assume that q_{mA} (mol/s) reactant A enter dV_R in unit time, $q_{mA} + dq_{mA}$ (mol/s) reactant A flow out dV_R in unit time and $(-r_A) \cdot dV_R$ (mol/s) is the amount of reactant A which disappears due to reaction in dV_R in unit of time. q_{mA_0} is the inlet flow rate (mol/s) of reactant A in the plug flow reactor, and x'_A is conversion efficiency of reactant A before it enters the micro element volume, dV_R .

$$q_{mA} = (q_{mA} + dq_{mA}) + (-r_A) \cdot dV_R$$

Similarly,

$$dq_{mA} = d[q_{mA_0}(1 - x'_A)] = q_{mA_0}d(1 - x'_A) = -q_{mA_0}dx'_A$$

After resorting, the following can be obtained:

$$q_{mA_0}dx'_A = (-r_A) \cdot dV_R$$

According to the boundary conditions:

$$V_R = 0, \quad x'_A = 0$$

$$V_R = V_R, \quad x'_A = x'_A$$

thus,

$$\int_0^{V_R} \frac{dV_R}{q_{mA_0}} = \int_0^{x'_A} \frac{dx'_A}{(-r_A)}$$

For steady operation at a fixed volume, q_{mA_0} and V_R are constants, so

$$\frac{V_R}{q_{mA_0}} = \int_0^{x'_A} \frac{dx'_A}{(-r_A)} \quad (5.76)$$

And it is known that,

$$q_{mA_0} = q_V c_{A_0}$$

In the equation, q_V is the volume flow rate of the plug flow reactor because

$$\begin{aligned} \frac{V_R}{q_V c_{A_0}} &= \int_0^{x'_A} \frac{dx'_A}{(-r_A)} \\ \tau &= \frac{V_R}{q_V} = c_{A_0} \int_0^{x'_A} \frac{dx'_A}{(-r_A)} \end{aligned} \quad (5.77)$$

For a constant volume process, $x'_A = 1 - c_A/c_{A_0}$, and thus

$$dx'_A = -\frac{dc_A}{c_{A_0}}$$

Therefore, Eq. (5.77) becomes

$$\tau = \frac{V_R}{q_V} = - \int_{c_{A_0}}^{c_A} \frac{dc_A}{(-r_A)} \quad (5.78)$$

Both Eqs. (5.77) and (5.78) are the basic equation for the calculation of a plug flow reactor running in a continuous mode.

(1) For an irreversible first-order reaction equation, there is

$$(-r_A) = kc_A = kc_{A_0}(1 - x'_A)$$

Substitute the above equation into Eq. (5.77) to obtain

$$\begin{aligned} \tau &= \frac{V_R}{q_V} = c_{A_0} \int_0^{x'_A} \frac{dx'_A}{(-r_A)} = c_{A_0} \int_0^{x'_A} \frac{dx'_A}{kc_{A_0}(1 - x'_A)} \\ \tau &= \frac{V_R}{q_V} = \frac{1}{k} \ln \frac{1}{1 - x'_A} \end{aligned} \quad (5.79)$$

Table 5.3 Reaction times of different chemical reactions in the plug flow reactor

The reaction order	Dynamic differential equation	Time to reach a certain conversion efficiency	Time to get to a certain concentration
Zero-order reaction	$-r_A = k$	$\tau = \frac{x'_A c_{A0}}{k}$	$\tau = \frac{c_{A0} - c_A}{k}$
First-order reaction	$-r_A = kc_A$	$\tau = \frac{1}{k} \ln \frac{1}{1-x'_A}$	$\tau = \frac{1}{k} \ln \frac{c_{A0}}{c_A}$
Second-order reaction	$-r_A = kc_A^2$	$\tau = \frac{1}{k} \frac{x'_A}{c_{A0}(1-x'_A)}$	$\tau = \frac{1}{k} \left(\frac{1}{c_A} - \frac{1}{c_{A0}} \right)$

- (2) For different reactions, the reaction time τ can be obtained at a certain conversion efficiency by integrating Eq. (5.77) according to the kinetic differential equation. The boundary condition of the integral is:

$$\tau = 0, x'_A = 0$$

$$\tau = \tau, x'_A = x'_A$$

Table 5.3 lists the reaction time required for several typical reactions to reach a certain conversion efficiency and a certain outflow concentration in a plug flow reactor running in continuous mode.

- (3) The relationship between various parameters can also be expressed graphically, as shown in Fig. 5.22.

Above discussion demonstrates that the equations corresponding to the continuous operation of the plug flow reactor listed in Table 5.3 are the same as the intermittent operation of the batch stirred reactor listed in Table 5.2, and the graphical expression of the relationship between various parameters is also the same, as shown in Figs. 5.21 and 5.22.

By comparison, it can be found that for the constant volume process, the time required for the same reaction to reach the same reaction progress in the continuous plug flow reactor and the batch stirred reactor is equal. In addition, since there is no backmixing in either reactor, the change history of molecular concentration is exactly the same from the beginning to the end over time. Therefore, when the effective volume is equal, the production capacity of the two reactors is the same. However, the two reactors are essentially different. First, there are essential differences in material flow states. The material in the plug flow reactor is a stable and unmixed continuous flow, while the material in the batch stirred reactor is a uniform and unstable nonflow system. For batch reactors, reaction time can be directly integrated from the definition of reaction rate. Reaction time is a direct reflection of the degree of reaction. The reaction time of the continuously running plug flow reactor changes with the axial length of the reactor. In any given cross section, any properties of materials do not change with time. Therefore, the parameters of the intermittently

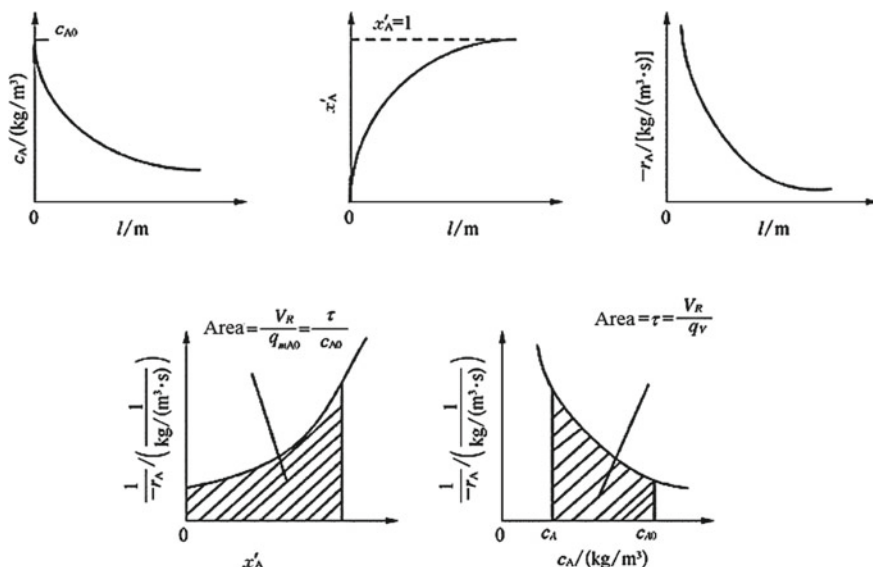


Fig. 5.22 Performance of an ideal push-flow reactor and graphical representation of its equation

running batch stirred reactor, such as concentration and conversion efficiency, are all functions of time and have nothing to do with the size of the reactor. The parameters of a continuously running plug flow reactor are a function of space and vary with space.

By taking the similarity of these two types of reactors the batch stirred reactor data can be directly used in the continuous plug flow reactor during the amplification design. In the same way, the effective volume of a continuously running plug flow reactor can also be calculated directly by means of microdynamic equations.

Example 5.4 The reaction shown in Example 5.3 is conducted in a continuously running plug flow reactor, and the operating conditions and output are the same as those in Example 5.3. Please calculate the effective volume of the reactor.

Solution:

$$\tau = \frac{1}{k} \ln \frac{1}{1 - x'_A} = \frac{1}{0.231} \ln \frac{1}{1 - 0.9} = 9.97 \text{ h}$$

$$V_R = \frac{\text{Daily treatment volume}}{24} \times \tau = \frac{189}{24} \times 9.97 = 78.5 \text{ m}^3$$

Answer: The effective volume of the reactor is 78.5 m^3 .

The effective volume of the continuous operation of the plug flow reactor is smaller than that of the intermittent operation of the batch stirred reactor. The difference is that the intermittent operation of batch stirred reactor has extra auxiliary time.

5.2.3.3 Continuous Stirred Tank Reactor

Figure 5.23 shows the performance and graphical equation of a continuous stirred tank reactor in continuous operation. The reactor is an ideal reactor. It is assumed that materials enter the reactor continuously and then immediately mix evenly. All reactants and products are evenly distributed inside the reactor. The concentrations of all components, temperature and other parameters both in the reactor and in outlet are all the same. For stable operation, the material balance equation without material accumulation is:

material input per unit of time = material output per unit of time + the amount of material disappeared due to reaction per unit of time,

namely,

$$q_{mA_0} = q_{mA} + (-r_A) \cdot V_R$$

$$q_{mA} = q_{mA_0}(1 - x'_A)$$

For a constant volume process,

$$q_{mA_0} = q_V c_{A_0}, q_{mA} = q_V c_A, \text{ so}$$

$$q_{mA_0} = q_{mA_0}(1 - x'_A) + (-r_A) \cdot V_R$$

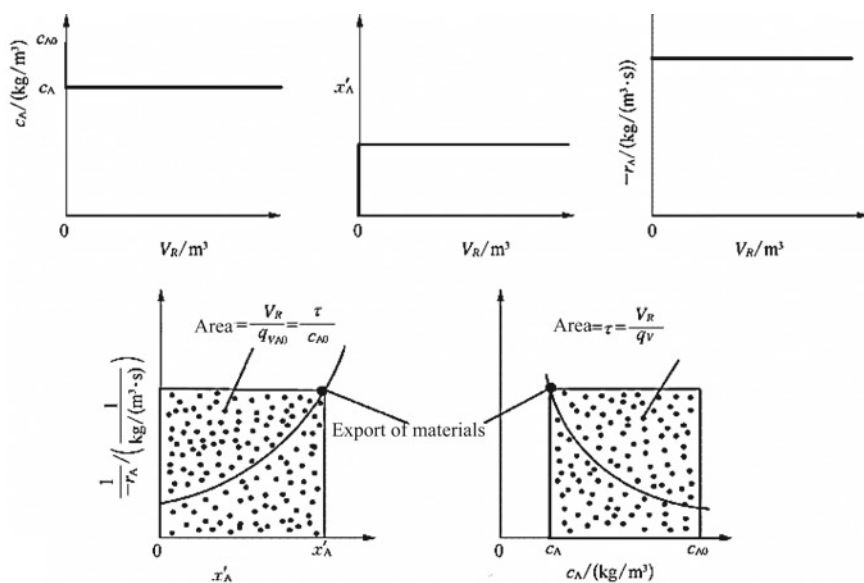


Fig. 5.23 Performance of a continuous stirred tank reactor and graphical representation of its equation

$$\frac{V_R}{q_{mA_0}} = \frac{x'_A}{(-r_A)} \quad (5.80)$$

$$q_V c_{A_0} = q_V c_A + (-r_A) \cdot V_R$$

$$\tau = \frac{V_R}{q_V} = \frac{c_{A_0} - c_A}{(-r_A)} \quad (5.81)$$

$$\tau = \frac{V_R}{q_V} = \frac{c_{A_0} x'_A}{(-r_A)} \quad (5.82)$$

Because the residence time of different material particles in the reactor is different, it is generally called the mean residence time τ . The above are the basic equations for calculating the continuous stirred tank reactor.

- (1) For an irreversible first-order reaction,

$$(-r_A) = k c_A = k c_{A_0} (1 - x'_A)$$

$$\tau = \frac{V_R}{q_V} = \frac{x'_A}{k(1 - x'_A)} = \frac{c_{A_0} - c_A}{k c_A} \quad (5.83)$$

- (2) For different reactions, the reaction time required to achieve a certain conversion efficiency or concentration can still be obtained through integration according to different kinetic equations. Table 5.4 shows the results of several typical reactions.
- (3) The relationship between various parameters can also be expressed graphically, as shown in Fig. 5.23.

Example 5.5 The reaction of Example 5.3 is conducted in a continuous stirred tank reactor, and the operating conditions and output are the same as those of Example 5.3. Please calculate the effective volume of the continuous stirred tank reactor.

Table 5.4 Reaction times of different chemical reactions in continuous stirred tank reactors

The reaction order	Dynamic differential equation	Time to reach a certain conversion efficiency	Time to obtain a certain concentration
Zero-order reaction	$-r_A = k$	$\tau = \frac{x'_A c_{A_0}}{k}$	$\tau = \frac{c_{A_0} - c_A}{k}$
First-order reaction	$-r_A = k c_A$	$\tau = \frac{x'_A}{k(1 - x'_A)}$	$\tau = \frac{c_{A_0} - c_A}{k c_A}$
The secondary reaction	$-r_A = k c_A^2$	$\tau = \frac{1}{k} \frac{x'_A}{c_{A_0}(1 - x'_A)^2}$	$\tau = \frac{1}{k} \left(\frac{c_{A_0}}{c_A^2} - \frac{1}{c_A} \right)$

Solution:

$$\begin{aligned}\tau &= \frac{c_{A0} - c_A}{kc_A} = \frac{x'_A}{k(1 - x'_A)} \\ &= \frac{0.9}{0.231 \times 0.1} = 39 \text{ h} \\ V_R &= \frac{189}{24} \times 39 = 307 \text{ m}^3\end{aligned}$$

Answer: The effective volume of the continuous stirred tank reactor is 307 m^3 .

The calculation results for the effective volume demonstrate that the effective volume of the continuous stirred tank reactor required to achieve the same conversion efficiency or material outlet concentration under the same conditions is much larger than those of both the batch reactor and the continuous plug flow reactor.

5.2.3.4 Multiple Continuous Stirred Tank Reactor

Continuous plug flow reactor and continuous stirred tank reactor are both typical ideal flow reactor. Because these two types of reactors can be combined in various ways to form a variety of reactors, their performances are also different. Here, only the multiple continuous stirred tank reactor is discussed.

When reactants enter the ideal continuous stirred tank reactor in a single reactor, their concentration decreases rapidly due to rapid mixing and diffusion, and the reaction rate also slows down, so it takes a long time to reach the same reaction progress. Therefore, when the reaction rate is the same and the final conversion efficiency of the reactor is the same, the volume of the ideal continuous stirred tank reactor with a single reactor is much larger than that of the ideal plug flow reactor. As shown in Fig. 5.24, the actual production process often adopts a multiple continuous

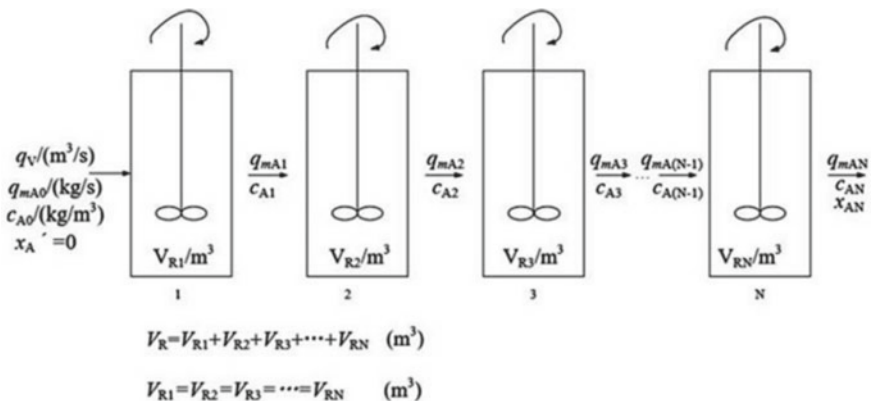


Fig. 5.24 Multiple continuous stirred tank reactor

stirred tank reactor to overcome this shortcoming. For example, an ideal continuous stirred tank reactor with a volume of V_R is divided into N ideal continuous stirred tank reactors with a volume of (V_R/N) . The latter reactant concentration decreases step by step under the condition of consistent temperature, initial reactant concentration and other conditions, so the latter means reaction driving force is higher than the former. Thus, the reaction rate is higher, the time required to reach the same conversion efficiency is less, and the total volume of the reactor is smaller. The larger the number of reactors, the closer to the ideal plug flow reactor.

Assuming a reactor composed of N reactors, the material balance of reactant A can be performed for any reactor as follows:

$$q_{mA_{i-1}} = q_{mA_i} + (-r_{A_i}) \cdot V_{Ri}$$

namely,

$$q_{mA_0}(1 - x'_{i-1}) = q_{mA_0}(1 - x'_i) + (-r_{A_i}) \cdot V_{Ri}$$

In the stable process, the volume flow rate q_V does not change. Divide by q_V on both sides of this equation to obtain:

$$c_{A_0}(1 - x'_{i-1}) = c_{A_0}(1 - x'_i) + (-r_{A_i}) \cdot \frac{V_{Ri}}{q_V}$$

$$\tau_i = \frac{V_{Ri}}{q_V} = \frac{c_{A_0}[(1 - x'_{i-1}) - (1 - x'_i)]}{(-r_{A_i})} = \frac{c_{A_0}(x'_i - x'_{i-1})}{(-r_{A_i})} \quad (5.84a)$$

$$\text{or } \tau_i = \frac{V_{Ri}}{q_V} = \frac{c_{A_{i-1}} - c_{A_i}}{(-r_{A_i})} \quad (5.84b)$$

where

x'_{i-1} , $c_{A_{i-1}}$ —Conversion efficiency and concentration of component A entering the i th reactor

x'_i , c_{A_i} —Conversion efficiency and concentration of component A entering the $(i + 1)$ th reactor or the conversion efficiency and concentration of component A when flowing out of the i th reactor

$q_{mA_{i-1}}$, q_{mA_i} —Mass flow rate of component A entering and leaving the i th reactor

$-r_{A_i}$ —Reaction rate of the i th reactor.

The above two equations are the basic equations of multiple continuous stirred tank reactor.

In the process of designing multiple continuous stirred tank reactor, it is necessary to calculate the effective volume V_{Ri} of each reactor, the number of reactors N , the final conversion efficiency x'_{AN} and the final concentration c_{AN} by the algebraic method or graphical method. The detailed calculation is shown below.

1) Algebraic method

As shown in Fig. 5.24, the material exported from the first reactor in the multiple continuous stirred tank reactors system is the material imported for the second reactor. Therefore, the calculation method of an ideal single continuous stirred tank reactor is generally used to conduct iterative calculation by the reactor until the final required conversion efficiency is obtained.

If using an isothermal first-order reaction, then

$$-r_{Ai} = kc_{Ai}$$

Substitute this into Eq. (5.84b) to obtain

$$\tau_i = \frac{c_{Ai-1} - c_{Ai}}{kc_{Ai}}$$

Sort out the following:

$$c_{Ai} = \frac{c_{Ai-1}}{1 + k\tau_i} \quad (5.85)$$

For the first reactor:

$$c_{A1} = \frac{c_{A0}}{1 + k\tau_1}$$

For the second reactor:

$$c_{A2} = \frac{c_{A1}}{1 + k\tau_2} = \frac{c_{A0}}{1 + k\tau_1} \frac{1}{1 + k\tau_2}$$

When the volume of each reactor and the flow rate q_V are both equal, then

$$\tau_1 = \tau_2 = \tau_3 = \cdots = \tau_N$$

Thus, for the N th reactor,

$$c_{AN} = \frac{c_{A0}}{(1 + k\tau)^N} \quad (5.86a)$$

$$1 - x'_{AN} = \frac{1}{(1 + k\tau)^N}$$

$$x'_{AN} = 1 - \frac{1}{(1 + k\tau)^N} \quad (5.86b)$$

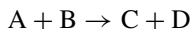
If k , V , and V_{Ri} are known, a series of concentrations can be calculated for each reactor to obtain c_{A1} , c_{A2} , ..., c_{AN} or x'_{A1} , x'_{A2} , ..., x'_{AN} . If the final concentration c_{AN}

or the final conversion efficiency x'_{AN} as well as k , V , and N are known, V_{Ri} can be directly calculated for each reactor.

If the temperature and volume of each reactor are different, the corresponding V_{Ri} and $(-r_{Ai})$ should be used during the calculation of that reactor.

For non-first-order reactions, if the number of reactors is not great, a similar method can be used.

Example 5.6 In a continuous flow reactor with two stirred tank reactors in series, the following gas phase reaction is carried out:



If the kinetic equation of the reaction is:

$$-r_A = k_c c_A c_B = k_c c_A^2 (c_A = c_B)$$

$k_c = 1.00 \text{ dm}^3/(\text{mol}\cdot\text{s})$, $c_{A0} = c_{B0} = 1.00 \text{ mol/dm}^3$, the time for materials to pass through the space of the two reactors is 1.0 s, and the volume of the two reactors is equal. Please determine the conversion efficiency of the reaction through the two reactors in series.

Solution: Because N is equal to 2,

$$\bar{\tau} = \frac{1.0}{2} = 0.5 \text{ s}$$

For the first reactor, the conversion efficiency can be obtained from Eq. (5.84b)

$$c_{A1}^2 = \frac{c_{A0} - c_{A1}}{k_c \bar{\tau}}$$

namely,

$$c_{A0} x'_{A1} - k_c \bar{\tau} [c_{A0} (1 - x'_{A1})]^2 = 0$$

$$\begin{aligned} x'_{A1} - 1.00 \times 0.5 \times (1 - x'_{A1})^2 &= 0 \\ x'_{A1} &= 0.27 = 27\% \end{aligned}$$

For the second reactor, the conversion efficiency can be obtained from Eq. (5.84b)

$$c_{A2}^2 = \frac{c_{A1} - c_{A2}}{k_c \bar{\tau}}$$

$$c_{A0} (x'_{A2} - x'_{A1}) - k_c \bar{\tau} [c_{A0} (1 - x'_{A2})]^2 = 0$$

$$x'_{A2} - 0.27 - 1.00 \times 0.5 \times (1 - x'_{A2})^2 = 0$$

$$x'_{A2} = 0.43 = 43\%$$

Answer: The conversion efficiency is 43% after the two reactors in series.

2) Graphic method

Some homogeneous reactions are difficult to establish corresponding kinetic models, and only a group of kinetic data $x'_A \sim (-r_A)$ can be obtained under isothermal conditions. At this time, a graphical method can be used to calculate the outlet conversion efficiency or the number of reactors. The material balance equation of this reactor (5.84a) can be rewritten as:

$$(-r_{Ai}) = \frac{c_{A0}}{\tau_i} x'_{Ai} - \frac{c_{A0}}{\tau_i} x'_{Ai-1} \quad (5.87)$$

The above equation indicates that the reaction rate $(-r_{Ai})$ of any reactor (such as reactor i) of in the multiple continuous stirred tank reactor has a linear relationship with the outlet conversion efficiency x'_{Ai} . That is, one straight line can be drawn on the $x'_A \sim (-r_A)$ graph with a slope of c_{A0}/τ_i and intercept of $(c_{A0}x'_{Ai-1}/\tau_i)$. Equation (5.87) is also called the equation of the operating line. The outlet conversion efficiency should not only satisfy the material balance equation but also satisfy the kinetic equation $(-r_A) = f(x'_A)$. Therefore, if these two equations on the $x'_A \sim (-r_A)$ graph are plotted, the value of x'_A corresponding to the intersection point of two lines is the outlet conversion efficiency of the reactor. The specific drawing steps are as follows if the final conversion efficiency is x'_{AN} known.

- (1) Plot $x'_A \sim (-r_A)$ curve with known kinetic data, as in AB in Fig. 5.25.
- (2) Determine the desired final conversion efficiency (x'_{AN}) to achieve on axis x'_A .
- (3) Apply a step-by-step graphical method according to the material balance Equation (5.87)

For the first reactor, $i = 1$, $x'_{Ai} = x'_{A1}$, $x'_{Ai-1} = x'_{A0} = 0$, where the intercept is zero, plot one straight line from the original point with a slope of c_{A0}/τ_1 , which intersects with line AB at point R_1 . Then, plot a vertical line from R_1 that crosses the abscissa at point O_1 , from which the outlet conversion efficiency (x'_{A1}) of the first reactor can be read.

For the second order, $i = 2$, plot one line from point O_1 with a slope of c_{A0}/τ_2 that intersects AB at point R_2 . Then, a vertical line is plotted from point R_2 , which crosses the abscissa at O_2 , from which the conversion efficiency (x'_{A2}) can be read.

Thus, for $x'_{Ai} \geq x'_{AN}$, the number of intersection points on the AB line is the required N of the series kettles.

If the residence time at all reactors is equal, that is, $\tau_1 = \tau_2 = \tau_3 = \dots = \tau_N$, then the slope of the line to AB should be the same. In this case, it just needs to follow the slope c_{A0}/τ_i to make the first reactor straight line; then, the other reactors can be obtained by making parallel lines according to the first reactor.

Fig. 5.25 Multiple continuous stirred tank reactor

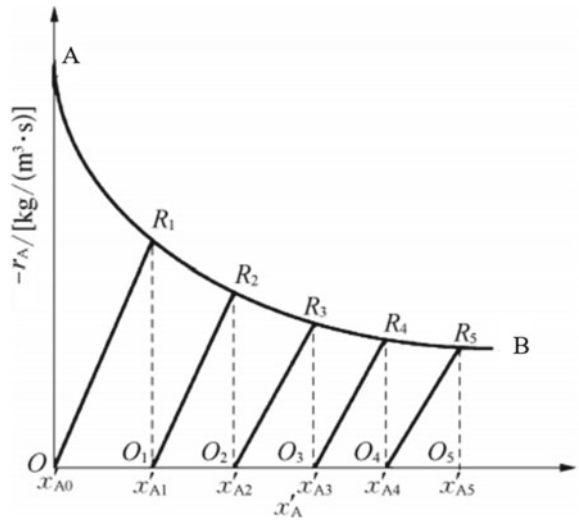
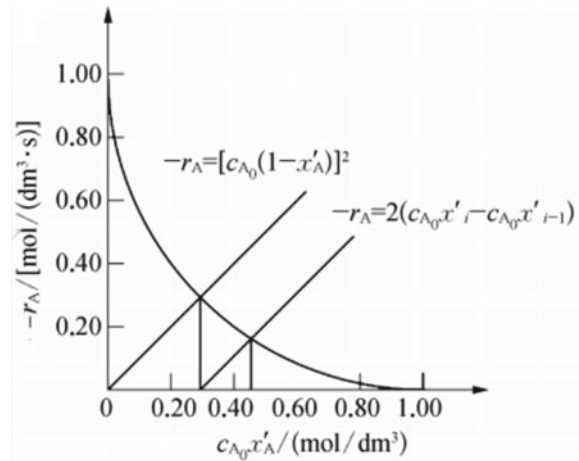


Fig. 5.26 Graphical result of Example 5.7



Example 5.7 Please calculate the final conversion efficiency of the chemical reaction of Example 5.6 through two continuous stirred tank reactors in series by a graphical method.

Solution:

$$\bar{\tau} = \frac{1.0}{2} = 0.5 \text{ s}$$

Because $k_c = 1.0 \text{ dm}^3/(\text{mol} \cdot \text{s})$,

$$-r_A = c_A^2$$

$$c_{A_0} = 1.00 \text{ mol/dm}^3$$

As shown in Fig. 5.26, the curve $-r_A = [c_{A_0}(1 - x'_A)]^2$ is plotted on the graph ($-r_A \sim c_{A_0}x'_A$). From $c_{A_0}x'_A = 0$, plot one straight line with a slope of $1/\bar{\tau} = 2.0$, $x'_1 = 0.27$ is obtained from the intersection. Then, another straight line is plotted from $c_{A_0}x'_1 = 0.27$ with a slope of 2.0, which intersects with line $-r_A = [c_{A_0}(1 - x'_A)]^2$ at a point with abscissa $c_{A_0}x'_A = 0.43$; thus, $x'_2 = 0.43$.

The graphical result is the same as that calculated by the algebraic method in Example 5.6.

5.2.4 Optimal Selection of Ideal Homogeneous Reactor

For the optimal selection of an ideal homogeneous reactor, different optimization objectives will result in different results. Even if the same optimization objectives but different reaction processes are adopted, the optimization results will be different. Due to the diversity of homogeneous reaction processes, it is impossible to study the optimization problems of their reactors one by one. This section only discusses the optimization objectives of production intensity and yield.

5.2.4.1 Take Production Intensity as the Optimization Objective

Production intensity refers to the production capacity of a unit volume reactor. The minimum effective volume required for the reactor represents its production strength when the material processing capacity and the final conversion efficiency are fixed. Under the same conditions, the volume of the reactor with higher production intensity is smaller.

In the previous examples, the volume required for isothermal homogeneous reactions in various ideal reactors has been discussed. The reaction volume of the plug flow reactor is the smallest, and that of the continuous stirred tank reactor is the largest. To make the comparison more intuitive, a graphical method is used, as shown in Fig. 5.27. As seen in the figure, to complete a certain production task and achieve a certain final conversion efficiency, the minimum volume is required by using the plug flow reactor, the maximum volume is required by using the continuous stirred tank reactor, and the multiple continuous stirred tank reactor lies in between. The reaction volume required by the multiple continuous stirred tank reactor decreases with the increase of series and tends to the reaction volume of the plug flow reactor. When the series is infinite, the reaction volume of the multiple continuous stirred tank reactor is equal to that of the plug flow reactor, which can be more vividly reflected from the Fig. 5.27. In addition, Fig. 5.27 shows that for reactors of the same

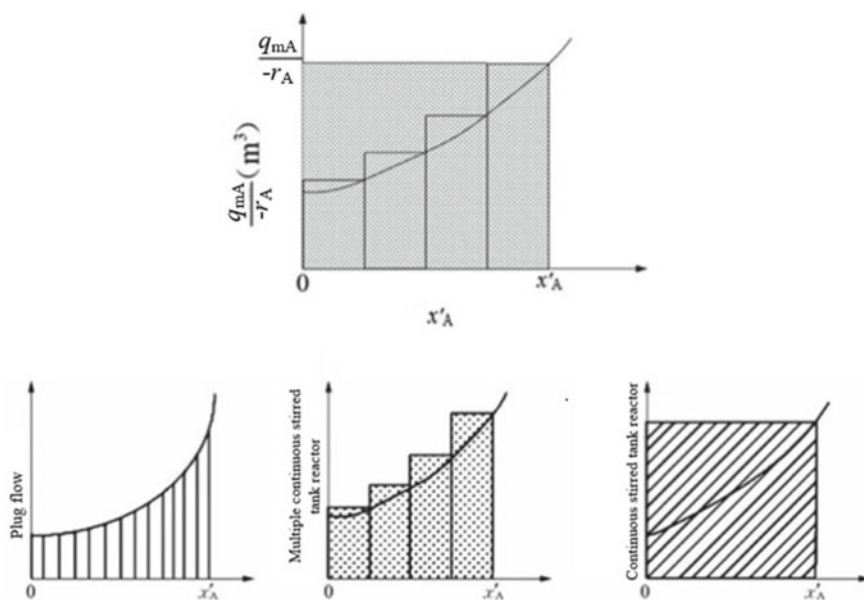
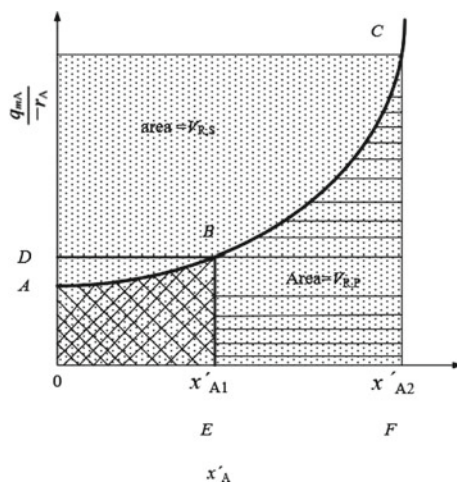


Fig. 5.27 Comparison of reaction volumes in ideal continuous flow reactors (Lin, 1999)

reaction type, the greater the final conversion efficiency of the reaction is, the larger the reaction volume required by the reactor.

Figure 5.28 graphically demonstrates the effective volume ($V_{R,P}$) of an ideal plug flow reactor required to achieve a certain conversion efficiency and effective volume ($V_{R,S}$) of an ideal continuous stirred tank reactor. When the conversion efficiency reaches x'_{A1} , $V_{R,P}$ is just the area of (*AOEB*) of a small piece below the curve, while $V_{R,S}$ is the area (*DOBE*) of the left rectangle of x'_{A1} . When the conversion efficiency

Fig. 5.28 The volume ratio of the reactor (Wuhan University, 2016)



reaches x'_{A2} , both of them have a large increase in area, but the increase of $V_{R, S}$ is greater than $V_{R, P}$.

If the dynamic equations are combined, more accurate and general conclusions can be obtained. Assume that for the same reaction, the ratio of the effective volume of a plug flow reactor to the effective volume of a continuous stirred tank reactor under the conditions of the same temperature, output and conversion efficiency is called the volume efficiency (or the effective utilization coefficient), expressed by η_e :

$$\eta_e = \frac{V_{R,P}}{V_{R,S}} = \frac{\tau_P}{\tau_S} \quad (5.88)$$

For a zero-order reaction,

$$\eta_e = \frac{V_{R,P}}{V_{R,S}} = \frac{c_{A0}x'_A/k}{c_{A0}x'_A/k} = 1 \quad (5.89)$$

For a first-order reaction,

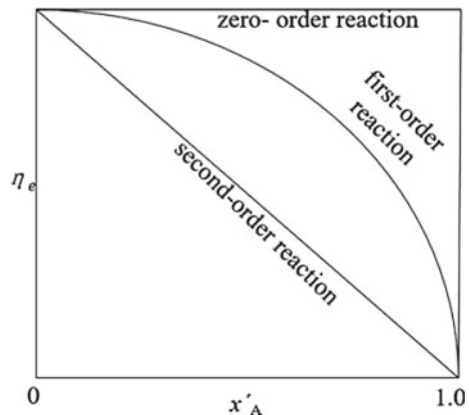
$$\eta_e = \frac{V_{R,P}}{V_{R,S}} = \frac{\ln\left(\frac{1}{1-x'_A}\right)/k}{\frac{x'_A}{1-x'_A}/k} = \frac{\ln\left(\frac{1}{1-x'_A}\right)}{\frac{x'_A}{1-x'_A}} \quad (5.90)$$

For a second-order reaction,

$$\eta_e = \frac{V_{R,P}}{V_{R,S}} = \frac{\frac{x'_A}{1-x'_A}/kc_{A0}}{x'_A/[kc_{A0}(1-x'_A)^2]} = 1 - x'_A \quad (5.91)$$

Figure 5.29 shows the relationship between η_e and x'_A for the above equations. The following can be obtained in the figure:

Fig. 5.29 Volumetric efficiency of a single reactor at a certain conversion efficiency (Lin, 1999)



- (1) For the zero-order reaction, $\eta_e = 1$, the reaction rate is only related to temperature, so the effective volumes required by the ideal continuous stirred tank, the ideal plug flow and the batch reactor (represented by reaction time) are equal; that is, the reaction rate has no direct correlation with the type of reactors.
- (2) Except for the zero-order reaction, the volume efficiency of reactors with first-, second- and other positive-order reactions is less than 1, and the volume efficiency decreases with increasing conversion efficiency. That is, when the conversion efficiency is large, the minimum effective volume required by the ideal continuous stirred tank reactor is much larger than that of the ideal plug flow or batch reactor.
- (3) Assuming that the conversion efficiency remains unchanged, the volume efficiency of the reactor decreases with increasing reaction order. That is, the minimum effective volume required for an ideal continuous stirred tank reactor increases significantly with increasing reaction order.
- (4) When the conversion efficiency is small and the reaction order is high, the volumetric efficiency is relatively large. Therefore, if the conversion efficiency of the reaction is required to be small, the effective volume of the ideal continuous stirred tank reactor is only slightly larger than that of the ideal plug flow or batch reactor. For stirred reactors, the operation of batch reactor requires some auxiliary time, so the ideal continuous stirred tank reactor has its particularly superior.

The effect of the number of multiple continuous stirred tank reactors on the total effective volume of the reactor can be similarly compared. For example, for the first-order reaction, the volumetric efficiency of several equal-volume multiple continuous stirred tank reactors is

$$\eta_e = \frac{\tau_P}{\tau_S} = \frac{\ln\left(\frac{1}{1-x'_A}\right)/k}{N\left[\left(\frac{1}{1-x'_A}\right)^{1/N} - 1\right]/k}$$

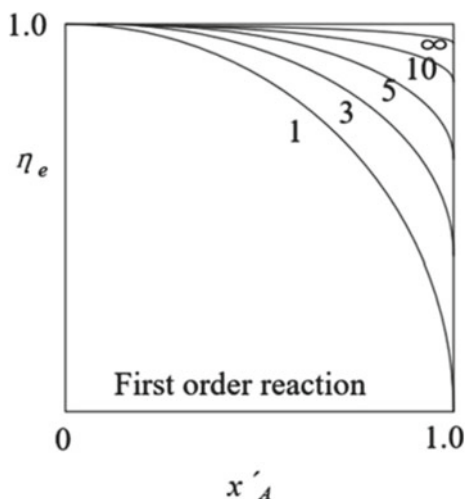
namely,

$$\eta_e = \frac{\ln\left(\frac{1}{1-x'_A}\right)}{N\left[\left(\frac{1}{1-x'_A}\right)^{1/N} - 1\right]} \quad (5.92)$$

The expression referenced in $\tau_s = N\left[\left(\frac{1}{1-x'_A}\right)^{1/N} - 1\right]/k$ is derived from Eq. (5.87).

If the number of reactors N becomes infinite, the time τ_S of multiple continuous stirred tank reactors can be derived mathematically, which is equal to the time of the

Fig. 5.30 Volumetric efficiency of constant volume multiple continuous stirred tank reactors (Lin, 1999)



ideal plug flow reactor τ_P , i.e. $\tau_S = \tau_P = \ln\left(\frac{1}{1-x'_A}\right)/k$; thus, $\eta_e = 1$. The above relationship is shown in Fig. 5.30.

Figure 5.30 illustrates that the volumetric efficiency increases with the number of equal-volume multiple continuous stirred tank reactors. The volumetric efficiency of a single reactor is the lowest, while the effect of increasing the number of multiple continuous stirred tank reactors is significant, but the marginal effect gradually decreases. Therefore, in practical applications, except for a few exceptions, the number of multiple continuous stirred tank reactors is generally no greater than 4.

5.2.4.2 Take Yield and Selectivity as Optimization Objectives

Sometime, from the perspective of cost investment, the proportion of reactor is very small, so it is not the primary consideration in the actual production process. Relatively speaking, the product yield has a greater impact on the selection of the technology flow, equipment design and overall investment accounting, and the yield is too low to conduct industrial production. Therefore, the reactor type and size are sometimes selected according to the yield and selectivity as the optimization objectives.

For a simple reaction with only one product, the product composition is single, and there is no a problem of the distribution of the final product. Therefore, only the production capacity of the reactor is generally considered to be optimized. However, side reactions often occur in many complex reactions, resulting in the formation of a variety of products. Reaction product distribution refers to the composition and proportion of the target product and various byproducts, which often changes with the conditions of the reaction process. Therefore, when selecting reactor type, the yield and selectivity of target products are more important than the production intensity.

In the actual production process, the yield and the reactor production capacity are contradictory. When both sides cannot be addressed, the appropriate reactor type and

operation mode should be chosen from the perspective of technical and economic considerations.

Here, the definitions of several commonly used characterization product distributions and their relations are introduced:

$$\text{Yield}(\varphi_y) = \frac{\text{Amount of reactant converted to target product}}{\text{Amount of reactants entering the reactor}}$$

$$\text{Selectivity}(\beta_s) = \frac{\text{Amount of reactant converted to the target product}}{\text{Amount of reactant converted to target product and byproduct}}$$

The conversion efficiency that is often used before is:

$$\begin{aligned} \text{Conversion efficiency}(x''_A) &= \frac{\text{Amount of reactant converted to target product and byproduct}}{\text{Amount of reactants entering the reactor}} \\ &= \frac{\text{Amount consumed by reaction}}{\text{Initial amount of reactant}} \end{aligned}$$

If the raw material does not circulate back to the reactor after the reaction, the relationship of the above three is:

$$\text{Yield} = \text{Selectivity} \times \text{Conversion efficiency}$$

namely,

$$\varphi_y = \beta_s x''_A$$

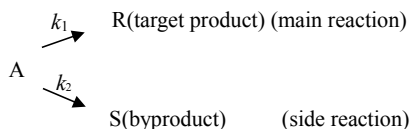
The yield (φ_y) usually represents the utilization efficiency of raw materials in industrial production, but selectivity (β_s) is often used in the process of theoretical research and discussion.

There are various types of complex reactions. Parallel reactions and consecutive reactions are typical complex reactions, and their combined processes can also constitute new complex reactions. In the following, how to improve the yield of the above two kinds of reactions by some factors, such as reactant concentration at a certain temperature, will be discussed.

1) Parallel reactions

Parallel reaction refers to the reaction in which one reactant is known to produce the target product and another byproduct at the same time. For example, the isomerization and dimerization of a hydrocarbon compound occur simultaneously under isothermal conditions. Many substitution reaction, addition reactions, and decomposition reactions have parallel reactions.

Assume there is a parallel reaction:



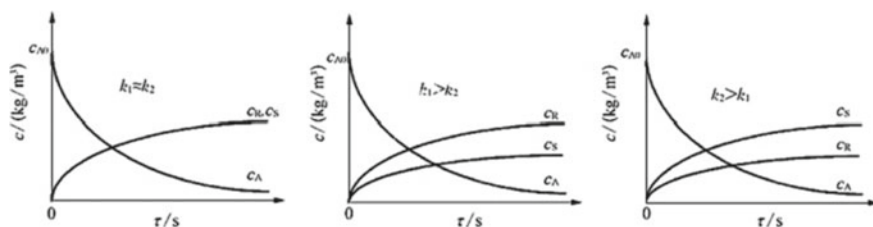


Fig. 5.31 Variety of the concentration of reactants and products over time in parallel reaction

Their reaction rates equation are

$$r_R = \frac{dc_R}{d\tau} = k_1 c_A^{n'_1}$$

$$r_S = \frac{dc_S}{d\tau} = k_2 c_A^{n'_2}$$

The relationship of the changes in the concentration of reactants and products over time τ is shown in Fig. 5.31.

If r_R/r_S represents the yield of target product R, the following equation can be obtained by dividing the above two equations:

$$\frac{r_R}{r_S} = \frac{k_1 c_A^{n'_1}}{k_2 c_A^{n'_2}} = \frac{k_1}{k_2} c_A^{n'_1 - n'_2} \quad (5.93)$$

Under certain reaction conditions, k_1 and k_2 and reaction orders n'_1 and n'_2 are all constant, and only the concentration c_A is a variable. The yield of target product R is positively correlated with the value of $c_A^{n'_1 - n'_2}$. As mentioned above, the concentration of reactant A in the ideal plug flow and batch reactor is higher than that in the ideal continuous stirred tank reactor. The higher the concentration of the reactant, the more favorable it is for the higher the reaction order, and vice versa.

To enlarge $c_A^{n'_1 - n'_2}$, different types of reactors can be selected, or other measures can be taken according to different circumstances.

When $n'_1 < n'_2$, it is advisable to choose a continuous stirred tank reactor.

When $n'_1 > n'_2$, the plug flow reactor or batch reactor is appropriate. If for some reasons, a continuous stirred tank reactor must be used, a multiple continuous stirred tank reactors should also be used.

To improve the yield of R, a larger or smaller c_A can be maintained in addition to choosing different reactors.

The following measures can be taken to maintain a larger c_A : ① Adopt a small one-way conversion efficiency. ② Use a higher concentration of feed. ③ For a gas phase reaction, increase the pressure of the system.

The following measures can be taken to maintain a smaller c_A : ① Design a higher conversion efficiency. ② Use the material cycle after the reaction to reduce the

concentration of reactants in the feed. ③ Add an inert diluent. ④ For a gas phase reaction, reduce the system pressure.

When $n'_1 = n'_2$, Eq. (5.93) becomes $\frac{r_R}{r_S} = \frac{k_1}{k_2} = \text{constant}$, and the change in concentration will not affect the yield of target product R, so the type of reactor will not affect the yield of R. In this case, k_1 and k_2 can only be altered by changing the temperature or by using a catalyst to increase the yield of R.

For following type of parallel reactions



The reaction rate equations are as follows.

$$r_R = \frac{dc_R}{d\tau} = k_1 c_A^{n'_1} c_B^{\gamma_1}$$

$$r_S = \frac{dc_S}{d\tau} = k_2 c_A^{n'_2} c_B^{\gamma_2}$$

After dividing the above two equations:

Table 5.5 Suitable operation selected according to reaction order

The magnitude of the reaction order	The concentration requirement	Suitable reactor type and operation method
$n'_1 > n'_2, \gamma_1 > \gamma_2$	c_A, c_B are all high	Batch reactor, ideal plug flow reactor or multiple continuous stirred tank reactors. A and B are added at the same time
$n'_1 > n'_2, \gamma_1 < \gamma_2$	c_A is high; c_B is low	Divide B into smaller strands, and then add into multiple continuous stirred tank reactors, respectively. Either B is added along the length of the pipe in a plug flow reactor, or B is supplied to the semi-continuous reactor successively
$n'_1 < n'_2, \gamma_1 > \gamma_2$	c_A is low; c_B is high	A is divided into small strands and added into the multiple continuous stirred tank reactors, or A is added along the length of the pipe in a plug flow reactor, or A is added successively to the semi-continuous reactor
$n'_1 < n'_2, \gamma_1 < \gamma_2$	c_A, c_B are all low	Ideal continuous stirred tank reactor; either slowly drop A and B into a batch reactor, or use a diluent to lower the concentration of both c_A and c_B

$$\frac{r_R}{r_S} = \frac{\frac{dc_R}{d\tau}}{\frac{dc_S}{d\tau}} = \frac{k_1}{k_2} c_A^{n'_1 - n'_2} c_B^{\gamma_1 - \gamma_2} \quad (5.94)$$

To improve the yield of R, the right term of Eq. (5.94) should be made as large as possible, and an appropriate operation method can be developed according to the level of each reaction order, as shown in Table 5.5. In addition, during the reaction, a certain reactant is overly added to keep its concentration basically unchanged. After the reaction, the reactant can be separated and recycled to convert the second-order reaction into a pseudo-first-order reaction.

Figure 5.32 shows a schematic diagram of various types of reactors and feeding

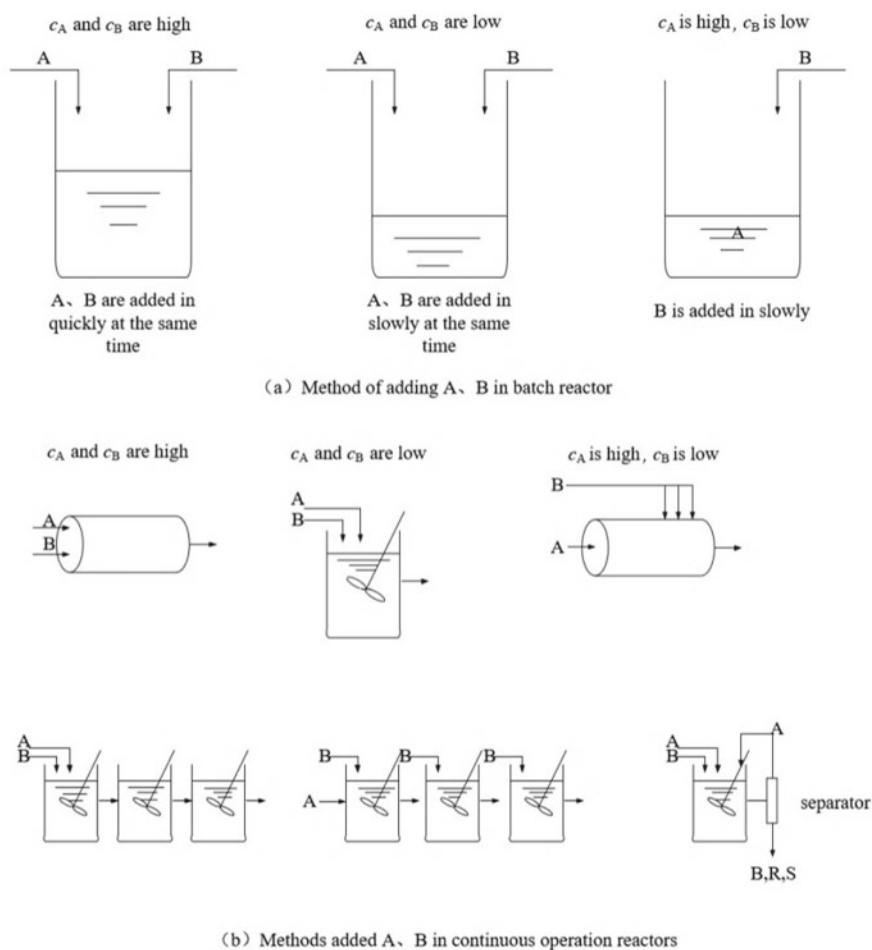
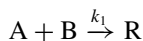


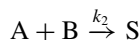
Fig. 5.32 Schematic diagram of various types of reactors and feeding operation methods

operation methods. However, for parallel reactions, at some temperature, the concentration of the reactants is directly related to the distribution of the products. The higher the concentration of the reactants is, the better the higher the reaction order in parallel reactions, and vice versa. For the same orders of reactions, the concentration does not affect the distribution of products, that is, it does not affect the yield.

Example 5.8 The following liquid-phase parallel reactions exist:



$$\frac{dc_R}{d\tau} = k_1 c_A c_B^{0.3} \quad (\text{Main reaction, target product})$$



$$\frac{dc_S}{d\tau} = k_2 c_A^{0.5} c_B^{1.8}$$

Which reactor and operation method are most appropriate?

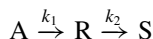
Solution: According to Eq. (5.94)

$$\frac{r_R}{r_S} = \frac{k_1}{k_2} c_A^{1-0.5} c_B^{0.3-1.8} = \frac{k_1}{k_2} c_A^{0.5} c_B^{-1.5}$$

Answer: To increase the yield of product R, the ratio of r_R/r_S should be improved by increasing the concentration of A (c_A) and reducing the concentration of B (c_B). The concentration of reactant B has a greater effect on the yield of R than reactant A. According to the above principles, an ideal plug flow reactor should be adopted, with reactant A as the main stream and reactant B added from different parts of the reactor, or it is most appropriate to use a batch reactor by adding all reactant A to the reactor and inputting reactant B gradually through semicontinuous operation.

2) Consecutive reaction

A consecutive reaction is one in which after a target product is formed, a byproduct will be formed further. Many hydrolysis reactions, halogenation reactions, chlorination reactions and radiation reactions are consecutive reactions. Assume the consecutive reaction to be:



The rate equation for each reaction is:

$$-r_A = -\frac{dc_A}{d\tau} = k_1 c_A$$

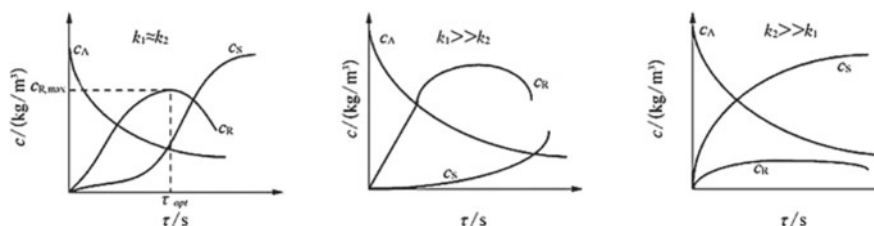


Fig. 5.33 Schematic diagram of concentration change of consecutive reaction

$$r_R = \frac{dc_R}{d\tau} = k_1 c_A - k_2 c_R$$

$$r_S = \frac{dc_S}{d\tau} = k_2 c_R$$

The relationship between the concentration of various materials and time is shown in Fig. 5.33. In this diagram, the concentration of reactant A (c_A) decreases continuously with time. The concentration of byproduct S increases with the reaction time, and the concentration of target product R (c_R) increases first and then decreases with the reaction time. In the earlier stage of the reaction, because c_A is high, c_R is low, and $k_1 c_A > k_2 c_R$, leading to an increase of c_R . In the latter period of the reaction, because c_A is low, c_R is higher, and $k_1 c_A < k_2 c_R$, leading to the reduction of c_R . Therefore, there must be an optimal time in the whole reaction process when the concentration of the target product c_R reaches the highest value, as shown in the figure with the $c_{R, \max}$ value of τ_{opt} .

Therefore, to achieve the maximum yield of the target product R, the reaction time should be strictly controlled. In the batch reactor, there is no material backmixing, all material particles experience the same time in the reactor, and the reaction time can be strictly controlled. In the plug flow reactor, the residence time of the material can be appropriately controlled because the material does not backmix at all, so the yield of the target product is close to the maximum. In a continuous stirred tank reactor, the material particles with different residence times are fully mixed, which leads to complete backmixing. Even if the material is maintained under the condition that the mean residence time is equal to the optimal time, the residence time of each particle in the material is still different; some are longer than the optimal time, and some are shorter. In this way, the target product yield will inevitably deviate from the maximum. Therefore, for the consecutive reaction mentioned above, it is advisable to choose a batch kettle reactor and a plug flow reactor.

5.3 Microbial Reactor

In environmental engineering facilities, especially large wastewater treatment plants, biological treatment is adopted in general, which relies on microorganisms (especially all kinds of bacteria) to convert pollutants into harmless or useful final products

(such as CO_2 , H_2O , N_2 , or CH_4) in the reactor. The operation mode of the reactor can be categorized as a batch, continuous flow, semi-continuous flow or sequential batch type. According to the flow and mixing state of reactants, reactors can also be classified as ideal flow reactors or nonideal flow reactors. An ideal flow reactor can be a continuous stirred tank reactor or a plug flow reactor. The reaction mixture can occur in a homogeneous reactor or a heterogeneous reactor according to its phase state. Homogeneous reactors are characterized by reactions occurring only in one phase, usually in a gas or liquid. When there are more than two phases in the reactor, it is called a heterogeneous reactor. In microbial reactors, such as activated sludge in the aeration tank, microbial bacteria (mainly) are solid, pollutants are mostly in the liquid phase, and air (oxygen) is supplied in the gas phase. Thus, the process is actually a heterogeneous reaction, and microbial cells themselves are a complex heterogeneous reactor, but it usually is simplified as an ideal homogeneous reaction in the liquid phase. The material balance equation based on the law of conservation of both matter and energy is still the basic equation of microbial reactor calculation. However, the corresponding reaction rate equation, which needs to be substituted into the material balance equation, is usually Monod equation. Then, boundary conditions are determined according to the characteristics of the operation mode of the reactor to obtain the time required to reach a certain pollutant concentration or removal efficiency. Finally, the corresponding volume of the microbial reactor can be obtained according to the amount of treatment.

5.3.1 Batch Microbial Reactor

The process of microbial growth in batch culture is complex, and it is difficult to describe the whole process with a simple model. Under the condition that there is only one restricted substrate, Monod equation can be used to describe the three growth stages of exponential, stationary, decline phases.

It is assumed that the growth of microorganisms conforms to Monod equation and that the cell yield coefficient $Y_{X/S}$ is a constant. Then, the material balance equation of microbial cell growth and substrate S is as follows:

$$\frac{d[X_C]}{d\tau} = r_X = \mu_B[X_C] = \frac{\mu_{B\max}[S]}{K_S + [S]}[X_C] \quad (5.95)$$

$$-\frac{d[S]}{d\tau} = -r_S = \frac{r_X}{Y_{X/S\max}} + m_k[X_C] \quad (5.96)$$

where

$[X_C]$ —Microbial concentration in the reactor at a certain time, kg/m^3

τ —Time, h

r_X —Microbial growth rate, $\text{kg}/(\text{m}^3 \cdot \text{h})$

μ_B —Specific growth rate of microorganism, h^{-1}

$\mu_{B\max}$ —Maximum specific growth rate of microorganism, h^{-1}

$[S]$ —Substrate (or matrix) concentration in the reactor, kg/m^3

K_S —Saturation coefficient, kg/m^3

r_S —Substrate consumption rate, $\text{kg}/(\text{m}^3 \cdot \text{h})$

$Y_{X/S}$ —Apparent microbial yield coefficient, $\text{kg-microorganism}/\text{kg-substrate}$ utilized

$Y_{X/S\max}$ —Maximum microbial yield coefficient without metabolism, $\text{kg-microorganism}/\text{kg-substrate}$ utilized

m_k —Endogenous respiration coefficient or maintenance coefficient, the amount of microorganisms consumed by endogenous respiration per unit time, $\text{kg-substrate}/(\text{kg-microorganism} \cdot \text{h})$

Under the condition of $\tau = 0$, $[X_C] = [X_C]_0$, $[S] = [S]_0$, the simultaneous equations of Eqs. (5.95) and (5.96) can be solved to find the variation of $[X_C]$ and $[S]$ with time. Since the above differential equations are difficult to solve, numerical analysis is generally required to obtain the solution. To facilitate the analysis, some simplifications are often made in practical applications.

In most cases, the value of the endogenous respiration coefficient (m_k) is very small, and the term $m_k[X_C]$ can be ignored. If $Y_{X/S}$ is used as a substitute for $Y_{X/S\max}$, Eq. (5.96) can be simplified as follows:

$$-\frac{d[S]}{d\tau} = \frac{r_X}{Y_{X/S}} \quad (5.97)$$

Equations (5.95) and (5.97) can lead to

$$-\frac{d[S]}{d\tau} = \frac{1}{Y_{X/S}} \frac{d[X_C]}{d\tau} \quad (5.98)$$

$$-d[S] = \frac{1}{Y_{X/S}} d[X_C] \quad (5.99)$$

Assuming that $Y_{X/S}$ does not change with time during the culture process, the integral of Eq. (5.99) can be obtained as follows:

$$[S]_0 - [S] = \frac{1}{Y_{X/S}} ([X_C] - [X_C]_0) \quad (5.100)$$

$$[S] = \frac{[S]_0 Y_{X/S} - [X_C] + [X_C]_0}{Y_{X/S}} \quad (5.101)$$

By defining $[X_C]'' = [X_C]_0 + [S]_0 Y_{X/S}$, Eq. (5.101) can then be rewritten as

$$[S] = \frac{[X_C]'' - [X_C]}{Y_{X/S}} \quad (5.102)$$

By substituting Eq. (5.102) into Eq. (5.95) and integrating under the conditions of $\tau = 0$, $[X_C] = [X_C]_0$, and Eq. (5.103a) can be obtained.

$$\left(1 + \frac{K_S Y_{X/S}}{[X_C]''}\right) \ln \frac{[X_C]}{[X_C]_0} - \frac{K_S Y_{X/S}}{[X_C]''} \ln \frac{[X_C]'' - [X_C]}{[X_C]'' - [X_C]_0} = \mu_{B_{\max}} \tau \quad (5.103a)$$

If $K_S \ll [S]_0$ and $[X_C]'' \approx [X_C]_0$, Eq. (5.103a) can be simplified to

$$\ln \frac{[X_C]}{[X_C]_0} = \mu_{B_{\max}} \tau \quad (5.103b)$$

According to Eqs. (5.103a) or (5.103b), different times τ and $[X_C]$ values can be calculated. By substituting $[X_C]$ into Eqs. (5.101) or (5.102), the corresponding $[S]$ value can be obtained.

Example 5.9 Glycerol is used as the substrate for a batch culture of *Aerobacter cloacae*. When $\tau_0 = 0$ and $[X_C]_0 = 0.1 \text{ g/L}$, $[S]_0 = 50 \text{ kg/m}^3$, and the reaction equation can be expressed by Monod equation, in which $\mu_{B_{\max}} = 0.85 \text{ h}^{-1}$, $K_S = 1.23 \times 10^{-2} \text{ kg/m}^3$, and $Y_{X/S} = 0.53 \text{ kg/kg}$. Assuming that there is no induction or decay period in the bacterial culture process, please determine the concentration of *Aerobacter cloacae* in the culture medium after 6 h of cultivation.

Solution:

The definition of bacterial yield can be substituted with the known values as follows:

$$Y_{X/S} = \frac{[X_C] - [X_C]_0}{[S]_0 - [S]} = \frac{[X_C] - 0.1}{50 - [S]} = 0.53 \quad (1)$$

Therefore, the following relationship exists between the substrate and bacterial concentration:

$$[S] = -1.89[X_C] + 50.2 \quad (2)$$

From the deformation of equation ①,

$$[S] = [S]_0 - \frac{[X_C] - [X_C]_0}{Y_{X/S}} \quad (3)$$

Because

$$r_X = \frac{d[X_C]}{d\tau} = \mu_B [X_C] = \frac{\mu_{B_{\max}} [S]}{K_S + [S]} \cdot [X_C] \quad (4)$$

Substituting equation ③ into equation ④ and then integrating obtains:

$$\tau - \tau_0 = \frac{1}{\mu_{B_{\max}}} \left[\left(\frac{Y_{X/S} K_S}{Y_{X/S} [S]_0 + [X_C]_0} + 1 \right) \ln \frac{[X_C]}{[X_C]_0} - \left(\frac{Y_{X/S} K_S}{Y_{X/S} [S]_0 + [X_C]_0} \right) \ln \frac{[S]}{[S]_0} \right] \quad (5)$$

Then, substituting the known conditions into equation ⑤, that is,

$$6 = \frac{1}{0.85} \left[\left(\frac{0.53 \times 1.23 \times 10^{-2}}{0.53 \times 50 + 0.1} + 1 \right) \ln \frac{[X_C]}{0.1} - \left(\frac{0.53 \times 1.23 \times 10^{-2}}{0.53 \times 50 + 0.1} \right) \ln \frac{-1.89[X_C] + 50.2}{50} \right]$$

The following can be obtain: $[X_C] = 16.4 \text{ kg/m}^3$.

Example 5.10 A batch culture of *Escherichia coli* is carried out in a bioreactor with effective volume of 1 m^3 . It is assumed that the growth of the bacteria and the utilization of substrates conform to Monod equation. It is known that $\mu_{B_{\max}} = 0.935 \text{ h}^{-1}$,

$K_S = 0.71 \text{ kg/m}^3$ and the initial concentration of the restricted substrate is 50 kg/m^3 . In addition, the inoculation concentration of *E. coli* is $[X_C]_0 = 0.1 \text{ kg/m}^3$, $Y_{X/S} = 0.6 \text{ kg/kg}$ (in terms of cells/substrate). Please calculate how long it takes for the substrate concentration to drop to one-fifth of its initial value.

Solution:

When the substrate concentration drops to the one-fifth of its initial value,

$$[S] = (1/5)[S]_0 = (1/5) \times 50 = 10 \text{ kg/m}^3.$$

$$Y_{X/S} = ([X_C] - [X_C]_0)/([S]_0 - [S])$$

$$0.6 = ([X_C] - 0.1)/(50 - 10)$$

$$[X_C] = 24.1 \text{ kg/m}^3$$

In the whole reaction process, $[S] \gg K_S = 0.71 \text{ kg/m}^3$, Monod equation can be simplified as:

$$r_X = d[X_C]/d\tau = \mu_{B\max}[X_C] = 0.935[X_C]$$

$$d\tau = d[X_C]/0.935[X_C]$$

$$\begin{aligned} \int_0^\tau d\tau &= \int_{[X_C]_0}^{[X_C]} \frac{1}{0.935[X_C]} d[X_C] \\ \tau &= \int_{0.1}^{24.1} \frac{d[X_C]}{0.935[X_C]} = \frac{1}{0.935} \ln\left(\frac{24.1}{0.1}\right) = 5.87 \text{ h} \end{aligned}$$

Answer: It takes 5.87 h for the substrate concentration to drop to one-fifth of its initial value.

5.3.2 Continuous Microbial Reactor

Compared with batch culture operations, continuous operations have many advantages. The activated sludge treatment of wastewater is a typical example. There are two main types of continuous operation, namely, continuous stirred tank reactors and plug flow reactors. As discussed earlier, the time required for a plug flow reactor to reach a certain pollutant concentration or removal efficiency is the same as that required for a batch reactor. This book will not discuss the plug flow reactor, but the continuous stirred tank reactor is introduced below.

After defining the boundary conditions of the reactor system at any time of culture, the corresponding material balance of bacteria, restricted substrate or products can be done according to the following equation:

$$\begin{aligned} \text{System accumulation} &= \text{System input} - \text{System output} \\ &\quad \pm \text{Product generated/reactant consumption} \end{aligned}$$

If the material to be balanced is a product of the system (microorganisms or metabolites), the positive sign is used. If the material to be balanced is the reactant of

the system (the restricted substrate), the negative sign is used. For the microorganisms, the concentration in influent is too low and can be basically ignored. Generally, the concentration of microorganisms in the reactor can be calculated by the following mathematical expression:

$$V \frac{d[X_C]}{d\tau} = V\mu_B[X_C] - q_V[X_C] \quad (5.104)$$

For the substrate, its concentration can be calculated as follows:

$$V \frac{d[S]}{d\tau} = q_V([S]_{in} - [S]) - V\gamma[X_C] \quad (5.105)$$

Generally, microbial metabolites are not included in the influent, so the product concentration can be calculated by the following mathematical expression:

$$V \frac{d[P]}{d\tau} = V\Gamma[X_C] - q_V[P] \quad (5.106)$$

where

V —Effective volume of the reactor, m^3

q_V —Inflow and outflow rate, m^3/h

$[S]_{in}$ —Restricted substrate concentration in influent, kg/m^3

$[S]$ —Restricted substrate concentration in the reactor or effluent, kg/m^3

$[X_C]$ —Concentration of the microorganisms in the reactor or effluent, kg/m^3

τ —Time, h

γ —Specific substrate consumption rate, $-\gamma = -r_S/X_C$, $kg(\text{substrate})/[kg(\text{microorganisms}) \cdot h]$

$[P]$ —Concentration of product in the reactor or effluent, kg/m^3

Γ —Specific formation rate of product, $\Gamma = Y_{P/X} \cdot \frac{\mu_{Bmax}[S]}{K_s + [S]}$, h^{-1}

μ_B —Specific growth rate of microorganism, h^{-1} .

In Eqs. (5.104) ~ (5.106), divide both sides by V , then,

$$\frac{d[X_C]}{d\tau} = \mu_B[X_C] - D_{dilution}[X_C] \quad (5.107)$$

$$\frac{d[S]}{d\tau} = D_{dilution}([S]_{in} - [S]) - \gamma[X_C] \quad (5.108)$$

$$\frac{d[P]}{d\tau} = \Gamma[X_C] - D_{dilution}[P] \quad (5.109)$$

where

$D_{dilution}$ —Dilution rate, h^{-1} ,

$$D_{\text{dilution}} = \frac{q_V}{V} \quad (5.110)$$

According to the definitions of microbial yield ($Y_{X/S}$) and product yield ($Y_{P/S}$), as well as Monod equation, Eqs. (5.107) ~ (5.109) can be rewritten as:

$$\frac{d[X_C]}{d\tau} = \left(\frac{\mu_{B\max}[S]}{K_s + [S]} - D_{\text{dilution}} \right) [X_C] \quad (5.111)$$

$$\frac{d[S]}{d\tau} = D_{\text{dilution}}([S]_{\text{in}} - [S]) - \frac{X_c}{Y_{X/S}} \frac{\mu_{B\max}[S]}{K_s + [S]} \quad (5.112)$$

$$\frac{d[P]}{d\tau} = \frac{Y_{P/X}\mu_{B\max}[S]}{K_s + [S]} [X_C] - D_{\text{dilution}}[P] \quad (5.113)$$

where

$$Y_{P/X} = \frac{Y_{P/S}}{Y_{X/S}}$$

K_s —Saturation coefficient, kg/m^3

$\mu_{B\max}$ —Maximum specific growth rate of microorganism, h^{-1}

$Y_{X/S}$ —Apparent microbial yield coefficient, $\text{kg}(\text{microorganism})/\text{kg}(\text{substrate utilized})$

$[P]$ —Concentration of product in the reactor or effluent, kg/m^3

$[S]$ —Concentration of limited substrates in the reactor or effluent, kg/m^3

$Y_{P/S}$ —Product yield coefficient, $\text{kg}(\text{product})/\text{kg}(\text{substrate utilized})$

$Y_{P/X}$ —Microbial yield coefficient, $\text{kg}(\text{product})/\text{kg}(\text{microorganism})$

D_{dilution} —Dilution rate, h^{-1} .

The above three equations Eqs. (5.111 ~ 5.113) show the concentration of microorganisms, substrates, metabolites are related to the specific growth rate of microorganisms in a continuous stirred tank reactor. If the continuous stirred tank reactor runs in a steady state, then

$$\frac{d[X_C]}{d\tau} = \frac{d[S]}{d\tau} = \frac{d[P]}{d\tau} = 0 \quad (5.114)$$

At this moment, the concentrations of microorganisms, substrates and metabolites can be expressed as follows:

$$\overline{[X_C]} = Y_{X/S} \left([S]_{\text{in}} - \frac{K_s D_{\text{dilution}}}{\mu_{B\max} - D_{\text{dilution}}} \right) \quad (5.115)$$

$$\overline{[S]} = \frac{K_s D_{\text{dilution}}}{\mu_{B\max} - D_{\text{dilution}}} \quad (5.116)$$

$$\overline{[P]} = Y_{P/S} \left([S]_{in} - \frac{K_S D_{dilution}}{\mu_{Bmax} - D_{dilution}} \right) \quad (5.117)$$

These equations show the relationship between the dilution rate and the concentration of each material. Under a steady state, it can be seen from Eq. (5.107) that

$$\overline{\mu}_B = D_{dilution} \quad (5.118)$$

Since $D_{dilution}$ is equal to q_V/V , when V is constant, the value of μ_B can control the flow rate q_V for getting some values of materials. The concentrations of microorganisms and metabolites in the stable state are given by Eqs. (5.115) and (5.117), that is,

$$D_{dilution} \overline{[X_C]} = Y_{X/S} D_{dilution} \left([S]_{in} - \frac{K_S D_{dilution}}{\mu_{Bmax} - D_{dilution}} \right) \quad (5.119)$$

$$D_{dilution} \overline{[P]} = Y_{P/S} D_{dilution} \left([S]_{in} - \frac{K_S D_{dilution}}{\mu_{Bmax} - D_{dilution}} \right) \quad (5.120)$$

The dilution rate of $D_{dilution \max}$ at the highest yield is obtained according to Eq. (5.120).

$$D_{dilution \max} = \mu_{Bmax} (1 - \sqrt{K_S / (K_S + [S]_{in})}) \quad (5.121)$$

Substitute Eq. (5.121) into Eqs. (5.115) and (5.117) respectively, metabolite concentration $[P]_{\max}$ and the concentration of bacteria $\overline{[P]}_{\max}$ can be obtained at the highest yield.

$$\overline{X}_{C \max} = Y_{X/S} \left[[S]_{in} + K_S - \sqrt{K_S \times (K_S + [S]_{in})} \right] \quad (5.122)$$

$$\overline{[P]}_{\max} = Y_{P/S} \left[[S]_{in} + K_S - \sqrt{K_S \times (K_S + [S]_{in})} \right] \quad (5.123)$$

where

$\overline{[X_C]}$ —Microorganism concentration in the reactor or effluent in the stable state, kg/m³

$\overline{[P]}$ —Concentration of product in the reactor or effluent in the stable state, kg/m³

μ_B —Specific growth rate of microorganisms in the stable state, h⁻¹

$[S]_{in}$ —Concentration of limited substrates in the influent, kg/m³

$D_{dilution \max}$ —Dilution rate at the highest yield in the stable state, h⁻¹

$\overline{[P]}_{\max}$ —Maximum of concentration of product at the highest yield in the stable state, kg/m³

$\bar{X}_{C\max}$ —The concentration of microorganisms at the highest yield in the stable state, kg/m^3 .

Example 5.11 Please determine the dilution rate $D_{\text{dilution max}}$, the concentration of microorganisms $\bar{X}_{C\max}$ and the substrate concentration $[\bar{S}]$ and product concentration $[\bar{P}]_{\max}$ at the highest yield for an antibiotic fermentation in a continuous stirred tank reactor running under the stable state. It is known that the growth of microorganisms can be expressed by Monod equation under the conditions of $[S]_{\text{in}} = 30 \text{ kg}/\text{m}^3$, $Y_{X/S} = 0.45$, $Y_{P/S} = 0.1$, $\mu_{B\max} = 0.18 \text{ h}^{-1}$, $K_S = 1.0 \text{ kg}/\text{m}^3$.

Solution:

According to Eq. (5.121), the dilution rate $D_{\text{dilution max}}$ at the highest yield can be obtained as follows:

$$D_{\text{dilution max}} = \mu_{B\max} (1 - \sqrt{K_S / (K_S + [S]_{\text{in}})}) = 0.18 \times (1 - \sqrt{1 / (1 + 30)}) = 0.15 \text{ h}^{-1}$$

According to Eq. (5.122), the concentration of microorganisms $\bar{X}_{C\max}$ at the highest yield can be obtained as follows:

$$\begin{aligned} \bar{X}_{C\max} &= Y_{X/S} [[S]_{\text{in}} + K_S - \sqrt{K_S \times (K_S + [S]_{\text{in}})}] \\ &= 0.45 \times (30 + 1 - \sqrt{1 \times (1 + 30)}) = 11.44 \text{ kg}/\text{m}^3 \end{aligned}$$

According to Eq. (5.116), the substrate concentration $[\bar{S}]$ at the highest yield can be obtained as follows:

$$[\bar{S}] = \frac{K_S D_{\text{dilution max}}}{\mu_{B\max} - D_{\text{dilution max}}} = \frac{0.15 \times 1.0}{0.18 - 0.15} = 5.0 \text{ kg}/\text{m}^3$$

According to Eq. (5.123), product concentration $[\bar{P}]_{\max}$ at the highest yield can be obtained as follows:

$$\begin{aligned} [\bar{P}]_{\max} &= Y_{P/S} [[S]_{\text{in}} + K_S - \sqrt{K_S \times (K_S + [S]_{\text{in}})}] \\ &= 0.1 \times (30 + 1 - \sqrt{1 \times (1 + 30)}) = 2.54 \text{ kg}/\text{m}^3 \end{aligned}$$

Answer: The dilution factor $D_{\text{dilution max}}$ is 0.15 h^{-1} . The concentrations of microorganisms $\bar{X}_{C\max}$, the substrate concentration $[\bar{S}]$, and product $[\bar{P}]_{\max}$ at the highest yield are $11.44 \text{ kg}/\text{m}^3$, $5.0 \text{ kg}/\text{m}^3$, and $2.54 \text{ kg}/\text{m}^3$, respectively.

Exercises

Part I Multiple choice and fill in the blanks

- For a steady flow, when the chemical reaction order is zero, the volume of the ideal plug flow reactor is () the volume of the continuous stirred tank reactor.
 - larger than
 - smaller than
 - the same as

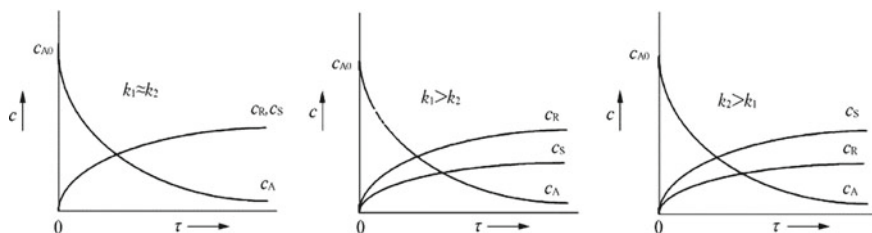


Fig. 5.34 Schematic diagram of the change of reaction concentration with reaction time

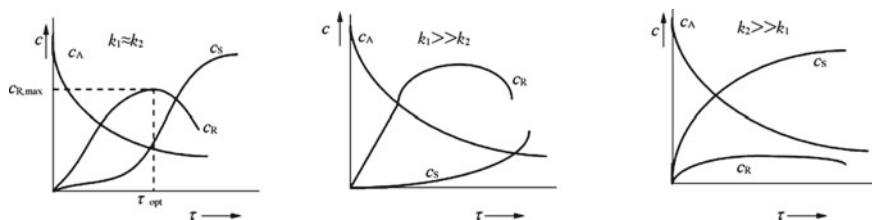


Fig. 5.35 Schematic diagram of the change of reaction concentration with reaction time

- For zero-order reaction, the reaction rate constant is () with the reactant concentration. For the first-order reaction, the reaction rate constant is () with the reactant concentration.
 - related
 - irrelevant
- Figure 5.34 shows the change of reactant concentration with time for () reaction. Figure 5.35 shows the change of reactant concentration with time for () reaction.
 - first-order
 - second-order
 - consecutive
 - parallel
- In the production process, if the raw material does not circulate back to the reactor after the reaction, the relationship between the yield, selectivity and conversion efficiency is ().
 - yield = selectivity \times conversion efficiency
 - selectivity = yield \times conversion efficiency
- The solution for the order of a chemical reaction does not include the ().
 - graphical differential method
 - differential method

- C. half-life method
D. material balance method
6. Figure 5.36 shows a schematic diagram of the performance and equation of the () reactor, and Fig. 5.37 shows a schematic diagram of the performance and equation of the () reactor.
- A. ideal plug flow
B. batch
C. continuous stirred tank
D. multiple continuous stirred tank
7. Please fill in the corresponding equation for calculating the time required to achieve a certain conversion efficiency in Tables 5.6, 5.7 and 5.8.
- A. $\tau = \frac{x'_A c_{A0}}{k}$
B. $\tau = \frac{1}{k} \frac{x'_A}{c_{A0}(1 - x'_A)}$
C. $\tau = \frac{1}{k} \frac{x'_A}{1 - x'_A}$
D. $\tau = \frac{1}{k} \ln \frac{1}{1 - x'_A}$
E. $\tau = \frac{1}{k} \frac{x'_A}{c_{A0}(1 - x'_A)^2}$

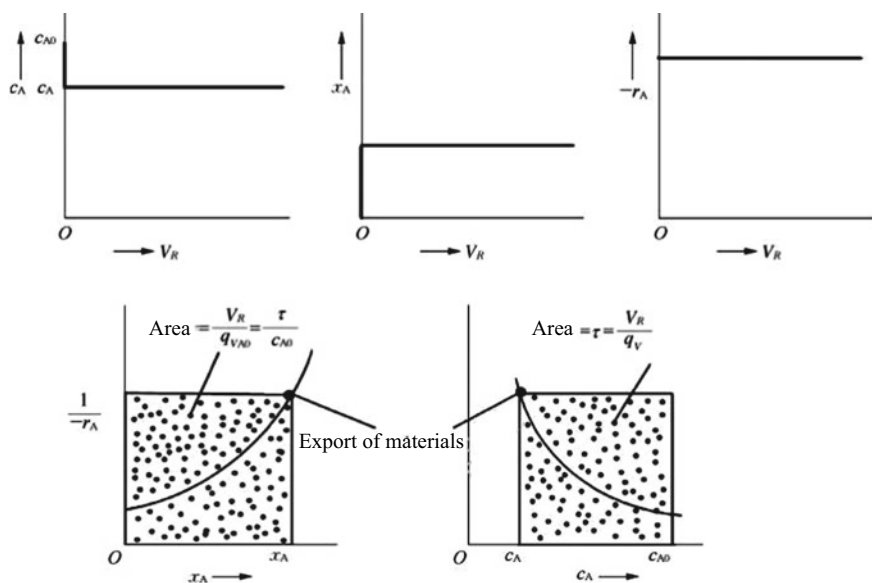


Fig. 5.36 Schematic diagram of the performance and equation of a reactor

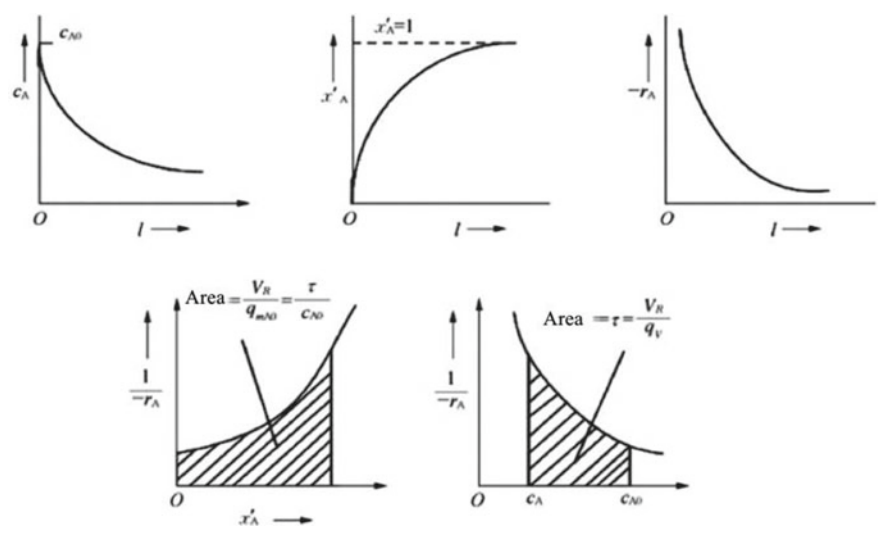


Fig. 5.37 Schematic diagram of the performance and equation of a reactor

Table 5.6 Reaction times of different chemical reactions in a batch reactor

The reaction order	Dynamic differential equation	Time to reach a certain conversion efficiency
Zero-order reaction	$-r_A = k$	()
First-order reaction	$-r_A = kc_A$	()
Second-order reaction	$-r_A = kc_A^2$	()

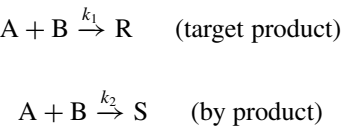
Table 5.7 Reaction time of different chemical reactions in a continuous stirred tank reactor

The reaction order	Dynamic differential equation	Time to reach a certain conversion efficiency
Zero-order reaction	$-r_A = k$	()
First-order reaction	$-r_A = kc_A$	()
Second-order reaction	$-r_A = kc_A^2$	()

Table 5.8 Reaction times of different chemical reactions in a plug flow reactor

The reaction order	Dynamic differential equation	Time to reach a certain conversion efficiency
Zero-order reaction	$-r_A = k$	()
First-order reaction	$-r_A = kc_A$	()
Second-order reaction	$-r_A = kc_A^2$	()

8. If the auxiliary time of feeding, discharging and cleaning in the operation of a batch reactor is not taken into account and the chemical reaction is a first-order reaction, to achieve a certain conversion efficiency within the same time, the ratio of the effective volume of the batch reactor to the effective volume of the plug flow reactor is (). The ratio of the effective volume of the batch reactor to the effective volume of the continuous stirred tank reactor is (). The ratio of the effective volume of the plug flow reactor to the effective volume of the continuous stirred tank reactor is ().
- A. 1
B. > 1
C. < 1
D. 0
9. Please fill in the appropriate selection in Table 5.9. For a parallel reaction:



The reaction rate equation is:

$$r_R = \frac{dc_R}{d\tau} = k_1 c_A^{n'_1} c_B^{\gamma_1}$$
$$r_S = \frac{dc_S}{d\tau} = k_2 c_A^{n'_2} c_B^{\gamma_2}$$
$$\frac{r_R}{r_S} = \frac{\frac{dc_R}{d\tau}}{\frac{dc_S}{d\tau}} = \frac{k_1}{k_2} c_A^{n'_1 - n'_2} c_B^{\gamma_1 - \gamma_2}$$

- A. c_A is low, c_B is high
B. c_A is high, c_B is low
C. c_A, c_B are both high

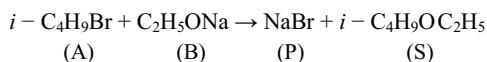
Table 5.9 Select the appropriate operations according to the reaction order

The magnitude of the reaction order	The concentration requirement	Suitable reactor type and operation method
$n'_1 > n'_2, \gamma_1 > \gamma_2$	()	()
$n'_1 > n'_2, \gamma_1 < \gamma_2$	()	()
$n'_1 < n'_2, \gamma_1 > \gamma_2$	()	()
$n'_1 < n'_2, \gamma_1 < \gamma_2$	()	()

- D. c_A, c_B are both low
 - E. A and B are added at the same time in a batch reactor, ideal plug flow reactor or multiple stirred tank reactors.
 - F. Divide A into small strands and add them to each reactor of multiple stirred tank reactors. Either the continuous addition of A is performed along the length of the reaction pipe or A is added successively to the semicontinuous operation of the reactor.
 - G. Divide B into small strands. The reactor is added to each reactor of multiple stirred tank reactors. Either the continuous addition of B is performed along the length of the reaction pipe or B is added successively to the semicontinuous operation of the reactor.
 - H. Use an ideal continuous stirred tank reactor. Either slowly add A and B into the batch reactor or use diluent to reduce both c_A and c_B .
10. According to the operation of the reactor, the reactor can be classified as:_____, continuous reactor, semicontinuous reactor, and so on.
 11. Write the Michaelis–Menten equation_____. In this equation, the characteristic constant of the enzyme is_____, the Michaelis constant is_____, and the parameter related to the concentration of the enzyme is_____. The maximum reaction rate is_____.

Part II Questions + answers

1. According to the following equation, bromoisobutane reacts with sodium ethanolate in an ethanol solution.



The initial concentrations of the reactants are $c_{A0} = 50.5 \text{ mol/m}^3$ and $c_{B0} = 76.2 \text{ mol/m}^3$, and there is no product in the raw material. After reaction for a certain time at 95°C , the analysis results show that $c_B = 37.6 \text{ mol/m}^3$. Please determine the concentration of the remaining components.

Answer: The concentration of A is 11.9 mol/m^3 , and the concentrations of P and S are both 38.6 mol/m^3 .

2. In the above question, both bromoisobutane and sodium ethanol are second-order reactions $(-r_A) = kc_A c_B$. Please determine the kinetic equation of the reaction by the concentration of reactants A and B.

Answer: The kinetic equation of the reaction expressed by the concentration of A is $(-r_A) = kc_A(c_A + 25.7)$. The reaction kinetic equation expressed by the concentration of B is $(-r_B) = kc_B(c_B - 25.7)$.

3. Please derive the Michaelis–Menten equation.

4. An irreversible first-order reaction is conducted in a batch reactor to generate product P from reactant A. The reaction temperature is 100 °C. The reaction rate constant k is known to be 0.01994 min^{-1} . The initial concentration of reactant A is $c_{A0} = 8 \text{ mol/L}$. The daily operation time is no more than 12 h with an output $n_P = 4752 \text{ mol/d}$. The time required for feeding and heating is 54 mins for each run, while the time required for cooling and unloading is 15 mins for each run. The loading coefficient $\varphi_{R/T}$ is 0.6. Please calculate: (1) the time required for the conversion efficiency of 99%; (2) the required effective and total volume of the reactor respectively.

Answer: (1) 230 min; (2) 300 L, 500 L.

5. A liquid phase reaction ($A + B \rightarrow P$) is conducted in an ideal batch reactor with a volume of 2.5 m^3 . The reaction is maintained at isothermal operation at 75 °C, and the reaction rate equation measured by the experiment is $(-r_A) = kc_A c_B \text{ kmol}/(\text{L} \cdot \text{S})$, in which k is $2.78 \times 10^{-3} \text{ L}/(\text{mol} \cdot \text{s})$. When the initial concentrations of reactants A and B are both 4 mol/L and the conversion efficiency of A (x_A) is 0.8, the batch reactor can process 0.684 kmol of reactant A per minute on average. If the reaction is moved to an ideal plug flow reactor with a pipe diameter of 125 mm, isothermal operation is still maintained at 75 °C, and the processing capacity and required conversion efficiency are the same. Please calculate the required length of the reactor.

Answer: 83.6 m.

6. If the reaction mentioned above is moved to a continuous stirred tank reactor and the reaction temperature, the initial concentration of materials, the reaction conversion efficiency and the material processing capacity remain unchanged, what is the effective volume of the reactor? It is also assumed that the reaction is carried out in the batch reactor, and 20 min is needed for discharging, feeding and heating between two batches of reactions. The material loading coefficient of the reactor is 0.8. At this time, to achieve the same production capacity, what is the volume of the batch reactor?

Answer: 5.13 m^3 ; 5.56 m^3 .

7. The hydrolysis reaction is conducted in four reactors of continuous stirred tank reactors, and the reactions are all first-order reactions. The temperature and rate constants of each reactor are as follows:

No. of kettles	1	2	3	4
$T/^\circ\text{C}$	10	15	25	40
$k(\text{min}^{-1})$	0.0567	0.0806	0.158	0.380

The volume of each reactor is 0.8 m^3 , influent concentration c_{A0} is 4.5 mol/L and flow rate is 100 L/min. Please calculate the following: (1) What is the concentration at the outlet of each reactor?

(2) If all the reactors are kept running at 15 °C with the same volume of 0.8 m³, how many reactors are needed to reach the same outlet concentration of the above four reactors in series?

Answer: (1) 3.09 mol/L, 1.88 mol/L, 0.833 mol/L; (2) 0.206 mol/L, 7 reactors are needed.

8. Please deduce the time required for a zero-order reaction, first-order reaction and second-order reaction to reach a certain conversion efficiency and concentration in a batch reactor, plug flow reactor, and continuous stirred tank reactor, respectively, and clarify the relationships among them and give the reasons.
9. Please briefly describe why it takes the same time for the ideal continuous stirred tank reactor reactor and the plug flow reactor to achieve a certain conversion efficiency for a zero-order reaction, but not for a first-order reaction or second-order reaction.

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Appendix A

Physical Properties of Water (Cardarelli, 2018)

Temperature $T/^{\circ}\text{C}$	Saturated vapor pressure p/kPa	Density $\rho/(\text{kg}/\text{m}^3)$	Enthalpy $H/(\text{kJ}/\text{kg})$	Specific heat capacity $C_p/[\text{kJ}/(\text{kg}\cdot\text{K})]$	Thermal conductivity $\sigma/[10^{-2}\text{ W}/(\text{m}\cdot\text{K})]$	Viscosity $\mu/[10^{-5}/(\text{Pa}\cdot\text{s})]$	Volume expansion coefficient $\beta_g/[10^{-4}/\text{K}^{-1})]$	Surface tension $[\times 10^{-3}/(\text{N}/\text{m})]$
0	0.608 2	999.9	0.00	4.212	55.13	179.21	0.63	75.6
10	1.226 2	999.7	42.04	4.191	57.45	130.77	0.70	74.1
20	2.334 6	998.2	83.90	4.183	59.89	100.50	1.82	72.6
30	4.247 4	995.7	125.69	4.174	61.76	80.07	3.21	71.2
40	7.376 6	992.2	167.51	4.174	63.38	65.60	3.87	69.6
50	12.31	988.1	209.30	4.174	64.78	54.94	4.49	67.7
60	19.923	983.2	251.12	4.178	65.94	46.88	5.11	66.2
70	31.164	977.8	292.99	4.178	66.76	40.61	5.70	64.3
80	47.379	971.8	334.94	4.195	67.45	35.65	6.32	62.6
90	70.136	965.3	376.98	4.208	67.98	31.65	0.95	60.7
100	101.33	958.4	419.10	4.220	68.04	28.38	7.52	58.8
110	143.31	951.0	461.34	4.238	68.27	25.89	8.08	56.9
120	198.64	943.1	503.67	4.250	68.50	23.73	8.64	54.8
130	270.25	934.8	546.38	4.266	68.50	21.77	9.17	52.8
140	361.47	926.1	589.08	4.287	68.27	20.10	9.72	50.7
150	476.24	917.0	632.20	4.312	68.38	18.63	10.3	48.6
160	618.28	907.4	675.33	4.346	68.27	17.36	10.7	46.6
170	792.59	897.3	719.29	4.379	67.92	16.28	11.3	45.3
180	1 003.5	886.9	763.25	4.417	67.45	15.30	11.9	42.3
190	1 255.6	876.0	807.63	4.460	66.99	14.42	12.6	40.8
200	1 554.77	863.0	852.43	4.505	66.29	13.63	13.3	38.4

(continued)

(continued)

Temperature $T/^{\circ}\text{C}$	Saturated vapor pressure p/kPa	Density $\rho/(\text{kg}/\text{m}^3)$	Enthalpy $H/(\text{kJ}/\text{kg})$	Specific heat capacity $C_p/[\text{kJ}/(\text{kg}\cdot\text{K})]$	Thermal conductivity $\sigma/[\times 10^{-2} \text{ W}/(\text{m}\cdot\text{K})]$	Viscosity $\mu/[\times 10^{-5}/(\text{Pa}\cdot\text{s})]$	Volume expansion coefficient $\beta_g/(\times 10^{-4}/\text{K}^{-1})$	Surface tension $[\times 10^{-3}/(\text{N}/\text{m})]$
210	1 917.72	852.8	897.65	4.555	65.48	13.04	14.1	36.1
220	2 320.88	840.3	943.70	4.614	64.55	12.46	14.8	33.8
230	2 798.59	827.3	990.18	4.681	63.73	11.97	15.9	31.6
240	3 347.91	813.6	1 037.49	4.756	62.80	11.47	16.8	29.1
250	3 977.67	799.0	1 085.64	4.844	61.76	10.98	18.1	26.7
260	4 863.75	784.0	1 135.04	4.949	60.84	10.59	19.7	24.2
270	5 503.99	767.9	1 185.28	5.070	59.96	10.20	21.6	21.9
280	6 417.24	750.7	1 236.28	5.229	57.45	9.81	23.7	19.5
290	7 443.29	732.3	1 289.95	5.485	55.82	9.42	26.2	17.2
300	8 592.94	712.5	1 344.80	4.736	53.96	9.12	29.2	14.7
310	9 877.96	691.1	1 402.16	6.071	52.34	8.83	32.9	12.3
320	1 130 0.3	667.1	1 462.03	6.573	50.59	8.53	38.2	10.0
330	1 287 9.6	640.2	1 526.19	7.243	48.73	8.14	43.3	7.82
340	1 461 5.8	610.1	1 594.75	8.164	45.71	7.75	53.4	5.78
350	1 563 8.5	574.4	1 671.37	9.504	43.03	7.26	66.8	3.89
360	1 866 7.1	528.0	1 761.39	13.984	39.54	6.67	109.0	2.06
370	2 104 0.9	450.5	1 892.43	40.319	33.73	5.69	264.0	0.48

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Appendix B

Important Physical Properties of Some Gases

(Cardarelli, 2018)

Name	Molecular equation	Density ρ (kg/m ³) (0 °C, 101.3 kPa)	Specific heat capacity C_p [kJ/(kg·K)]	Viscosity μ [× 10 ⁻⁵ /(Pa·s)]	Boiling point T_b /°C (101.3 kPa)	Vaporization heat r /(kJ/kg)	Critical point Temperature T_c /°C	Pressure p_c /kPa	Thermal conductivity σ [W/(m·K)]
Air		1.293	1.009	1.73	−195	197	−140.7	3 768.4	0.024 4
Oxygen	O ₂	1.429	0.653	2.03	−132.98	213	−118.82	5 036.6	0.024 0
Nitrogen	N ₂	1.251	0.745	1.70	−195.78	199.2	−147.13	3 392.5	0.022 8
Hydrogen	H ₂	0.089 9	10.13	0.842	−252.75	454.2	−239.9	1 296.6	0.163
Helium	He	0.178 5	3.18	1.88	−268.95	19.5	−267.96	228.94	0.144
Argon	Ar	1.782 0	0.322	2.09	−185.87	163	−122.44	4 862.4	0.017 3
Chlorine	Cl ₂	3.217	0.355	1.29 (16 °C)	−33.8	305	144.0	7 708.9	0.007 2
Ammonia	NH ₃	0.771	0.67	0.92	−33.4	137 3	132.4	1 129.5	0.021 5
Carbon monoxide	CO	1.25	0.754	1.66	−191.48	211	140.2	3 497.9	0.022 6
Carbon dioxide	CO ₂	1.976	0.653	1.37	−78.2	574	31.1	7 384.8	0.013 7
Sulfur dioxide	SO ₂	2.927	0.502	1.17	−10.8	394	157.5	3 879.1	0.007 7
Nitrogen dioxide	NO ₂	–	0.615	–	21.2	712	158.2	10 130	0.040 0
Hydrogen sulfide	H ₂ S	1.539	0.804	1.17	−60.2	548	100.4	19 136	0.013 1
Methane	CH ₄	0.717	1.7	1.03	−161.58	511	−82.15	4 619.3	0.030 0
Ethane	C ₂ H ₆	1.357	1.44	0.850	−88.50	486	32.1	4 948.5	0.018 0
Propane	C ₃ H ₈	2.02	1.65	0.795 (18 °C)	−42.1	427	95.6	4 335.9	0.014 8
N-butane	C ₄ H ₁₀	2.673	1.73	0.810	−0.5	386	152	3 798.8	0.013 5
N-pentane	C ₅ H ₁₂	–	1.57	0.874	−36.08	151	197.1	3 342.9	0.012 8

(continued)

(continued)

Name	Molecular equation	Density ρ (kg/m ³) (0 °C, 101.3 kPa)	Specific heat capacity C_p [kJ/(kg·K)]	Viscosity μ [× 10 ⁻⁵ (Pa·s)]	Boiling point T_b /°C (101.3 kPa)	Vaporization heat r (kJ/kg)	Critical point Temperature T_c /°C	Pressure p_c /kPa	Thermal conductivity σ [W/(m·K)]
Ethylene	C ₂ H ₄	1.261	1.222	0.935	103.7	481	9.7	5 135.9	0.016 4
Propylene	C ₃ H ₆	1.914	1.436	0.835 (20 °C)	−47.7	440	91.4	4 559.0	–
Acetylene	C ₂ H ₂	1.171	1.352	0.935	−83.66	829	35.7	6 240.0	0.018 4
Chloromethane	CH ₃ Cl	2.303	0.582	0.989	−24.1	406	148	6 685.8	0.008 5
Benzene	C ₆ H ₆	–	1.139	0.72	80.2	394	288.5	4 832.0	0.008 8

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Appendix C

Important Physical Properties of Some Liquids (Cardarelli, 2018)

Name	Molecular equation	Density ρ (kg/m ³) (20 °C)	Specific heat capacity C_p /(kJ/(kg·K)) (20 °C)	Viscosity μ /($\times 10^{-5}$ /(Pa·s)) (20 °C)	Boiling point T_b /°C (101.3 kPa)	Vaporization heat r /(kJ/kg) (101.3 kPa)	Coefficient of volume expansion β_g /($\times 10^{-4}$ /K ⁻¹) (20 °C)	Surface Tension/($\times 10^{-3}$ /(N/m)) (20 °C)	thermal conductivity σ /(W/(m·K)) (20 °C)
Trichloromethane	CHCl ₃	1 489	61.2	253.7	0.992	0.58	0.133 (30 °C)		28.5 (10 °C)
Carbon tetrachloride	CCl ₄	1 584	76.8	195	0.850	1.0	0.12		26.8
Dichloroethane-1,2	C ₂ H ₄ Cl ₂	1 253	83.6	324	1.260	0.83	0.14 (50 °C)		30.8
Benzene	C ₆ H ₆	879	80.10	393.9	1.704	0.737	0.148	12.4	28.6
Toluene	C ₇ H ₈	867	110.63	363	1.70	0.675	0.138	10.9	27.9
O-xylene	C ₈ H ₁₀	880	144.42	347	1.74	0.811	0.142		30.2
M-xylene	C ₈ H ₁₀	864	139.10	343	1.70	0.611	0.167	0.1	29.0
P-xylene	C ₈ H ₁₀	861	138.35	340	1.704	0.643	0.129		28.0
Styrene	C ₈ H ₉	911 (15.6 °C)	145.2	(352)	1.733	0.72			
Chlorobenzene	C ₆ H ₅ Cl	1 106	131.8	325	1.298	0.85	0.14 (30 °C)		32
Nitrobenzene	C ₆ H ₅ NO ₂	1 203	210.9	396	396	2.1	0.15		41
Aniline	C ₆ H ₅ NH ₂	1 022	184.4	448	2.07	4.3	0.17	8.5	42.9
Phenol	C ₆ H ₅ OH	1 050 (50 °C)	181.8 (Melting point 40.9 °C)	511		3.4 (50 °C)			
Naphthalene	C ₁₀ H ₈	1 145 (solid state)	271.9 (Melting point 80.2 °C)	314	1.8 (110 °C)	0.59 (100 °C)			
Methanol	CH ₃ OH	791	64.7	1101	2.48	2.48	0.212	12.2	22.6
Ethanol	C ₂ H ₅ OH	789	78.3	846	2.39	2.39	0.172	11.6	22.8
Ethanol (95%)		804	78.2		1.4				
Glycol	C ₂ H ₄ (OH) ₂	1 113	197.6	780	2.35	23			47.7
Glycerol	C ₃ H ₅ (OH) ₃	1 261	290 (decomposition)	–		1499	0.59	5.3	63
Ether	(C ₂ H ₅) ₂ O	714	34.6	360	2.34	0.24	0.140	16.3	18
Acetaldehyde	CH ₃ CHO	783 (18 °C)	20.2	574	1.9	1.3 (18 °C)			21.2

(continued)

(continued)

Name	Molecular equation	Density ρ (kg/m ³) (20 °C)	Specific heat capacity C_p [kJ/(kg·K)] (20 °C)	Viscosity μ [($\times 10^{-5}$)/(Pa·s)] (20 °C)	Boiling point T_b /°C (101.3 kPa)	Vaporization heat r [(kJ/kg) (101.3 kPa)	Coefficient of volume expansion β_g [($\times 10^{-4}$)/K ⁻¹) (20 °C)	Surface Tension [($\times 10^{-3}$)/(N/m)] (20 °C)	thermal conductivity σ [W/(m·K)] (20 °C)
Furfural	C ₅ H ₄ O ₂	1168	161.7	452	1.6	1.15 (50 °C)			43.5
Acetone	CH ₃ COCH ₃	792	56.2	523	2.35	0.32			23.7
Formic acid	HCOOH	1 220	100.7	494	2.17	1.9			27.8
Acetic acid	CH ₃ COOH	1 049	118.1	406	1.99	1.3	0.17	10.7	23.9
Ethyl acetate	CH ₃ COOC ₂ H ₅	901	77.1	368	1.92	0.48	0.14 (10 °C)		
Kerosene		780 ~ 820				3	0.15	10.0	
Petrol		680 ~ 800				0.7 ~ 0.8	0.19 (30 °C)	12.5	

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Appendix D

Physical Properties of Dry Air (101.33 kPa)

(Cardarelli, 2018)

Temperature $T/^{\circ}\text{C}$	Density $\rho/(\text{kg}/\text{m}^3)$	Specific heat capacity $C_p/[\text{kJ}/(\text{kg}\cdot^{\circ}\text{C})]$	Thermal conductivity $\sigma/[\times 10^{-2} \text{ W}/(\text{m}\cdot\text{K})]$	Viscosity $\mu/[\times 10^{-5}/(\text{Pa}\cdot\text{s})]$
−50	1.584	1.013	2.035	1.46
−40	1.515	1.013	2.117	1.52
−30	1.453	1.013	2.198	1.57
−20	1.395	1.009	2.279	1.62
−10	1.342	1.009	2.360	1.67
0	1.293	1.009	2.442	1.72
10	1.247	1.009	2.512	1.76
20	1.205	1.013	2.593	1.81
30	1.165	1.013	2.675	1.86
40	1.128	1.013	2.756	1.91
50	1.093	1.017	2.826	1.96
60	1.060	1.017	2.896	2.01
70	1.029	1.017	2.966	2.06
80	1.000	1.022	3.047	2.11
90	0.972	1.022	3.128	2.15
100	0.946	1.022	3.210	2.19
120	0.898	1.026	3.338	2.28
140	0.854	1.026	3.489	2.37
160	0.815	1.026	3.640	2.45
180	0.779	1.034	3.780	2.53
200	0.746	1.034	3.931	2.60
250	0.674	1.043	4.268	2.74
300	0.615	1.047	4.605	2.97

(continued)

(continued)

Temperature $T/^{\circ}\text{C}$	Density $\rho/(\text{kg}/\text{m}^3)$	Specific heat capacity $C_p/[\text{kJ}/(\text{kg}\cdot^{\circ}\text{C})]$	Thermal conductivity $\sigma/[\times 10^{-2} \text{ W}/(\text{m}\cdot\text{K})]$	Viscosity $\mu/[\times 10^{-5}/(\text{Pa}\cdot\text{s})]$
350	0.566	1.055	4.908	3.14
400	0.524	1.068	5.210	3.30
500	0.456	1.072	5.745	3.62
600	0.404	1.089	6.222	3.91
700	0.362	1.102	6.711	4.18
800	0.329	1.114	7.176	4.43
900	0.301	1.127	7.630	4.67
1000	0.277	1.139	8.071	4.90
1100	0.257	1.152	8.502	5.12
1200	0.239	1.164	9.153	5.35

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Appendix E

Absolute Roughness of Pipe Wall (Cardarelli, 2018)

Pipe material and its use	Absolute roughness ε/mm
Clean drawn copper, brass, lead and glass pipes	0.0015 ~ 0.01
Rubber hose	0.01 ~ 0.03
Cement paste powder pipe	0.45 ~ 3.0
Pottery drainage pipe	0.35 ~ 6
New seamless steel pipe	0.04 ~ 0.07
Seamless steel pipe used in gas pipeline for a year	~0.12
Slightly corroded seamless steel pipe	0.2 ~ 0.3
Old stainless steel pipe	0.6 ~ 0.7
Galvanized pipe or new cast iron pipe	0.25 ~ 0.4
Corroded old cast iron pipe	>0.85

Note In general calculations, for clean glass, copper, lead and other drawn pipes, it can be regarded as smooth pipe ($\varepsilon = 0$), new seamless steel pipe, $\varepsilon = 0.1$ mm, slightly corroded seamless steel pipe and new seamless steel pipe, $\varepsilon = 0.35$ mm, old cast iron pipe or strongly corroded pipe, $\varepsilon = 1$ mm
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Reference

Cardarelli, F. (Eds.). (2018). *Materials Handbook*, Springer London, Springer, Springer Science & Business Media, London

Symbols and Units

Symbol number	Name and definition	Unit	Correlation equation	Page number
A	1. Area 2. Force area, which is equal to the cross-sectional area of the fluid perpendicular to the flow direction in the pipe at that time 3. Effective area of gas-liquid contact (A is related to those factors, such as the cross-sectional area of the tower and the tower height, etc.)	m^2	$A = \pi r^2$ $A = a A_T H'$	13, 187
A_0	Heat transfer area, total surface area of tube bundle	m^2	$A_0 = n \pi d_o l$	144
A_1	The heat transfer area on the hot fluid side of the interwall	m^2		133
A_2	The heat transfer area on the cold fluid side of the interwall	m^2		133
A_a	Absorption factor: the geometric meaning is the ratio of the slope of the operating line L/V to the slope of the equilibrium line m_e . The greater the A_a value is, the better the absorption effect is in absorption tower	Dimensionless	$A_a = \frac{L}{m_e V}$	192
(continued)				

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
$\frac{1}{A_a}$	Desorption factor: the reciprocal of the absorption factor, $\frac{1}{A_a} = \frac{m_e V}{L}$. This called the desorption factor; the greater $\frac{1}{A_a}$ is, the better the desorption effect is in desorption tower	Dimensionless	$\frac{1}{A_a} = \frac{m_e V}{L}$	192	
A_d	Diffusion area	m ²		156	
A_h	Heat conduction area: cross-sectional area perpendicular to the heat flow direction	m ²		106	
A_j	Force area equal to the contact area of the fluid between the adjacent two layers in the pipe at that time	m ²	$A_j = 2\pi r l$	14, 24	
A_m	Logarithmic mean of the cylinder wall	m ²	$A_m = 2\pi r_m l$	113	
A_p	Projection area: the projection area of spherical particles in the flow direction	m ²	$A_p = \frac{\pi}{4} d_p^2$	73	
A_T	Cross section area of packed tower	m ²		187	
(continued)					

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
A'	Pre-exponential factor: a constant independent of temperature	Dimensionless		236
a	Effective contact area between gas and liquid phases in unit volume packing	m^2/m^3		187
a_{BET}	Specific surface area of packings	m^2/m^3		200
a_j a_{jx}, a_{jy}, a_{jz}	Acceleration of an object in the direction of force	m/s^2	$a_{jx} = du_x/d\tau$, $a_{jy} = du_y/d\tau$, $a_{jz} = du_z/d\tau$	6 9
a_t	Temperature coefficient: negative for most metallic materials and positive for most non-metallic materials	K^{-1}		107
B	Channel width	m		69
B_m	Constant: B_m of the Lacy ring is 0.022, B_m of arc saddle packing is 0.26	Dimensionless		200
b_w	Channel bottom width	m		57
C_c	Chézy coefficient	$\text{m}^{1/2}/\text{s}$	$C_c = \frac{1}{n} R_h^{\frac{1}{6}}$	56
(continued)				

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Symbol number	Name and definition	Unit	Correlation equation	Page number
C_p	Specific heat capacity at constant pressure, or specific heat capacity, the heat absorbed per unit mass of a material by 1 K increase in temperature at constant pressure	J/(kg·K) or m ² /(s ² ·K)		118
c_A	Concentration of diffusive components A	kmol/m ³ or kg/m ³		156
c_A	Reaction time concentration at τ : zero-order reaction time concentration at τ	mol/dm ³ or kg/m ³	$c_A = c_{A0} - k \tau$	225
c_A	Reaction time concentration at τ : first-order reaction time concentration at τ	mol/dm ³ or kg/m ³	$c_A = c_{A0}e^{-k\tau}$	226
c_{AL}	The concentration of absorbate A in the main body of liquid phase	kmol/m ³ or kg/m ³		209
c_{Ai}	The concentration of absorbate A at the gas–liquid interface	kmol/m ³ or kg/m ³		209
c_{Bm}	The logarithmic mean of the concentration of the component B between the interface and the main body of the liquid phase	kmol/m ³ or kg/m ³	$c_{Bm} = \frac{c_{B2} - c_{B1}}{\ln \frac{c_{B2}}{c_{B1}}}$ $= \frac{c_T - c_{A2} - (c_T - c_{A1})}{\ln \frac{c_T - c_{A2}}{c_T - c_{A1}}}$	159
(continued)				

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
c_P	Material P concentration at reaction time τ ; Material P concentration at time τ of parallel reaction	mol/dm ³ or kg/m ³	$c_P = \frac{k_2 c_{A0}}{k_1 + k_2} (1 - e^{-(k_1 + k_2)\tau})$	231
c_G^*	The absorbate G concentration of the liquid phase in equilibrium with the partial pressure of the absorbate p_G in the main body of the gas phase	kmol/m ³ or kg/m ³		176
c_i	The concentration of absorbent at the interface	kmol/m ³ or kg/m ³		175
c_L	The concentration of absorbate in the main body of the liquid phase	kmol/m ³ or kg/m ³		175
c_S	Material S concentration at τ time of reaction: Material S concentration at τ time of parallel reaction	mol/dm ³ or kg/m ³	$c_S = \frac{k_1 c_{A0}}{k_1 + k_2} (1 - e^{-(k_1 + k_2)\tau})$	231
(continued)				

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
D	1. Molecular diffusion coefficient 2. Represents the diffusion coefficient of gas diffusion component A in the gas 3. Solute diffusion coefficient in liquid	m^2/s or m^2/h	1. Represents the molecular diffusion coefficient: $N_A = \frac{J_A}{A_d} = -D \frac{dc_A}{dz}$ 2. Represents the diffusion coefficient of gas diffusion component A in the gas $D = \frac{4.36 \times 10^{-6} T^{3/2}}{p \left(V_{mA}^{1/3} + V_{mB}^{1/3} \right)^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$ 3. Indicates the diffusion coefficient of the solute in the liquid: $D = 1.859 \times 10^{-18} \frac{(\alpha_s M)^{1/2} T}{\mu V_m^{0.6}}$	1. 161 2. 162 3. 163
D_a	Axial diffusion coefficient	m^2/s		248
D_w	Eddy diffusion coefficient	m^2/s	$N_A = -D \frac{dc_A}{dz} - D_w \frac{dc_A}{dz}$ $= -(D + D_w) \frac{dc_A}{dz}$	170
D_{dilution}	Dilution rate	h^{-1}	$D_{\text{Dilution}} = q_V/V$	279
d	Inner diameter of tube or pipe: indicates pipe internal diameter	m		16
d_i, d_o, d_m	The inner diameter, outer diameter and average diameter of the pipe wall, respectively	m	$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{\delta_p \Lambda_1}{\sigma \Lambda_m} + \frac{\Lambda_1}{\alpha_2 \Lambda_2}$	134
d'_1, d'_2	Impeller diameter of pump	m		84

(continued)

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
dA	Heat transfer area of the microelement perpendicular to the heat transfer direction	m^2		122
dc_A/dl	Axial concentration gradient	kg/m^4		248
$\frac{dT}{d\delta}$	Temperature gradient	K/m		109
$\frac{du}{dy}$	Normal velocity gradient: the change rate of the fluid velocity in the y direction perpendicular to the flow direction	s^{-1}		14
$d\ddot{U}$	Reaction progress	mol or kg	$d\ddot{U} = \frac{dn_L}{dt}$	229
d_{cr}	Critical diameter of insulation layer	m	$d_{cr} = \frac{2\sigma}{\alpha}$	131
d_e	Equivalent diameter: the diameter of a pipe with equal hydraulic radius	m	$d_e = \frac{4 \times \text{Circulation cross sectional area}}{\text{Wet periphery length}}$	17
d_t	Tower diameter	m	$d_t = \sqrt{\frac{4q_g}{\pi u}}$	200
$[E]$	Free enzyme concentration	kg/m^3	$[E] = [E_0] - [ES]$	245
$[E_0]$	Total concentration of enzyme	kg/m^3		245
E_a	Arrhenius activation energy: a constant independent of temperature	J/mol		242
				(continued)

(continued)					
Symbol number	Name and definition	Unit	Correlation equation	Page number	
E_n	Internal energy: the total internal energy of the equilibrium state of the system	J		3	
E_{n1}	Initial internal energy: the total internal energy of the initial equilibrium state of the system	J		3	
E_{n2}	Final internal energy: the total internal energy of the final equilibrium state of the system	J		3	
Eu	Euler number: the ratio of the pressure drop to the inertia force caused by loss along the course	Dimensionless	$Eu = \frac{\Delta p_f}{\rho u^2}$	27	
[ES]	Enzyme-substrate complex concentration	kg/m ³	[ES] = k_1 [E] [S]	245	
F	Surface force, the force acting perpendicularly on the surface of any fluid element	kg·m/s ² , N			
F_d	Drag force, action force on surface of particle by fluid caused by the relative motion between fluid and particle	kg·m/s ² , N		73	
F_b	Buoyance, the force on which an object immersed in a fluid is supported vertically by the fluid is called buoyance	kg·m/s ² , N		74	
					(continued)

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
F_g	Gravity, the force on an object near the ground due to the attraction of the earth	kg·m/s ² , N		74
F_n	Internal friction force: the interaction between two adjacent fluid layers in the a moving fluid	kg·m/s ² , N		14
Gr	Grashof number, dimensionless number, represents the characteristic number of natural convection influence	Dimensionless	$Gr = \frac{L^3 \rho^2 g \beta_s \Delta t}{\mu^2}$	120
G_τ	Generation time, the time needed for completing once of cell division	s or h	$G_\tau = \frac{1}{R_C} = \frac{t_2 - t_1}{3.3 \lg \frac{X_{CI}}{X^* - X_{CI}}}$	249
g	Gravity acceleration, 9.81 m/s ²	m/s ²		38
\mathcal{H}	Enthalpy, a thermodynamic energy state function of a body	kJ/mol	$\mathcal{H} = U + pV$	4
H	Height	m		77
H'	Packing layer height	m	$H' = \frac{L}{K_X a A_T} \int_{X_2}^{X_1} \frac{dX}{\bar{X}^* - \bar{X}}$	188
			$H' = \frac{V}{K_Y a A_T} \int_{Y_2}^{Y_1} \frac{dY}{\bar{Y} - \bar{Y}^*}$	188
			$H' = \frac{L}{k_X a A_T} \int_{X_2}^{X_1} \frac{dX}{\bar{X}_i - \bar{X}}$	188
			$H' = \frac{V}{k_Y a A_T} \int_{Y_2}^{Y_1} \frac{dY}{\bar{Y} - \bar{Y}_i}$	188
(continued)				

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
$HETP$	Height equivalent to theoretical plate	m	$HETP = \frac{H'}{N_T}$	201
H_a	Local atmospheric pressure at pump installation area	m Liquid column		90
H_e	Lift head, effective pressure head of the pump, namely, the energy obtained from the pump per unit weight of liquid	m Liquid column		37
H_{eg}	The energy required to lift per unit weight of liquid	m Liquid column	$H_{eg} = \Delta Z + \frac{\Delta p}{\rho g} + \frac{\Delta u^2}{2g} + \Sigma H_{tf,1-2}$	84
H_f	The straight pipe friction loss, mechanical energy consumed for fluid flow to overcome resistance	m Liquid column	$H_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2}{2g}$	25, 2756
H_G	Height of gas phase mass transfer unit	m	$H_G = \frac{V}{k_y a A_T}$	195
H_g	Allowable installation height of the centrifugal pump	m		89
H_L	Height of liquid phase mass transfer unit	m	$H_L = \frac{L}{k_x a A_T}$	195
H_{OG}	Height of overall gas phase transfer unit	m	$H_{OG} = \frac{V}{K_y a A_T}$	194
H_{OL}	Height of overall liquid phase transfer unit	m	$H_{OL} = \frac{L}{K_x a A_T}$	195
(continued)				

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
H_{pc}	Solubility coefficient: The higher the H_{pc} value is, the higher the gas solubility, and the easier the solute can be absorbed by the solvent. The solubility coefficient H_{pc} decreases with increasing temperature	$\text{kmol}/(\text{m}^3 \cdot \text{kPa})$	$p_A^* = \frac{c_A}{H_{pc}}$ $H_{pc} = \frac{1}{H_{pm}} \frac{\rho_0}{M_0}$		173
H_{pm}	Henry's coefficient: an empirical equation based on experimental results. The value of H_{pm} indicates the degree of difficulty or easiness of gas absorption	kPa	$p_A^* = H_{pm} x_A$		173
H_s	The allowable vacuum height of centrifugal pump	mH ₂ O	$H_s = \frac{p_a - p_1}{\rho g}$		89
H'_s	Allowable vacuum height for transporting liquid under operating conditions	mH ₂ O	$H'_s = \left[\frac{H_s + (H_a - 10) - \frac{1000}{\rho}}{\left(\frac{p_v}{9.81 \times 10^3} - 0.24 \right)} \right]$		89
H_w	Head above weir or blockage height above weir, the height between the crest of the weir and the surface of upstream flow	m			68, 69
H_{w1}	Height of the rectangular thin wall weir	m			68, 69
h	Height, water depth, etc.	m			57

(continued)

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
h_f	Straight pipe friction loss, pipe head loss, major loss, Moody-type friction loss, the energy loss caused by the internal friction of the fluid when the fluid flows in a straight pipe	J/kg	$h_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2}{2}$		25, 27
h'_f	Local loss, minor loss, the energy loss caused by the separation of the boundary layer resulted from the change in flow direction or cross section through pipe fittings, valves, sudden expansion or reduction of a cross section, other devices, etc	J/kg	$h'_f = \xi \cdot \frac{u^2}{2}$ or $h'_f = \lambda_e \cdot \frac{u^2}{d} \cdot \frac{1}{2}$		23, 29
h_R	Indicate the height difference between the two sides of the liquid	m	$q_V = K_V A_0 \sqrt{\frac{2h_R g(\rho_A - \rho)}{\rho}}$		70
h_{tf}	The total resistance loss, the mechanical energy consumed by overcoming the resistance along the way and locally during the flow of the fluid	J/kg	$h_{tf} = h_f + h'_f$ or $\Sigma h_{tf} = \Sigma h_f + \Sigma h'_f$		23, 31
i	Bottom slope or slope	Dimensionless	$i = \frac{\Delta Z_h}{l} = \sin \varphi_d$		56
J'	Absorption mass transfer rate of the whole absorption tower	kmol/h	$J' = N_A A = K_Y (Y - Y^*) A = K_X (X^* - X) A$		189
(continued)					

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
J_A	Diffusion rate of diffusive component A	kmol/s or kmol/h	$J_A = \frac{DA_d}{\delta_d} (c_{A1} - c_{A2})$		161
J_s	Hydraulic gradient: head loss per unit the length of the flow	Dimensionless	$J_s = H_f / l_f$		56
K	Overall heat transfer coefficient: an important parameter that reflects the heat transfer capacity of a heat exchanger, and it is also the basis for heat transfer calculation of a heat exchanger	W/(m ² ·K)	$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{\delta_p A_1}{\sigma A_m} + \frac{A_1}{\alpha_2 A_2}$		137
K_0	Overall heat transfer coefficient calculated according to the outer surface	W/(m ² ·K)	$\frac{1}{K_0} = \frac{1}{\alpha_i} \frac{d_0}{d_i} + \frac{\delta_p d_0}{\sigma d_m} + \frac{1}{\alpha_o}$		137
K_1	Matrix inhibition coefficient	kg/m ³			250
K_C	Constant: for a general mixture or solution, $K_C = 1.0$. For an organic aqueous solution, $K_C = 0.9$	Dimensionless	$\sigma_m = K_C \sum_{i=1}^n \sigma_i v_i$		108
K_G	Overall absorption mass transfer coefficient of the driving force expressed by the partial pressure difference of the gas phase	kmol/(m ² ·s·kPa)	$N_A = \frac{p_G - p_L^*}{\frac{1}{k_G} + \frac{1}{H_{p \cdot k_L}}} = K_G (p_G - p_L^*)$		(continued)

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		57
K_i	Flow rate modulus: it reflects the influence of the shape, size and roughness of the open channel cross section on the flow rate of the channel	m ³ /s	$q_V = Au = AC_c\sqrt{Ri} = K_i\sqrt{i}$		
K_L	Overall absorption mass transfer coefficient of the driving force expressed by the liquid phase concentration difference	m/s	$N_A = \frac{c_G^* - c_L}{\frac{H_{pc}}{k_G} + \frac{1}{k_L}}$ $= K_L(c_G^* - c_L)$		
K_m	Michaelis constant, one of the characteristic constants of an enzyme, the value of which is related to the structure of the enzyme, the corresponding substrate and environmental factors (such as temperature, pH value, ionic strength), and has nothing to do with the concentration of the enzyme itself	kg/m ³	$r = \frac{r_m[S]}{K_m + [S]}$		245
(continued)					

(continued)

Symbol number	Name and definition	Unit	Correlation equation	Page number
K_P	Metabolite inhibition coefficient	kg/m ³	$\mu_B = \frac{\mu_{Bmax}}{\left(1 + \frac{K_S}{[S]} + \frac{[P_C]}{K_P}\right)}$ or $\mu_B = \frac{\mu_{Bmax} [S]}{K_S \left(1 + \frac{[P_C]}{K_P}\right) + [S]}$	251
K_S	Saturation coefficient: K_S is equal to $[S]$ when $\mu_B = \mu_{Bmax}/2$	kg/m ³	$\mu_B = \frac{\mu_{Bmax} [S]}{K_S + [S]}$	250
K_V	Flow rate coefficient, dimensionless, which can be measured experimentally or from the instrument manual and is generally $0.6 \sim 0.7$	Dimensionless	$q_V = K_V A_0 \sqrt{\frac{2\Delta p}{\rho}} = K_V A_0 \sqrt{\frac{2h_{RG}(\rho_A - \rho)}{\rho}}$	
K_w	Flow rate coefficient, dimensionless	Dimensionless	$q_V = K_w b_w \sqrt{2g} H_w^{3/2}$	
K'_w	Flow rate coefficient taking into account the kinetic energy of upstream velocity	Dimensionless	$q_V = K'_w b_w \sqrt{2g} H_w^{3/2}$	
K''_w	Flow rate coefficient of triangular thin wall weir	m ^{1/2} /s	$q_V = K''_w H_w^{5/2}$	

(continued)

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
K_Y, K_X	Overall absorption mass transfer coefficients driven by the molar ratio difference (molar ratio) ($Y - Y^*$) and ($X^* - X$), respectively	kmol/(m ² ·s)	$N_A = K_Y(Y - Y^*)$ $N_A = K_X(X^* - Y)$	177
K_{Xa}	Overall volume liquid mass transfer coefficient	kmol/(m ³ ·s)	$H' = \frac{L}{K_{Xa}A_T} \int_{X_2}^{X_1} \frac{dX}{Y^* - X}$	188
K_{Ya}	Overall volume gas mass transfer coefficient	kmol/(m ³ ·s)	$H' = \frac{V}{K_{Ya}A_T} \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*}$	188
k	Rate constant	Depends on the order of reaction	$k = A' \exp\left(-\frac{E_a}{RT}\right)$	229
k_1	Reaction rate constant for the generation reaction of ES	Depends on the order of reaction		236
k_2	Reaction rate constant of the decomposition reaction of ES into E + S	Depends on the order of reaction		236
k_G	Absorption mass transfer partial coefficient in gas film	kmol/(s·m ² ·kPa)	$k_G = \frac{D}{RT\delta'_G} \frac{p_T}{p_{bm}}$ $N_A = k_G(p_{A1} - p_{A2})$	165 166
k_L	Absorption mass transfer partial coefficient in liquid film	kmol/(s·m ² ·kmol/m ³) or m/s	$k_L = \frac{D_{CT}}{\delta'_L c_{Bm}}$ $N_A = k_L(c_{A1} - c_{A2})$	165 166
(continued)				

(continued)

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
k_Y, k_X	Absorption mass transfer partial coefficient driven by the gas film molar ratio difference ($Y - Y_i$) and liquid film molar ratio difference ($X_i - X$) respectively	kmol/(m ² ·s)	$N_A = k_Y(Y - Y_i)$ $N_A = k_X(X_i - X)$		178
$k_X a$	Liquid film volume mass transfer coefficient	kmol/(m ³ ·s)	$H' = \frac{L}{k_X a A_T} \int_{X_2}^{X_1} \frac{dX}{X_i - X}$		188
$k_Y a$	Gas film volume mass transfer coefficient	kmol/(m ³ ·s)	$H' = \frac{V}{k_Y a A_T} \int_{Y_2}^{Y_1} \frac{dY}{Y - Y_i}$		188
L	Molar flow rate of absorbent	kmol/h			187
L/V	The specific absorbent dosage refers to the molar flow rate of absorbent over molar flow rate of inert gas during the gas mixture treatment process, also known as the ratio of liquid over gas or liquid-gas ratio	Dimensionless			205
$(L/V)_{\min}$	Minimum liquid-gas ratio	Dimensionless	$(L/V)_{\min} = \frac{Y_1 - Y_2}{X_{1\max} - X_2}$ When $X_2 = 0$, $(L/V)_{\min} = m_e \eta_A$		189
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Symbol number	Name and definition	Unit	Correlation equation	Page number
l	Pipe length: the length of the pipe through which the fluid flows, the length of the capillary, the length of the dust chamber, the length of the pipe wall, the effective length of the pipe bundle	m		24, 70, 77, 127, 144
l_e	The equivalent length of fittings or valves, as determined by experiments and are usually expressed in terms of their ratio to diameter	m		29
l_f	The length of the flow	m		56
M	Molar mass	g/mol		15
M_A	Molar mass of component A	g/mol		167
M_i	Molar mass of component i	g/mol		112
m	Mass: the amount of matter	kg	$m = \rho V$	5
m_0		Dimensionless	$m_0 = \left(0.405 + \frac{0.0027}{H_w}\right) \times \left[1 + 0.55 \left(\frac{H_w}{H_w + H_{w1}}\right)^2\right]$	68
m_e	Phase equilibrium constant	Dimensionless	$y_A^* = m_e x_A$ $m_e = \frac{H_{pm}}{pT}$	169
m_{ej}	Phase equilibrium constant of absorbate component j	Dimensionless	$y_j^* = m_{ej} x_j, Y_j^* = m_{ej} X_j$	207
(continued)				

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Symbol number	Name and definition	Unit	Correlation equation	Page number
m_k	Maintenance coefficient	kg(Substrate)/(kg(Cell)·h)	$-r_s = \frac{r_x}{Y_G} + m_k X_C$	246
m_s	Slope coefficient	Dimensionless	$m_s = \cot \alpha$	57
m_t	Thermal diffusivity	m ² /s	$m_t = \frac{\sigma}{\rho C_p}$	121
N	Number of continuous stirred tank reactors	Dimensionless	$c_{AN} = \frac{c_{A0}}{(1+k\tau)^N}$	257
			$x'_{AN} = 1 - \frac{1}{(1+k\tau)^N}$	259
N_A	Diffusion flux, diffusion rate of component A per unit area	kmol/(m ² ·s) or kmol/(m ² ·h)	$N_A = \frac{J_A}{A_d} = -D \frac{dc_A}{dz}$	156
			Under steady conditions: $N_A = \frac{D}{\delta_d} (c_{A1} - c_{A2})$	158
			When waiting for the reverse diffusion of molecules: $N_A = \frac{D}{RT\delta_d} (p_{A1} - p_{A2})$	159
			In the case of unidirectional diffusion: $N''_A = \frac{Dc_T}{\delta_d C_{Bm}} (c_{A1} - c_{A2})$ $= \frac{D}{RT\delta_d} \frac{p_T}{p_{Bm}} (p_{A1} - p_{A2})$	159
N_A	Total absorption mass transfer flux	kmol/(m ² ·s)	$N_A = K_Y(Y - Y^*) = K_X(X^* - X)$	175
$N_{A,G}$	Gas film absorption mass transfer flux	kmol/(m ² ·s)	$N_{A,G} = k_G(p_G - p_i)$	175
$N_{A,L}$	Liquid film absorption mass transfer flux	kmol/(m ² ·s)	$N_{A,L} = k_L(c_i - c_L)$	175

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Symbol number	Name and definition	Unit	Correlation equation	Page number
N'_A	Chemical absorption flux	$\text{kmol}/(\text{m}^2 \cdot \text{s})$	$N'_A = k'_L (c_{Ai} - c_{AL}) = \beta_d k_L (c_{Ai} - c_{AL})$	209
n'	Reaction order: if the chemical experimental results show that the reaction rate of product M can be derived from the following equation: $r_M = \frac{d[M]}{dt} = k c_A^a c_B^b$ For the reactant A, the reaction is a -order reaction, for the reactant B, the reaction is b -order reaction, and the total reaction is $(a + b)$ -order reaction	Dimensionless	$n' = \frac{\lg r_1 - \lg r_2}{\lg c_1 - \lg c_2}$	234
n_A	Moles or kilograms of reactant A	mol or kg		222
n_{A0}	Number of moles of initial reactant A	mol or kg		229
n_g	Generation number: If a division is divided into "one generation", the number of bacterial colonies doubles with each generation	Dimensionless	$n_g = 3.31 g \frac{X'_{C2}}{X'_{C1}}$	243
n_r	Rough coefficient (or roughness, or Manning's n_r , Factor, or resistance factor): it comprehensively reflects the resistance of the river and channel wall to the flow	Dimensionless		57
(continued)				

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Symbol number	Name and definition	Unit	Correlation equation	Page number
[P]	Concentration of product in the reactor or effluent	kg/m ³		245
P'	Parameters	Dimensionless	$P' = \frac{\text{The initial temperature increase of cool fluid}}{\text{The initial temperature difference between two fluids}} = \frac{t_2 - t_1}{T_1 - t_1}$	142
[P̄]	Concentration of product in the reactor or effluent in the stable state	kg/m ³	$[\bar{P}] = Y_{P/S} \left[[S]_{\text{lin}} - \frac{K_s D_{\text{dilution}}}{\mu_{\text{Bmax}} - D_{\text{dilution}}} \right]$	274
p ₀	Pressure above the liquid surface to be transported	Pa		90
P _a	Shaft power of the pump: Power of a transport equipment, such as a pump. The power input from the motor to the centrifugal pump	W or kg·m ² /s ³	$P_a = P_e / \eta$	38
[P _C]	Metabolite concentration	kg/m ³		245
P _d	Motor power matching transport equipment (such as pump)	W or kg·m ² /s ³	$P_d = \frac{P_e}{\eta}$	38
P _e	Effective power: the effective power of a transport equipment (such as a pump) to work on a fluid	W or kg·m ² /s ³	$P_e = W_e q_m = W_e q_V \rho = H_e q_V \rho g$ $P_a = \frac{P_e}{\eta}$	38
				(continued)

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
$[P]_{\max}$	Maximum of concentration of product at the highest yield in the stable state	kg/m ³	$[P]_{\max} = Y_{p/s}([S]_{\text{in}} + K_S - \sqrt{K_S/(K_S + [S]_{\text{in}})})$		279
Pr	Prandtl number: the ratio of molecular momentum transfer capacity to molecular heat transfer capacity	Dimensionless	$Pr = \nu/m = \mu/\rho \times m = \mu C_p/\lambda$		121
p_A^*	Equilibrium partial pressure of Solute A in Gas Phase	kPa	$P_A^* = H_{pm} x_A = \frac{c_A}{H_{pc}}$		173
p_a	Atmospheric pressure	Pa			89
p_{Bm}	The logarithmic mean value of the partial pressure difference of inert gas between the main body of the gas phase and the interface	Pa	$p_{Bm} = \frac{p_{B2} - p_{B1}}{\ln \frac{p_{B2}}{p_{B1}}} (p_T - p_{A1})$ $= \frac{p_T - p_{A2}}{\ln \frac{p_T - p_{A2}}{p_T - p_{A1}}}$		164
p_G	Partial pressure of absorbate in the main body of the gas phase	kPa			180
p_L^*	The partial pressure of absorbate in the gas phase in equilibrium with the main body concentration of the liquid phase c_L	kPa	$p_L^* = \frac{c_L}{H_{pc}}$		181
p_i	Partial pressure of absorbate at the interface	kPa			180
(continued)					

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Symbol number	Name and definition	Unit	Correlation equation		
p_v	Saturated vapor pressure of the transported liquid at the operating temperature	Pa			90
Q	The total heat absorbed by the system from the outside is positive, and the opposite is negative	J	$Q = E_{n2} - E_{n1} + W$		3, 35
Q_e	The amount of heat per unit mass fluid obtained through a heat exchanger	J/kg			35
Q'_e	The energy, including heat and work, obtained by 1 kg fluid when it flows from cross section 1-1' to 2-2'	J/kg			35
q_g	Flow rate of gas mixture	m ³ /s			205
q_h	Average amount of materials processed per hour	m ³ /h			258
q_m	Mass flow rate: the mass of fluid passing through any cross section per unit time	kg/s	$q_m = \rho q_v$		12
q'_m	Mass flow of a condensation steam	kg/s			135
q_{mA}	Material output per unit of time	mol/s or kg/s			259

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(continued)						Page number
Symbol number	Name and definition	Unit	Correlation equation			
q_T	Heat flux or heat flow density, represents the heat transfer quantity per unit time and area	W/m ² or J/(m ² ·s)	$q_T = \frac{\phi}{A_h} = \frac{Q}{\tau A_h}$			110
q'_T	Heat flow rate per unit pipe length	W/m or J/(m·s)	$q'_T = \frac{\phi}{L} = \frac{2\pi(t_1 - t_2)}{\frac{1}{\sigma} \ln(r_2/r_1)}$			116
q_V	Volume flow rate: the volume of fluid passing through any cross section per unit time	m ³ /s	$q_V = uA$			12
q_{Vg}	Flow rate of pipeline system	m ³ /h				84
R	Molar gas constant: 8.314	J/(mol·K)				242
R'	Parameters	Dimensionless	$R' = \frac{\text{Temperature drop of thermal fluid}}{\text{Temperature rise of cold fluid}}$ $= \frac{T_1 - T_2}{t_2 - t_1}$			142
R_C	Growth rate: the generation number per unit time	s ⁻¹	$R_C = \frac{n_g}{t_2 - t_1} = \frac{\frac{x'}{x_{cl}}}{t_2 - t_1}$			249
Re	Reynolds number: represents the ratio of inertia force to viscous force, and the flow state and turbulence degree of the fluid, which is dimensionless	Dimensionless	$Re = \frac{du\rho}{u}$			16
(continued)						

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Symbol number	Name and definition	Unit	Correlation equation		
R_h	Hydraulic radius: the ratio of the overflowing area of a water conveyance cross section to the side length (wetted perimeter) of the water pipeline in contact with the water body	m	$R_h = \frac{A}{\chi}$		56, 57
r	1. Cylinder wall radius 2. The radius of any place between the center of the pipe and the wall of the pipe 3. Reaction rate 4. The rate of enzyme-catalyzed reaction	m m kg/(m ³ ·h) kg/(m ³ ·h)	$r = k c_A^a c_B^b$ $r = r_m \frac{[S]}{K_m + [S]}$		239
r_0	Radius from the center of the pipe to the wall of the pipe	m			24
r_1, r_2	Inner and outer wall radii of the cylinder wall	m			113
r_A	The reaction rate of the reactants A: the reaction rate of the components A in the chemical reaction	mol/(m ³ ·s)	$r_A = \frac{1}{V} \frac{dn_A}{dt}$		222
r_M	The reaction rate of the product M	mol/(m ³ ·h) or kg/(m ³ ·h)	$r_M = \frac{dc_M}{dt} = k c_A^a c_B^b$		223
r_m	Logarithmic mean radius of the cylinder wall	m	$r_m = \frac{r_2 - r_1}{\ln(r_2/r_1)}$		113
(continued)					

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Symbol number	Name and definition	Unit	Correlation equation	Page number
r_p	Reaction rate of product P	mol/(m ³ ·h) or kg/(m ³ ·h)	$r_p = \frac{d[p]}{dt} = k' c_A^{\alpha'} c_B^{b'}$	223
r_s	Substrate consumption rate	kg(matrix)/(m ³ ·h)	$-r_s = \frac{d[S]}{dt} = \frac{r_X}{Y_{X/S}}$	245
r_T	Reaction rate at temperature of T °C	kg/(m ³ ·h)		241
r_X	Microbial growth rate	kg/(m ³ ·h)	$r_X = \frac{dX_C}{dt} = \mu_B X_C$	244
Sc	Schmidt number: a dimensionless scalar, defined as the ratio of kinematic viscosity to diffusion coefficient, used to describe a fluid with both momentum and mass diffusion	Dimensionless	$Sc = \frac{\mu}{\rho D}$	166
Sh	Sherwood number: a dimensionless degree group that reflects the undetermined mass transfer coefficient, which characterizes the ratio of convective mass transfer to diffusion mass transfer	Dimensionless	$Sh = 0.023 Re^{0.83} Sc^{0.33}$ $= \frac{k_L d}{D} \frac{c_{Bm}}{c_T} = \frac{k_G d R T}{D} \frac{p_{Bm}}{p_T}$	166
[S]	Substrate concentration	kg/m ³		239
[S] _{lin}	Restricted substrate concentration in influent	kg/m ³		273
T	Temperature: a physical quantity that indicates the degree of cold and heat of an object	K		5

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Symbol number	Name and definition	Unit	Correlation equation	Page number	
T or t	Represents the temperature of hot and cold fluids, respectively	K		132	
t, t_w	The temperature of the heat transfer wall surface in contact with the fluid and the fluid, respectively	K		119	
$\tau_{1/2}$	The half-life ($\tau_{1/2}$), the time needed for the initial concentration of a material c_{A0} to be reduced to $\frac{1}{2} c_{A0}$	S	$\tau_{1/2} = A \frac{1}{d^{n-1}}$	226	
t_1	Main body temperature of the hot fluid	K		133	
t_2	Main body temperature of the cold fluid	K		133	
t_1, t_2	Temperature on both sides of the flat wall	K		133	
t_l	The temperature at the distance l	K		113	
t_s	Saturation temperature of condensate	K		132	
t_w	Wall temperature	K		132	
t_{w1}	Temperature of the interwall surface in the hot fluid side	K		133	
t_{w2}	Temperature of the interwall surface in the cold fluid side	K		133	
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Symbol number	Name and definition	Unit	Correlation equation		
U	Internal energy per unit mass	J			34
u	Average flow velocity or average velocity of a cross section, the distance of fluid passed in the direction of flow per unit of time	m/s	$u = q_v / A$		12
u_0	Gas velocity of empty tower: the empty tower velocity of the gas in the tower, the gas flow velocity calculated on the basis of the cross-sectional area of the empty tower	m/s			200
u_f	Gas velocity avoiding liquid overflow	m/s	$\lg \left[\frac{a_{BET} u_f^2}{g \varepsilon_V^2} \frac{\rho_G}{\rho_L} \frac{\mu_L^{0.2}}{\rho_L} \right]^{1/4} \left(\frac{\rho_G}{\rho_L} \right)^{1/8}$ $= B_m - 1.75 \left(\frac{L}{V} \right)^{1/4} \left(\frac{\rho_G}{\rho_L} \right)^{1/8}$		200
μ_w	Viscosity of fluid at the inner wall temperature	Pa · s			
u'	The maximum allowable flow velocity avoiding scouring, referred to as the allowable flow velocity for nonscouring or the maximum design flow velocity	m/s			63
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Symbol number	Name and definition	Unit	Correlation equation		
u''	The minimum allowable flow velocity without siltation, referred to as the allowable flow velocity without siltation or the minimum design flow velocity	m/s			63
V	1. Volume: the amount of space occupied by a material or object 2. Molar flow rate of inert gas	m ³ kmol/h	When indicating the mass flow rate of inert gas, there are: $Y = \frac{L}{V}X + (Y_1 - \frac{L}{V}X_1)$		34 182
V_{mA}	Molar volume of component A at the normal boiling point	m ³ /mol			162
V_R	Effective volume of reactor	m ³	$V_R = \frac{V_T \phi_R / T}{\text{daily treatment volume}}$ $V_R = \frac{24}{\times (\tau + \tau') = q_h (\tau + \tau')}$		250, 252
V_R, S	Effective volume of a continuous stirred tank reactor	m ³			263
V_R, P	Effective volume of a plug flow reactor	m ³			263
W	Work, the external work system does. The system receiving external work is negative, and the opposite is positive	J or kg·m ² /s ²			3
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Symbol number	Name and definition	Unit	Correlation equation		
W_e	The work received by a unit mass fluid during the process of passing through the system, which is positive and the opposite is negative	J/kg	$gz_{h1} + \frac{u_1^2}{2} + \frac{p_1}{\rho} + W_e = gz_{h2} + \frac{u_2^2}{2} + \frac{p_2}{\rho} + \Sigma h_{t,f}$		35
W_{e-pa}	The work received by a unit mass fluid in the process of passing through the system, calculate by Pa	Pa	$\rho gz_{h1} + \frac{\rho u_1^2}{2} + p_1 + W_{e-pa} = \rho gz_{h2} + \frac{\rho u_2^2}{2} + p_2 + \Delta p_{t,f}$		
w_i	Mole fraction: the mole fraction of component i in the liquid mixture	Dimensionless			108
X	Molar ratio	Dimensionless	Molar ratio X $= \frac{\text{Mass of solute/Molar mass of solute}}{\text{Quality of solvent/Molar mass of solvent}} = \frac{x}{1-x}$		168
X^*	The molar ratio in the liquid phase in which the absorbate molar ratio Y is in equilibrium with the main body of the gas phase	Dimensionless			177
X_2	The composition of the liquid entering the tower, expressed by the molar ratio	Dimensionless			197
X_A	Mole fraction of solute A in solution	Dimensionless			170
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Symbol number	Name and definition	Unit	Correlation equation	Page number
X'_{C1}	Number of cells at time τ_1	Dimensionless		243
X'_{C2}	Number of cells at time τ_2	Dimensionless	$X'_{C2} = X'_{C1} \cdot 2^{n_g}$	243
X_C	Cell concentration	kg/m ³		244
\overline{X}_C	Microorganism concentration in the reactor or effluent in the stable state	kg/m ³	$\overline{X}_C = Y_{X/S} \left[[S]_{lin} - \frac{K_S D_{dilution}}{\mu_{Bmax} - D_{dilution}} \right]$	274
X_{Cmax}	The concentration of microorganisms at the highest yield in the stable state	kg/m ³	$X_{Cmax} = Y_{X/S} \left[[S]_{lin} + K_S - \sqrt{K_S / (K_S + [S]_{lin})} \right]$	274
X_i	Liquid phase molar ratio of absorbate at the interface	Dimensionless		177
X_j	Molar ratio of j component in liquid phase	Dimensionless		207
x	Mole fraction	Dimensionless	$\begin{aligned} &\text{Mole fraction } x \\ &= \frac{\text{Mass of solute/molar mass of solute}}{\frac{\text{Mass of solute}}{\text{Molar mass of solute}} + X} = \frac{X}{1 + X} \end{aligned}$	168
x_i	Mole fraction: the mole fraction of the component i in mixed liquid	Dimensionless		15
x_A	Mole fraction of solute A in solution	Dimensionless		169
x'_A	Conversion efficiency of reactant A	Dimensionless	$x'_A = (n_{A0} - n_A) / n_{A0}$	251
(continued)				

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Symbol number	Name and definition	Unit	Correlation equation		
x''_A	Conversion efficiency	Dimensionless	$x''_A = \text{Amount of reactants converted to target products and by-products/amount of reactants entering the reactor}$		265
x'_{AN}	Final conversion efficiency: final conversion efficiency of multiple continuous stirred tank reactor	Dimensionless	$x'_{AN} = 1 - \frac{1}{(1+k\tau)^N}$		260
Y^*	The molar ratio in the gas phase in which the absorbate materials is in equilibrium with the main body of the liquid phase X	Dimensionless	$Y^* = m_e X$		177
Y_A^*	The molar ratio of solute A to inert components in the gas phase, which is in equilibrium with the X_A of solute A in the liquid phase	Dimensionless	$Y_A^* = m_e X_A$		170
Y_1	The composition of the gas entering the tower, expressed by the molar ratio	Dimensionless			197
Y_2	The composition of the gas out of the tower, expressed by the molar ratio	Dimensionless			297
Y_G	Maximum cell yield coefficient without maintenance metabolism	kg(Cell)/kg(Substrate)	$-r_s = \frac{r_X}{Y_G} + m_k X_C$		246
Y_i	Gas phase molar ratio of absorbate at the interface	Dimensionless			177
(continued)					

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Symbol number	Name and definition	Unit	Correlation equation		
Y_j^*	The molar ratio of the component j in the gas phase which is in equilibrium with the liquid phase X_j	Dimensionless	$Y_j^* = m_{ej} X_j$		207
$Y_j - Y_{j+1}$	Variation of gas phase composition	Dimensionless			189
$(Y - Y^*)_{\text{average}, j \rightarrow j+1}$	Mean driving force of mass transfer based on a overall mass transfer unit in the gas phase		$\frac{Y_j - Y_{j+1}}{(Y - Y^*)_{\text{average}, j \rightarrow j+1}} = 1$		189
$Y_{p/s}$	Product yield coefficient	kg(-product)/kg(-substrate utilized)			279
$Y_{x/s}$	Apparent microbial yield coefficient	kg(cell)/kg(matrix)			245
y_i	Mole fraction: the mole fraction of component i in gas mixture	Dimensionless			15
y_A^*	The equilibrium concentration of solute A in the gas phase corresponding to the concentration of the liquid phase x_A , expressed in mole fraction	Dimensionless	$y_A^* = m_e x_A = \frac{Y_A^*}{1 + Y_A^*}$		169
Z	Distance along the direction of diffusion	m			156

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Symbol number	Name and definition	Unit	Correlation equation	Page number
α	Convection heat transfer film coefficient or convection heat transfer coefficient, heat transfer coefficient, heat transfer film coefficient	$W/(m^2 \cdot K)$		119
α_1	Convection heat transfer film coefficient of hot fluid	$W/(m^2 \cdot K)$		133
α_2	Convective heat transfer film coefficient of cold fluid	$W/(m^2 \cdot K)$		133
α_a	Association parameters of a typical solvent	Dimensionless		164
α_f	Filling degree	Dimensionless	$\alpha_f = h/d$	59
β_a	Multiple of the absorption flux increased by chemical reaction is called the enhancement factor or reaction factor	Dimensionless	$N'_A = k'_L(c_{Ai} - c_{AL}) = \beta_a k_L(c_{Ai} - c_{AL})$	209
β_g	Gas expansion coefficient: the inverse of temperature T , the ratio of the increment of the volume of a gas to the volume of the original gas for every 1 K of temperature increase.	K^{-1}	$\beta_g = \frac{V_2 - V_1}{V_1 \Delta T}$	121
β_m	Width-depth ratio	Dimensionless	$\beta_m = b_w/h$	62

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Symbol number	Name and definition	Unit	Correlation equation	Page number
β_s	Selectivity	Dimensionless	$\beta = \frac{\text{The amount of reactants converted to the target products/the amount of reactants converted to the target products and by-products}}$	265
γ	Specific substrate consumption rate, the substrate consumption rate divided by the amount of bacteria	kg(matrix)/[kg(cell)·h]	$\gamma = r_s/X_c$	245
γ_{\max}	Maximum specific substrate consumption rate	kg(matrix)/[kg(cell)·h]		246
Δc	The driving force expressed by the molar concentration difference in the liquid film	Kmol/m ³	$\Delta c = c_G^* - c_L$	180
ΔH_c	The critical cavitation allowance ΔH_c is the minimum specified value, which is the sum when the hydrostatic head ($\frac{p_L}{\rho_g}$) and the dynamic head of the liquid $\frac{\mu_L^2}{2g}$ at the entrance of the centrifugal pump subtracts the saturated vapor head of the liquid $\frac{p_v}{\rho_g}$ at the operating temperature.	mH ₂ O	$\Delta H_c = \left(\frac{p_L}{\rho_g} + \frac{\mu_L^2}{2g} \right) - \frac{p_v}{\rho_g}$	90
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Symbol number	Name and definition	Unit	Correlation equation		
Δp	1. Pressure difference: the pressure difference between the two ends of the fluid flow in the pipe 2. The driving force expressed by the pressure difference in the gas film	Pa	$F_n = \Delta p \cdot A$ $\Delta p = p_G - p_L^*$		24, 180
Δp_f	The straight pipe friction loss, mechanical energy consumed for fluid flow to overcome resistance	Pa	$\Delta p_f = \lambda \cdot \frac{l}{d} \cdot \frac{u^2 \rho}{2}$		25
$\Delta p'_f$	The local resistance loss	Pa	$\Delta p'_f = \zeta \cdot \frac{u^2 \rho}{2}$ $\Delta p'_f = \lambda \cdot \frac{l_e}{d} \cdot \frac{u^2 \rho}{2}$		
$\Delta p_{t,f}$	The total resistance loss	Pa	$\rho g z_{h1} + \frac{\rho u_1^2}{2} + p_1 + W_{e-pa} = \rho g z_{h2} + \frac{\rho u_2^2}{2} + p_2 + \Delta p_{t,f}$		110
Δt	Temperature difference, driving force of heat conduction	K			
Δt_m	Mean temperature difference	K	$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\ln \frac{\Delta t_1}{\Delta t_2}}$		136
ΔX_m	Liquid phase mean driving force	Dimensionless	$\Delta X_m = \frac{X_1^* - X_1 - (X_2^* - X_2)}{\ln \frac{X_1^* - X_1}{X_2^* - X_2}}$		191
ΔY_m	Gas phase mean driving force	Dimensionless	$\Delta Y_m = \frac{Y_1 - Y_1^* - (Y_2 - Y_2^*)}{\ln \frac{Y_1 - Y_1^*}{Y_2 - Y_2^*}}$		191
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Symbol number	Name and definition	Unit	Correlation equation	Page number
δ	Thickness	m		157
δ_B	Thickness of flow boundary layer	m		118
δ_b	Actual thickness of heat conduction laminar sublayer	m		115, 116
δ_d	Thickness of diffusion layer	m		156
δ_f	Thickness of the virtual heat conduction layer outside the laminar sublayer, which represents the thermal resistance of the fluid particle with relative displacement and mixing	m		115, 116
δ_p	Thickness of flat wall	m		110, 132
δ_t	The equivalent thickness of the heat conduction boundary layer, which concentrates all thermal resistances and temperature differences and transfers the heat via heat conduction	m	$\delta_t = \delta_b + \delta_f$	116
δ'_b	Thickness of heat transfer boundary layer	m		165
δ'_G	Thickness of virtual stagnant film of the gas phase	m		165
δ'_L	Thickness of virtual stagnant film of the liquid phase	m		165
(continued)				

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
ε	Absolute roughness: represents the average height of the protruding part of the wall surface	m		26	
ε/d	Relative roughness	Dimensionless		27	
ε_V	Packing voidage	Dimensionless		200	
ζ	Local loss coefficient, measured experimentally	Dimensionless		29	
η	Efficiency: the efficiency of transport equipment, such as a pump	%	$\eta = P_e / P_a$	37, 79	
η_A	Absorption efficiency of absorbate or recovery of absorbate	%	$\eta_A = \frac{\text{The amount of solute absorbed}}{\text{The amount of solute in the mixture}} = \frac{Y_1 - Y_2}{Y_1}$	185	
η_A	Conversion efficiency of component A	%	$\eta_A = \frac{n_{A0} - n_A}{n_{A0}} = \frac{a\dot{U}}{n_{A0}}$	223	
η_e	Volume efficiency: the ratio of the effective volume (reaction volume) of a plug flow reactor to the effective volume of a continuous stirred tank reactor under the conditions of the same temperature, output and conversion efficiency	Dimensionless	$\eta_e = \frac{V_{R,P}}{V_{R,S}} = \frac{\tau_P}{\tau_S}$	263	
					(continued)

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
η_T	Total plate efficiency: the ratio of the number of equivalent of a theoretical plate to the actual number of plates	Dimensionless	$\eta_T = N_T/N_P$	197
η'	The efficiency of matching motor, the ratio of shaft power of the pump to power of the motor	Dimensionless		38, 102
θ	Angle of triangular weir	Radian		77
θ_t	Temperature coefficient: 1.02 ~ 1.25 in general	Dimensionless	$\theta_t = 1.02\text{--}1.25$	241
Λ	Specific volume: the volume per unit mass is called the specific volume of the fluid, which is the reciprocal of density	m ³ /kg	$\Lambda = V/m$	35
λ	Friction coefficient: the ratio of the resistance loss over kinetic energy, in which the resistance loss is along the flow path per unit mass of fluid flowing through a cross section of pipeline at the same distance as the diameter of the pipeline	Dimensionless	$\lambda = \frac{H_f/(l/d)}{u^2/2g}$	25
μ	Viscosity: Proportional coefficient, known as viscous coefficient or dynamic viscosity	kg/(m·s), Pa·s, N·s/m ²		16
(continued)				

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
μ_B	Specific growth rate	h^{-1}	$\mu_B = \frac{1}{X_C} \frac{dX_C}{dt} = \frac{\mu_{B\max}[S]}{K_s + [S]}$	244
$\mu_{B\max}$	Maximum specific growth rate	h^{-1}		244
μ_i	Viscosity: the viscosity of component i in mixed liquids	Pa·s		15
μ_{mix}	Mixed viscosity: the viscosity of mixed liquids or gases	Pa·s	$\log \mu_{\text{mix}} = \sum_{i=1}^n x_i \log u_i$ $\mu_{\text{mix}} = \frac{\sum_{i=1}^n y_i u_i M^{1/2}}{\sum_{i=1}^n y_i M^{1/2}}$	15
μ_w	Viscosity of the fluid at the inner wall temperature	Pa·s		
ν	Kinematic viscosity: indicates level of fluid viscosity, expressed by the ratio of viscosity μ to density ρ	m^2/s	$\nu = \frac{\mu}{\rho}$	14
(continued)				

(continued)				
Symbol number	Name and definition	Unit	Correlation equation	Page number
ξ	Drag coefficient: the ratio of the drag force acting on a particle to the product of the projected area of the particle in its direction of motion and the hydrodynamic pressure. Generally, it needs to be determined by experiment	Dimensionless	1. Laminar flow region: $Re_P < 2$ $\xi = \frac{24}{Re_P}$ 2. Transition zone: $2 < Re_P < 1000$ $\xi = \frac{18.5}{Re_P^{0.6}}$ 3. Turbulent region: $1000 < Re_P < 2 \times 10^5$ $\xi = 0.44$ 4. Turbulent boundary layer region $2 \times 10^5 < Re_P$ $\xi = 0.1$	73, 74
ρ	Fluid density, the mass of a fluid per unit volume	kg/m ³	$\rho = m/V$	10
$\frac{\rho_G}{\rho_L}$	Density ratio of gas over liquid	Dimensionless		200
$\rho g \beta_g \Delta t$	Buoyance	N		120
Σh_f	The sum of all straight pipe friction losses	J/kg		23
$\Sigma h'_f$	The sum of all local losses	J/kg		23
(continued)				

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
$\Sigma H_{tf, 0-1}$	Resistance head loss of liquid from cross section 0-0' to 1-1'	mLiquid column		89	
ΣH_{tf}	Total head loss	mLiquid column	$\Sigma H_{tf} = \Sigma h_{tf} / g$	37	
Σh_{tf}	Total friction loss, friction loss, the sum of all straight pipe friction losses and all local losses	J/kg	$\Sigma h_{tf} = \Sigma h_f + \Sigma h'_f$	23	
Σq_{m-out}	Total amount of output material: the total input materials of the system per unit time	kg/s		1	
$\Sigma q_{m-reaction}$	Total amount of reaction materials: the total reacted materials of the system per unit time	kg/s		1	
$\Sigma q_{m-accumulation}$	Total amount of accumulated materials: the total accumulated materials of the system per unit time	kg/s		1	
Σq_{m-in}	Total amount of input materials: the total input materials of the system per unit time	kg/s	$\Sigma q_{m-in} = \Sigma q_{m-out} + \Sigma q_{m-accumulation} + \Sigma q_{m-reaction}$	1	
σ	Thermal conductivity: represents heat conduction rate of a material per unit area and per unit temperature gradient	W/(m·K)	$\sigma = -\frac{\Phi}{A_h} \frac{dT}{d\delta} = -\frac{q_T}{d\delta}$	107	
(continued)					

(continued)

(continued)					Page number
Symbol number	Name and definition	Unit	Correlation equation		
σ, σ_0	Thermal conductivity of the solid at temperatures of t and t_0	W/(m·K)	$\sigma = \sigma_0(1 + a_t(t - t_0))$		107
σ_m, σ_i	Thermal conductivity of the liquid mixture and component i	W/(m·K)	$\sigma_m = K_C \sum_{i=1}^n \sigma_i w_i$		108
σ'_m, σ'_i	Thermal conductivity of the gas mixture and component i	W/(m·K)	$\sigma'_m = \frac{\sum_{i=1}^n \sigma'_i y_i M_i^{1/3}}{\sum_{i=1}^n y_i M_i^{1/3}}$		109
τ	1. Time 2. Reaction time: the reaction time necessary for reactants to reach a certain conversion efficiency in a batch reactor	s s	When indicating the reaction time, there are: $\tau = n_{A0} \int_0^{x_A'} \frac{dx_A'}{(-r_A)}$		5, 251
$\tau_{1/2}$	Half-life: the time needed for the initial concentration of a material c_{A0} to be reduced to $1/2c_{A0}$	s	$\tau_{1/2} = \hat{G} \frac{1}{c_{A0}^{n'-1}}$		225, 229
τ_p	Particle settling time	s	$\tau_p = \frac{H}{u_\tau}$		77
τ_g	Gas passage time	s	$\tau_g = \frac{l}{u}$		77
τ'	Shear stress: internal friction force per unit area	Pa	$\tau' = -\mu \frac{du}{dy}$		14
(continued)					

(continued)

Symbol number	Name and definition	Unit	Correlation equation	Page number
ϕ	Heat flow rate or heat transfer rate: heat transfer quantity per unit time	W or J/s	$\phi = \frac{Q}{\tau} = -\sigma A_h \frac{dt}{db}$ <p>For single-layer flat walls:</p> $\phi = \frac{\sigma}{\delta_p} A_h (t_1 - t_2)$ $\text{or } \phi = \frac{t_1 - t_2}{\frac{\delta_p}{\sigma A_h}} = \frac{\Delta t}{\Omega}$ <p>For the n flat wall of the status layer</p> $\phi = \frac{t_1 - t_{n+1}}{\sum_{i=1}^n \Omega_i} = \frac{t_1 - t_{n+1}}{\sum_{i=1}^n \frac{\delta_{pi}}{\sigma_i A_h}}$ <p>For cylinder walls:</p> $\phi = 2\pi l \sigma \frac{t_1 - t_2}{\ln(r_2/r_1)}$ $= \frac{2\pi l (t_1 - t_2)}{\frac{1}{\sigma} \ln(r_2/r_1)}$ <p>Newton's Law of Cooling:</p> $\phi = \alpha A \Delta t = \frac{\Delta t}{\frac{1}{\alpha A}} = \frac{\Delta t}{\Omega}$	106, 109, 110, 112
ϕ_e	Heat load of a heat exchanger: heat transfer quantity per unit time	W or J/s	$\phi_e = q'_{m,h} [t_e + C_{p,h}(t_s - T_2)] = q_{m,c} C_{p,c} (t_2 - t_1)$	132
ϕ_c	Central angle	Radian		58

(continued)

(continued)

Symbol number	Name and definition	Unit	Correlation equation	Page number
ϕ_d	The angle between the bottom slope of the open channel and the horizontal line	Radian		57
$\phi_{R/T}$	Loading coefficient, the ratio of the effective volume (V_R) of the reactor over the total volume (V_T) of the reactor	Dimensionless	$\phi_{R/T} = \frac{V_R}{V_T}$	250
ϕ_y	Yield, represents the utilization efficiency of raw materials	Dimensionless	$\phi_y = \beta_s x'_A$	265
ϕ_s	Sphericity, the ratio of the surface area of a sphere of the same volume to the surface area of an object	Dimensionless		75
χ	Wetted perimeter, wetted perimeter length	m	$\chi = b_w + 2h\sqrt{1 + m_s^2}$	57
Ω	Thermal resistance of heat conduction	K/W K/W or (K · s)/J	$\Omega = \frac{\delta_p}{\sigma A_h}$ $\Omega = \frac{1}{\alpha A}$	134
\ddot{U}	Reaction progress	Dimensionless	$\ddot{U} = \frac{n_{A0} - n_A}{a}$	223

Index

Basic Concepts and Terminology

A

Absolute pressure, 16, 17, 56, 157, 158, 160, 243, 261
Absolute roughness, 39, 40, 43, 66, 166, 467, 508
Absolute specificity, 389
Absolute temperature, 260
Absorbent dosage L (kmol/h), 301, 339, 355
Absorption and mass transfer flux, 282
Absorption and mass transfer flux equation, 281, 300
Absorption and mass transfer rate, 481
Absorption driving force, 276
Absorption factor, 313, 322, 331, 348, 356, 470, 471
Absorption factor method, 310, 312, 316
Absorption flux, 299, 340
Absorption flux equation, 307, 330
Absorption mass transfer coefficient, 308, 348
Absorption operation line equation, 299, 330
Absorption rate, 275, 339
Acceleration, 8, 15, 84, 115
Activation energy, 386–389
Activator, 388, 396
Actual number of plates, 320
Addition reaction, 431
Adiabatic process, 54
Adiabatic reaction, 364
Adsorption, 112, 245, 248, 286
Allowable velocity, 95
Allow suction vacuum, 124–126, 138, 139, 167

Along the course, 477
Amount of matter, The, 487
Annular boundary layer, 31
Apparent cell yield coefficient, 503
Area, 9, 21, 29, 33, 35, 36, 48, 50, 52, 55, 86, 88, 92–94, 96, 107, 109, 113, 114, 120, 122, 142, 147, 163, 165, 166, 172, 173, 178, 181, 185, 191–193, 205, 209, 212–214, 218, 222, 223, 230–232, 235, 236, 238, 242, 245, 252, 258, 300, 305, 308, 317, 325, 347, 470–472, 495, 498, 511–513, 515
Arrhenius activation energy, 386, 476
Arrhenius equation, 386
Association parameter, 262, 504
Atmospheric pressure, 16, 27, 56, 57, 62, 122, 138–141, 157, 158, 160, 163–167, 492
Atomic reaction, 364
Average density, 27
Average driving force, 308, 310
Average driving force of mass transfer, 309
Average flow velocity or average velocity, 18
Average residence time, 404, 419
Average temperature difference, 215, 242, 506
Average velocity, 18, 24, 25, 27, 31, 38, 39, 83, 84, 111, 159, 197
Axial backmixing model, 405
Axial concentration gradient, 406, 476
Axial diffusion, 405, 406
Axial diffusion coefficient, 406, 475

B

Backflow, 3
 Backmixing, 403, 405, 414, 416
 Bacteria, 146, 246, 396–398, 401, 402, 436, 437, 439–442, 444, 505
 Bacterial quantity, 397
 Balance, 1–4, 47, 48, 56, 209, 210, 218, 221, 333, 334, 408, 409, 446
 Balanced partial pressure, 280
 Base mass, 1
 Basic dimension, 38–40
 Batch reactor, 406, 407, 409–412, 416, 429, 432, 433, 449–451, 513
 Bernoulli equation, 50, 51, 55, 56, 58–67, 76, 78, 80, 109, 110, 134, 138, 157, 159
 Biochemical oxygen demand, 2
 Biochemical reaction, 364, 388, 403
 Biological treatment, 245, 395, 436
 Blade twist angle, 156
 Body resistance, 33, 35, 41
 Boiling point, 260, 458, 459, 462, 463, 499
 Bottom slope, 83–85, 91, 92, 94, 97, 98, 100, 102, 164
 Boundary layer at the entrance of circular pipe, 32
 Boundary layer separation, 31, 33, 34, 114
 Boundary layer theory, 29
 Boundary Reynolds number, 30
 Branch line, 74, 79
 Brownian motion, 117
 By-product, 502, 505

C

Carrier gas desorption, 329
 Carrying capacity, 408
 Catalytic activity, 389
 Cell concentration, 398, 402, 501
 Cell yield, 437, 502
 Cell yield coefficient, 437
 Central angle, 89
 Centrifugal compressor, 142–144
 Chain reaction, 369
 Characteristic constant of enzyme, 393, 449, 483
 Characteristic curve, 127, 132, 135, 136, 149
 Characteristic curve of centrifugal pump, 124, 129, 160
 Chemical absorption, 277, 278, 299, 336, 338–342, 350, 351, 355
 Chemical oxygen demand, 2, 6
 Chemical reaction rate, 339, 340, 342–344, 351–354, 364, 406, 409

Chemical reaction rate constant, 340
 Chézy coefficient, 84, 85, 89, 91, 96, 472
 Chlorination reaction, 435
 Clearance volume, 147
 Coenzyme, 396
 Compactness degree, 95
 Competitive inhibition, 400
 Competitive reaction, 379
 Complex flow, 217
 Complex reaction, 363, 368, 430, 431
 Components, 15, 22, 23, 83, 84, 139, 147, 153, 175, 176, 247, 248, 251–256, 258, 260, 261, 263, 267, 273, 277–279, 322, 332, 336–340, 347–350, 356, 366, 370, 371, 382, 383, 401, 402, 408, 409, 414, 418, 421, 449, 473, 475, 487, 495, 499–503, 508, 510
 Compressible fluid, 54
 Concentration, 2, 10, 117, 170, 245, 246, 251, 252, 255, 256, 258, 262, 263, 266, 267, 269, 270, 273–276, 278–282, 284–286, 289, 296, 297, 299, 305, 314, 315, 317, 330, 336–338, 340, 341, 346, 349, 355, 365–368, 370–373, 375–377, 379–382, 390, 392, 393, 398–401, 404, 407, 409, 411, 412, 414, 416–422, 431–445, 448–451, 473, 474, 477, 483, 492, 503
 Concentration difference, 7, 247, 251, 266, 275, 276, 282, 289, 290, 294, 295, 330, 357
 Concentric cylinder layer, 20
 Condensation heat of saturated steam, 210
 Conservation of energy, 4
 Constant temperature heat transfer, 215
 Contact surface area, 21
 Continuous flow reactor, 403, 406, 423, 427
 Continuous medium, 14, 48
 Continuous reaction, 5, 363, 364, 369, 382, 383
 Continuous steady state operation, 48
 Control body, 14, 47
 Convection diffusion, 264–266
 Convection-diffusion mass transfer coefficient, 264
 Convection-diffusion mass transfer flux, 265
 Convection heat transfer, 188, 189, 192–195, 197, 198, 202, 206, 209, 211, 215, 223, 235, 237–239, 264, 504

- Convection heat transfer film coefficient, 193, 195, 197–201, 204, 205, 214, 215, 223, 240, 241, 243, 504
- Convection heat transfer resistance, 206, 215
- Convective heat transfer, 197
- Convective heat transfer rate, 265
- Convective mass transfer, 264, 288, 496
- Conversion efficiency, 366, 409–414, 416, 417, 419–425, 428, 429, 431, 432, 445–447, 450, 451, 502, 508
- Countercurrent, 194, 217, 218, 220–222, 236, 239, 242, 276, 297, 298, 303, 305, 306, 314, 318, 321, 322, 329, 330, 349, 356
- Covalent modification of enzyme, 389
- Crank drive, 147
- Critical diameter of insulation layer, 204, 476
- Cross flow, 220, 231
- Cross section, 17–19, 26, 31, 35, 48, 49, 51, 52, 54–56, 58–62, 64–66, 71, 72, 74, 76, 78, 80, 83–89, 91–98, 100, 107–109, 112, 122, 128, 138, 159, 164, 173, 197, 223, 235, 274, 289, 297–299, 306, 310, 312, 313, 325, 337, 404, 414, 416, 483, 493–495, 512
- Culture temperature, 398
- Cutting law of centrifugal pump, 130
- D**
- Dalton's law of partial pressure, 272
- Death period, 395
- Death rate, 398
- Decay period, 398, 439
- Decay rate, 372
- Decomposition reaction, 387, 388, 391, 412, 431
- Degree of freedom, 267
- Degree of turbulence, 237
- Density, 3, 5, 10, 14–17, 21, 23–25, 27, 30, 39, 48, 54, 55, 57, 64, 66, 69, 109, 111–113, 116–118, 122, 123, 129, 138–140, 159, 164, 173, 174, 178, 179, 185, 191, 193, 195, 240, 257, 260, 264, 271, 273, 290, 326, 345, 348, 356, 454, 455, 458, 459, 462, 463, 465, 466, 509–511
- Design point, 127
- Desorption, 248, 266, 267, 272, 274, 275, 288, 296, 297, 313, 329–335, 347–349, 355–357
- Desorption driving force, 329
- Desorption factor, 313, 331, 471
- Diameter, 24, 26–28, 38–40, 45, 46, 48, 50, 51, 62, 64, 69, 72, 74, 98, 108, 110, 111, 113, 116–118, 124–126, 130, 133, 139, 158–160, 163, 165, 197, 200, 202, 204, 205, 208, 209, 214, 223, 231, 232, 236, 238, 241, 297, 317, 325, 408, 413, 475, 476
- Differential balance, 47
- Diffuser mass, 142
- Diffusion, 146, 252, 257–260, 263, 264, 278, 341–344, 347, 348, 351–354, 396, 403, 475, 488, 496, 503
- Diffusion area, 252, 471
- Diffusion coefficient, 258–264, 347, 475, 496
- Diffusion flow, 255
- Diffusion flux, 252, 254, 255, 263, 488
- Diffusion process, 247, 257
- Diffusion rate, 252, 264, 339, 340, 482
- Diffusion rate of reaction products, 339
- Diffusion resistance, 258, 338
- Dilution rate, 475
- Dimensional analysis, 35, 38, 40, 41, 113, 195
- Dimensionless number groups, 41
- Direct contact heat exchanger, 224
- Dissociation equilibrium constant, 341
- Dissolved oxygen, 146, 245, 262, 273
- Distance, 18, 20, 21, 29–31, 34, 35, 38, 55–57, 83, 84, 115, 138, 166, 179, 191, 230, 233, 252, 257, 258, 263, 299, 301, 321, 497, 503
- Disturbance, 25, 237
- Drag coefficient, 113, 114, 116, 511
- Drag force, 112–115, 281, 511
- Drift factor, 256
- Driving force, 6, 7, 72, 178, 180, 185, 247, 267, 275, 281, 282, 285, 288, 289, 291, 294, 295, 300, 301, 306, 310, 311, 321, 330, 346, 482
- Drying, 10, 250
- Dynamic equilibrium, 267
- Dynamic head, 140
- Dynamic viscosity, 21, 25, 30, 113, 191
- E**
- Eddy current diffusion, 263, 278
- Eddy current diffusion coefficient, 263, 264, 475
- Effective area, 300

- Effective mass transfer area, 339, 355
 Effective power, 57, 123, 160, 491
 Effective specific surface area, 314
 Effective volume, 409, 411–413, 416, 417, 419–421, 426, 428, 429, 439, 448, 499, 508
 Efficiency, 13, 57, 121, 123–127, 129, 130, 133–135, 137, 142, 144, 145, 154, 156, 157, 160, 162–164, 227, 238, 297, 303, 305, 322, 325, 326, 388–390, 394, 395, 403, 423, 428, 429, 431, 437, 440, 508, 509
 Elastic film, 152
 Electric current, 7, 9
 Electrodialysis, 247
 Electrostatic precipitator, 249
 Elementary reaction, 368–370
 Empirical equation (Wilke-Chang), 262
 Empty tower speed, 314
 Endothermic reaction, 364
 Energy, 1, 4, 8, 9, 20, 33, 35, 47, 50–54, 57, 74, 120–123, 133, 142, 144–146, 157, 169–171, 231, 238, 247, 402, 403, 408, 437, 478, 493
 Energy balance, 50, 51
 Energy losses, 5, 33, 35, 54, 55, 64, 72, 77, 108, 109, 112, 123, 129
 Enhancement factor (reaction factor), 340
 Enthalpy of fluid, 210
 Entropy, 247
 Enzyme, 388–396, 399, 449, 476, 477, 483, 495
 Enzyme activity, 389, 395, 399, 400
 Enzyme concentration, 388, 390, 392, 476
 Equation of total absorption and mass transfer flux, 281
 Equilibrium concentration, 267, 272–276, 503
 Equilibrium constant, 313, 389
 Equilibrium curve, 267, 288, 317, 348
 Equilibrium line, 299, 318, 330
 Equilibrium separation process, 247–249
 Equilibrium solubility, 267
 Equilibrium state, 247, 250, 276, 477
 Equipment, 169
 Equivalent diameter, 26, 164, 476
 Equivalent length, 44–46, 57, 64, 66, 72, 487
 Equivalent length method, 45
 Equivalent thickness of heat transfer boundary layer, 188, 507
 Euler method / flow field method, 14–17
 Euler number, 40, 477
 Exothermic reaction, 364
 Exponential growth period, 397
 External work, 5, 52, 53, 57, 64, 247
 Extraction, 249, 250, 330, 347

F
 Fanning equation, 113
 Feed flow rate, 422
 Fick's law, 251, 279, 405
 Filling depth, 89
 Film resistance, 282, 289, 290, 293–295, 355
 Film volume mass transfer coefficient, 308, 486
 Final concentration, 421, 422
 Final conversion efficiency, 421, 423, 424, 426, 502
 Final product, 363, 368, 430, 436
 First law of thermodynamics, The, 1, 4, 5, 47, 53
 First-order dissociation constant, 341
 First order reaction, 410, 445
 Fixed volume, 414
 Flat plate heat exchanger, 232, 233
 Flow, 3, 4, 13–21, 23–25, 29–31, 33–35, 38, 40, 43, 46–49, 51, 54–56, 59, 62, 64, 71, 72, 74, 76, 77, 79, 81–85, 89, 91–95, 105–107, 109–114, 120, 121, 123, 125–127, 130–133, 135–137, 145, 146, 150, 151, 155–161, 163–165, 167, 172, 173, 178, 179, 182, 184, 185, 189, 192–194, 196, 198, 210, 215, 217, 218, 220–222, 226, 230, 231, 233, 235–240, 242, 255, 263, 265, 301, 303, 305, 308, 329, 332, 334, 339, 356, 364, 403–408, 414–417, 420, 428–430, 432, 433, 435, 437, 440, 444, 446–448, 450, 451, 471, 481, 483, 484, 487, 490, 494, 498, 508
 Flow boundary layer, 29, 188, 189, 191, 192
 Flow depth, 86
 Flow rate, 2–5, 10, 13, 17, 48, 50, 66, 69, 71, 72, 74, 76, 77, 89, 96–98, 100, 102, 122, 124, 127, 129–137, 139, 142, 145–147, 149–155, 157, 160, 172, 173, 178, 181, 183, 191, 205, 210, 222, 238, 240, 242, 243, 278, 298, 300, 306, 320, 322, 325, 326, 330, 414, 421, 443
 Flow resistances, 35, 38, 51, 54, 55, 71, 92, 200, 231

Flow velocity, 13, 18, 20, 27, 30, 43, 49,
50, 60, 62, 72, 89, 91, 92, 95, 98, 99,
109, 112, 122, 134, 160, 161, 195,
196, 200, 231, 237, 264, 314, 498,
499

Fluid, 4, 13–18, 20–22, 24, 25, 27, 29–31,
33–39, 41, 47, 48, 50–49, 51–57, 64,
69, 71, 74, 83, 95, 108, 109,
111–118, 120, 122, 135, 142, 146,
147, 157–161, 169, 170, 188, 189,
191–195, 197–201, 209, 210,
212–218, 220, 222–225, 227,
230–233, 236–239, 242, 251, 263,
264, 279, 308, 470, 477, 478, 481,
493, 494, 496–498, 500, 504, 507

Fluid continuity hypothesis, 14

Fluid conveying equipment, 120, 146, 152

Fluid flow, 13–16, 20, 21, 23, 25, 26,
29–31, 33, 35, 36, 38, 48, 51, 52, 54,
55, 57, 67, 82, 112, 113, 149, 160,
188, 189, 191, 196, 218, 227, 231,
237, 264, 479, 487, 506

Fluid transport equipment, 52, 120

Force, 7–9, 13, 16, 20, 21, 35, 36, 52, 55,
83, 84, 112, 113, 122, 159, 247, 477,
478

Forced convection, 193, 194, 197

Force difference, 36

Fouling thermal resistance, 214–216

Fourier's law, 171–173, 177

Free radical, 368, 369

Friction loss, 44, 62, 69, 72, 74

G

Gas constant, 254

Gas film absorption mass transfer
coefficient, 345

Gas film absorption mass transfer flux, 488

Gas film molar ratio difference, 485

Gas flow rate, 27, 28, 314, 320, 332, 335

Gas-liquid contact area, 305, 472

Gas-liquid equilibrium, 266, 267, 273, 274,
280, 282, 288, 305, 310, 312, 319,
355, 356

Gas-liquid mass transfer process, 248, 249

Gas mass flux, 326

Gas phase, 6, 246–250, 252, 253, 255, 256,
265–267, 269, 271–274, 276, 277,
280–283, 297, 299, 300, 308–310,
318, 319, 325, 329, 332, 337, 340,
341, 355, 437, 474, 482, 492, 500,
502, 503, 506, 507

Gas phase concentration, 319

Gas phase driving force, 299

Gas phase mass transfer coefficient, 265

Gas phase partial pressure difference, 286

Gas phase reaction, 364, 423

Gas phase velocity, 326

Gas stripping desorption, 329

Gas thermal conductivity, 176

Gas velocity, 119, 325, 326, 329

Gas velocity of empty tower, 325

Gauge pressure, 16, 17, 56, 65, 76, 78, 128,
160

General balance / macro balance, 47

Generation rate, 391

Generation time, 397, 478

Glucose effect, 399

Graphic method, 288, 321, 324, 424

Gravity acceleration, 55, 57, 116, 478

Gravity action, 52

Gravity flow, 82, 83

Gravity settlement, 115, 119

Growth curve, 396

Growth rate, 397–399, 401, 437, 496

H

Halogenation reaction, 435

Handling capacity, 426, 450

Head loss, 56, 84, 85, 92, 138, 163, 165,
482

Heat, 4–6, 9, 13, 20, 24, 25, 50, 52–54, 144,
157, 169–173, 178, 179, 181–215,
218, 219, 221–227, 230–233,
235–243, 252, 347, 470, 482, 492,
493, 496, 497, 504, 507, 514

Heat carrier, 233

Heat conduction, 169, 171–174, 177–180,
182–185, 189, 190, 194, 209, 211,
236–239

Heat conduction ability, 173

Heat conduction area, 172, 173, 471

Heat conduction boundary layer, 507

Heat conduction of cylinder wall, 182–185

Heat exchange equipment/heat exchanger,
5, 51, 52, 54, 169, 171, 183, 202,
209, 210, 212–215, 218, 220–233,
235–237, 239, 240, 242, 482, 493

Heat exchangers, 211, 224

Heat flow rate, 494

Heat flux, 173, 178, 179, 185

Heat load of heat exchanger, 514

Heat transfer, 241

Heat transfer boundary layer, 188, 189,
191, 192, 237, 239

Heat transfer coefficients, 193, 206,
213–215, 218, 222, 223, 227,
231–233, 236–238, 241–243, 504
Heat transfer equipment, 225
Heat transfer process, 7, 169–171, 180,
181, 189–192, 194, 196, 199, 209,
210, 213, 215, 223, 235, 236, 239
Heat transfer rate, 193
Height, 39, 52, 64–66, 76, 85, 107, 108,
118–121, 133, 137–140, 142, 161,
165, 167, 202, 276, 297, 300, 301,
303–308, 314–317, 319, 322, 326,
329, 330, 332, 335, 348, 408, 409,
413, 478–481, 508
Height of gas phase mass transfer unit, 479
Henry coefficient, 269, 270, 273, 345–347,
355, 356, 480
Henry's law, 269, 270, 272, 273, 276, 280,
286, 287, 289, 301, 310, 312, 313,
321, 326, 331, 336, 341, 355
Heterogeneous phase, 247
Heterogeneous reaction, 437
Homogeneous phase, 247, 407, 437
Homogeneous reactor, 408, 424, 426, 437
Horizontal push flow model, 364
Humidification, 249
Hydraulic radius, 85, 87, 88, 91, 97, 476,
495
Hydraulic slope, 84
Hydrolysis reaction, 435, 450
Hydrostatic equation, 55

I

Ideal fluid, 29, 55
Incompressible fluid, 48, 50, 54, 64
Inducer, 389
Induction period, 439
Inertia centrifugal force, 121, 122
Inertia force, 24, 40, 159, 477, 494
Inflow velocity, 30
Inhibitor, 388, 389
Initial concentration, 370–372, 374,
376–379, 381, 412, 440, 449, 450,
497
Initial rate, 376, 390, 391
Initiation step, 369
Inner diameter, 25, 31, 38, 63, 198, 199,
223, 475
Inoculation concentration, 440
Inorganic reaction, 364
Instantaneous rate, 376, 377
Instantaneous velocity field, 15

Intercept, 394
Interfacial concentration, 341
Interference settlement, 117
Intermittent reaction, 364, 435
Internal energy, 4, 52, 53, 159, 477, 498
Internal friction of fluid flow, 158
Internal friction stress, 29
Interphase mass transfer, 245, 279, 288
Interphase mass transfer process, 249, 355
Ion exchange, 112, 248
Ionic strength, 365, 393, 483
Irreversible reaction, 338, 345, 346, 363,
364
Isothermal absorption, 278, 306
Isothermal process, 54, 278, 306
Isothermal reaction, 364

J

Jacket heat exchanger, 226, 227
Jet aerator, 142, 146
Jet type, 120

K

Kettle number, 421
Kinematic viscosity, 21, 22, 191, 496, 510
Kinetic energy, 38, 52, 55, 84, 105, 122,
137, 142, 158, 159, 171, 484
Kinetic energy difference, 130
Kinetics of enzyme-catalyzed reaction, 388
Klemseel (Kremser) equation, 322

L

Lagrangian method, 14, 16, 17
Laminar bottom layer/viscous bottom
layer/flow inner layer, 188
Laminar boundary layer, 29, 30
Laminar flow region, 42, 114, 511
Laminar fluid, 263, 264
Laminar sublayer, 190, 192, 507
Lateral contraction, 165
Leaching, 249
Length, 7, 9, 26, 31, 35, 38–40, 43, 44, 47,
50, 57, 64, 66, 72, 74, 83, 85, 119,
133, 158, 166, 183, 185, 186, 194,
199, 202, 204, 206, 221, 222, 224,
229–231, 240, 241, 243, 408, 409,
416, 433, 449, 450, 487, 494, 495,
515
Lift, 57, 136, 147, 149, 156
Liquid boiling, 194
Liquid equilibrium partial pressure, 280

- Liquid film absorption and mass transfer flux, 488
- Liquid film absorption mass transfer coefficient, 345
- Liquid film control, 282
- Liquid film driving force, 289
- Liquid film resistance, 282, 289, 290, 293–295, 355
- Liquid flow, 20, 24, 122, 152, 164, 339, 340
- Liquid-gas ratio, 300–303, 323, 337, 486
- Liquid phase, 6, 245, 247–250, 252, 253, 255, 256, 265–267, 269, 270, 272–274, 276–284, 287, 295, 299, 305, 306, 308–310, 314, 320, 321, 326, 329, 332, 337, 338, 340–343, 346, 350, 351, 355, 357, 408, 437, 472–474, 489, 492, 500–503, 506, 507
- Liquid phase concentration, 280, 282, 355
- Liquid phase concentration difference, 281, 282
- Liquid phase driving force, 299
- Liquid phase mass transfer coefficient, 266
- Liquid phase reaction, 364, 407, 450
- Liquid thermal conductivity, 175
- Loading coefficient, 409, 450
- Local acceleration, 15, 19
- Local atmospheric pressure, 479
- Local loss coefficient method, 44
- Local losses, 62, 72
- Local resistance, 45, 83, 132, 133, 231, 506
- Logarithmic phase, 396
- Luminous intensity, 7, 9
- Lumped reaction, 363
- M**
- Maintenance coefficient, 402, 438, 488
- Mass balance equation, 3, 4, 48
- Mass conservation, 2, 47
- Mass constant pressure heat capacity, 195
- Mass flow, 17, 48, 57, 64, 161, 210, 222, 242, 421, 493
- Mass flow of fluid, 48, 64, 210, 493
- Mass flow rate of inert gas, 499
- Mass flow rates, 218
- Mass transfer, 245, 247, 249–251, 263, 264, 277, 279, 282, 287–290, 293, 294, 299, 305, 308–310, 322, 330, 338, 340, 341, 346–348, 355, 482, 483, 489, 496
- Mass transfer area, 301
- Mass transfer coefficient, 266, 282–286, 288, 289, 291, 294, 295, 308, 338, 340, 355, 485, 486, 496
- Mass transfer driving force, 266, 287
- Mass transfer flux, 256, 257, 265–267, 280–282, 287–289, 292, 295, 340, 341, 345, 346
- Mass transfer flux equation, 265, 279, 280, 282, 285–287, 307
- Mass transfer process, 245–250, 263, 280, 282, 341, 346, 355
- Mass transfer rate, 25, 250, 339, 481
- Mass transfer rate equation, 250
- Mass transfer resistance, 279, 282, 341
- Mass transfer separation process, 247
- Mass transfer units, 312
- Material balance, 47, 48, 297, 299, 303–307, 312, 313, 316, 320, 327, 330, 333, 408, 414, 421, 440
- Material balance equation, 48, 307, 315, 323, 326, 337, 408, 414, 418, 424, 437
- Material concentration, 404, 497
- Material particle, 404, 405, 407, 414, 419
- Material volume, 410, 412, 414
- Matrix, 401, 402, 437, 438, 441, 496, 503, 505
- Matrix concentration, 438, 441
- Matrix consumption rate, 401, 402, 496
- Matrix inhibition coefficient, 400, 482
- Maximum cell yield coefficient, 502
- Maximum specific consumption rate of matrix, 402
- Maximum specific growth rate, 399, 438, 442, 510
- Maxwell-Gilliland empirical equation, 260
- Mean driving force, 310, 311, 506
- Mean temperature difference, 217, 220, 222, 231, 236, 239
- Mechanical energy, 20, 33, 38, 53–55, 123, 479, 481, 506
- Mechanical energy balance, 53–55
- Mechanical loss, 123, 124
- Metabolic activity, 395
- Metabolite, 245, 246, 398, 399, 440–442
- Metabolite concentration, 401, 491
- Metabolite inhibition coefficient, 484
- Michaelis constant, 391, 393, 449, 483
- Michaelis-Menten equation, 391, 393, 394, 449
- Microbial yield, 503
- Migration acceleration, 15
- Minimum gas-liquid ratio, 330

Minimum liquid-gas ratio, 301, 302, 323, 486
 Molar concentration, 257, 271, 294, 295, 357, 381
 Molar flow, 322, 486, 499
 Molar gas constant, 386, 494
 Molar mass, 23, 260, 262, 271, 487
 Molar ratio, 272, 273, 283, 285, 286, 297, 320, 337, 373, 485, 500–503
 Molar ratio difference, 282, 283, 295, 485
 Molar transfer coefficient, 289
 Molar volume, 260, 262, 263, 499
 Molecular diffusion, 251, 255, 263, 278, 279
 Molecular diffusion coefficient, 252, 260, 263, 264, 475
 Molecular diffusion rate, 252
 Molecular reaction, 364
 Molecular weight, 314
 Mole fraction, 271, 272, 285, 286, 288, 289, 318, 326, 501, 503
 Momentum conservation, 47
 Momentum transfer, 112, 191, 492
 Monod equation, 399, 401, 437, 439, 440, 442, 444
 Moody's friction coefficient, 42, 43, 67, 68
 Multicomponent absorption, 278, 336, 349, 356
 Multi-layer flat wall heat conduction, 179, 180, 237
 Multiphase reactor, 408, 437
 Multi-reactor series reactor, 433
 Multistage reaction, 364

N
 Nanofiltration, 247
 Natural convection, 193–195, 197, 199, 201, 202, 238
 Newton's law of cooling, 192, 193, 212, 265, 514
 Newton's law of viscosity, 21, 35
 Non-competitive inhibition, 400
 Non-electrolyte solution, 262
 Non-isothermal absorption, 278
 Non-positive displacement pump, 155
 Non-uniform flow, 20, 82, 83
 Normal velocity gradient, 21, 476
 Normal water depth, 93
 Number of moles, 409
 Number of theoretical plate, 322, 324
 Numerical integration method, 310, 317
 Nusselt number, 195, 489

O

Open channel, 81–85, 87, 89, 91–93, 95–97, 161, 483, 515
 Open channel flow, 20, 81, 82, 84, 159, 161
 Operating temperature, 139, 140, 297, 313, 356, 493
 Operation line, 300, 330, 356
 Operation line equation, 320, 424
 Organic reaction, 364
 Output power, 60, 62, 129
 Overall heat transfer coefficient, 213, 232
 Overflow device, 23
 Oxidation reaction, 364

P

Parallel connection, 135, 136
 Parallel pipeline, 71
 Parallel reaction, 363, 369, 379, 380, 431, 432, 435, 448, 474
 Partial pressure, 163, 253–257, 267, 269, 271, 274, 280–282, 285, 286, 299, 338, 340, 341, 347, 348, 355, 356, 474, 482, 492
 Partial pressure difference, 256, 281, 282, 290, 293, 492
 Particle, 14–17, 20, 23–25, 33, 35, 41, 47, 48, 83, 95, 112–120, 137, 159, 160, 164, 188, 189, 263, 356, 364, 404, 405, 511, 513
 Pasteur effect, 399
 PH, 365, 367, 388, 390, 393, 395, 408, 483
 Phase equilibrium, 250, 274–276, 279, 285, 287, 297, 299, 301, 303, 305, 309, 312, 313, 321, 326, 330, 331, 346, 348, 356
 Phase equilibrium constant, 272–274, 337, 355, 487
 Phase equilibrium curve, 286
 Phase equilibrium relation, 286
 PH of environment, 398
 Physical absorption, 277, 338–340, 342, 350
 Physical equation, 38, 39
 Physical quantity, 7–10, 38–41, 47, 48, 56, 496
 Pipe center velocity, 18
 Pipe diameter, 31, 64, 89, 90, 159, 194, 199, 200, 450
 Pipe flow, 18, 24, 47, 71, 74, 89, 167, 404
 Pipeline characteristic curve, 131–135, 149
 Piston stroke, 151
 Plate heat exchanger, 226, 232, 233, 235–237

Positive displacement characteristic, 147
Positive displacement pump, 147, 149, 152
Potential difference, 109, 247
Potential energy, 52, 55, 56, 78, 84, 136, 159
Power, 7, 9, 10, 13, 27, 28, 39, 42, 50, 57, 60, 121–126, 129, 130, 135, 142, 155, 160, 163, 170, 185, 200, 231, 246, 248, 260, 277, 336
Power type, 120
Pre-exponential factor, 386, 472
Pressure, 9, 10, 13–16, 21, 22, 27, 33, 35, 40, 47, 52, 55–59, 62, 64, 71, 82, 89, 108, 109, 112, 114, 120, 122–124, 128, 131, 137–140, 142, 144, 145, 147, 150, 153, 154, 156–159, 161, 163, 164, 167, 173, 176, 193, 198, 227, 230, 232, 233, 242, 256, 258–261, 266, 267, 269, 272, 273, 278, 297, 318, 325, 340, 345–347, 356, 364, 365, 367, 404, 432, 433, 454, 455, 458, 459, 477, 493, 511
Pressure difference, 7, 36, 56, 109, 122, 156, 247, 255, 289, 291, 294, 329, 506
Pressurized flow, 20
Principle of dimensional consistency, 38, 40
Product, 10, 48, 124, 139, 140, 246, 277, 308, 339, 363–368, 372, 373, 376, 379, 382, 390, 391, 399, 407, 412, 418, 430–433, 435, 436, 440–443, 449, 450, 490, 495, 496, 502, 503, 505, 511
Product inhibition, 400
Production intensity, 250, 426, 430
Productivity, 401, 402
Product yield, 430, 442, 503
Proportion law of centrifugal pump, 130
Pseudo first order reaction, 368, 434
Pump cavitation, 137

Q

Qualitative temperature, 196–200, 202
Quality, 92, 216, 227, 247, 262, 407, 412
Quantity and concentration of a substance, The, 2, 372

R

Radiation reaction, 435
Radius, 31, 35, 156, 182, 183, 204, 205, 241, 495
Rate, 235, 239

Rate constant, 367–369, 372, 374, 379, 385–387, 393, 398, 450, 485
Rate-controlled separation process, 247
Rate control step, 383
Rate equation of absorption and mass transfer, 250
Reaction kinetic constant, 370, 373
Reaction order, 367, 368, 375–377, 410, 411, 416, 419, 429, 432–434, 444, 447, 448, 490
Reaction progress, 366, 476, 515
Reaction rate, 340, 364–370, 374, 376, 379, 382–386, 389–393, 395, 408, 409, 411, 416, 421, 429, 432, 437, 448–450, 490, 495, 496
Reaction rate constant, 381, 384–386, 391, 410, 445, 450, 485
Reaction time, 365, 401, 404, 407, 409–412, 416, 419, 429, 436, 445, 447, 473, 513
Reaction volume, 427, 508
Reactor, 6, 13, 152, 170, 183, 403–409, 411–438, 440–451, 499, 502
Reciprocating compressor, 146–148
Reciprocating pump, 146, 147, 149–152
Rectangular Weir, 105, 107
Regenerative heat exchanger, 225, 226
Relative roughness, 40, 42, 43, 64, 69, 70, 158, 508
Relative specificity, 389
Residence time, 403–405, 414, 419, 424
Resistance along the way / straight pipe, 14, 31, 37, 41, 42, 44, 45, 83, 158, 160, 197, 199, 201, 226, 481
Resistance loss, 54, 63, 64, 72, 84, 128, 133
Resistances, 6, 7, 13, 29–31, 33, 35, 36, 38, 39, 41, 44, 54, 67, 83, 85, 112, 115, 116, 131, 132, 135, 136, 138, 139, 159, 160, 178, 180, 181, 183–186, 188, 190–192, 204–206, 214, 215, 227, 231, 236–239, 242, 243, 282, 346, 479, 490, 506, 512
Restricted substrate, 398, 440
Restricted substrate concentration, 399, 496
Reverse osmosis, 247
Reversible reaction, 345, 363, 369, 380
Reynolds experiment, 23
Reynolds number, 24, 25, 29, 31, 35, 113, 114, 116, 117, 159, 196, 266, 494
Roots blower, 142, 144, 145
Roughness coefficient, 83, 92

S

Saturation coefficient, 399, 402, 484

- Saturation concentration, 390
 - Schmidt number, 266, 496
 - Second law of thermodynamics, The, 169, 196, 247
 - Second-order reaction, 364, 373, 374, 378, 411, 428, 429, 434, 451
 - Second-order reaction kinetic constant, 372, 373
 - Section, 229, 237
 - Selectivity, 299, 389, 430, 431, 445, 505
 - Semi-continuous reactor, 406, 433, 449
 - Semi-intermittent reaction, 364
 - Sequencing batch reactor, 406, 407
 - Series connection, 135, 136
 - Shaft power, 123–126, 129, 133–135
 - Shaft power of pump, 123, 135
 - Sharp edge weir, 104
 - Shearing stress, 21, 35, 513
 - Shell side, 231, 232
 - Simple pipeline, 63
 - Simple reaction, 363, 368, 370, 430
 - Simpson method, 317, 318
 - Simultaneous reaction, 363
 - Single acting pump, 149
 - Single component absorption, 278, 336
 - Single layer flat wall heat conduction, 514
 - Single-phase mass transfer flux equation, 280
 - Single phase medium, 245
 - Single pipe path, 242
 - Sinusoidal feature, 147
 - Sleeve type heat exchanger, 221
 - Slope, 84, 85, 92, 164, 288, 299, 301, 305, 310, 313, 320, 331, 356, 373, 376, 378, 387, 394, 424, 426, 470
 - Slope coefficient, 87, 93, 97, 100, 488
 - Smooth pipe, 43, 158, 159, 236, 467
 - Solid boundary, 112
 - Solid content of sewage, 3
 - Solid phase, 248, 249, 364
 - Solid thermal conductivity, 173, 513
 - Solubility, 267, 271, 272, 274, 282, 299, 317, 336, 337, 341, 345–348, 355, 480
 - Solubility coefficient, 271, 273, 274, 282, 289, 341, 345, 355, 356, 480
 - Solubility curve, 267–269
 - Solvent, 259, 262, 270, 271, 273, 302, 330, 336, 365, 480
 - Specific gravity of fluid, 16
 - Specific growth rate, 398–400, 438, 441–443, 510
 - Specific heat, 10, 242, 348, 454, 455, 458, 459, 462, 463, 465, 466
 - Specific matrix consumption rate, 402
 - Specific surface area, 306, 326, 472
 - Specific volume, 16, 54, 509
 - Specific volume of fluid, 16, 52, 509
 - Speed, 3, 120, 122, 124, 133, 137, 142, 151, 156
 - Spray density, 302
 - Square law, 44
 - Stable period, 396
 - Stagnant film, 264, 507
 - Stagnant-film model, 264
 - Stagnant flow/laminar flow, 24, 25, 29–31, 35–37, 47, 111–114, 116–118, 159, 160, 188–190, 192, 194, 197
 - Stagnant gas film, 279
 - Stagnant liquid film, 279
 - Static fluid, 20, 29
 - Static head, 56
 - Static pressure, 52, 124
 - Static pressure energy, 52, 55, 137, 159
 - Steady flow, 14–17, 54, 82, 161
 - Steady state, 252, 279, 280, 442, 443
 - Steady-state molecular diffusion, 252, 279, 347
 - Steady uniform flow, 82, 85
 - Stereoisomerism specificity, 389
 - Streamline, 14, 17, 19, 82, 83, 161
 - Streamline of a fluid, The, 14
 - Stroke, 147, 152
 - Submerged flow, 108
 - Substance concentration, 245, 340, 372, 377, 443
 - Substitution reaction, 431
 - Substrate concentration, 370, 388, 390–393, 398, 399, 401, 402, 440, 444, 496
 - Substrate inhibition, 400
 - Suppression step, 369
- T**
- Temperature, 5–7, 9, 10, 22, 27, 52, 60, 66, 72, 120, 139, 140, 158, 160, 161, 169–182, 185–200, 202, 204–206, 209, 210, 212, 213, 215, 217, 218, 220, 221, 223, 230, 233, 235–243, 253, 254, 258–262, 266, 267, 269–273, 278, 299, 318, 340, 341, 345–349, 356, 364, 365, 367, 378, 384–388, 390, 393–395, 404, 407, 409, 412, 414, 418, 423, 428, 429, 431, 433, 435, 450, 454, 455, 458, 459, 465, 466, 472, 476, 480, 483, 496–498, 507, 512, 513
 - Temperature coefficient, 174, 395, 472, 509
 - Termination step, 369
 - Theoretical board (theoretical level), 322

- Theoretical plates, 319
Theoretical series, 322
Thermal conductivity, 172–178, 181, 183–185, 187–189, 191–193, 195, 205, 212, 214, 223, 225, 235, 237–241, 454, 455, 458, 459, 462, 463, 465, 466, 488, 512, 513
Thermal conductivity coefficient, 192
Thermal effect, 278, 364
Thermal motion of molecules, 20
Thermal resistance of heat conduction, 515
Thermodynamic temperature, 7
Thickness of flow boundary layer, 507
Thickness of heat transfer boundary layer, 507
Thin-walled weir, 104–107, 484
Time, 2, 4, 5, 7, 9, 14–20, 31, 35, 36, 39, 40, 42, 48, 51, 53, 56, 83, 111, 114, 115, 117–119, 121, 122, 127, 136, 137, 158–160, 167, 170–173, 199–202, 213–215, 223, 231, 235, 237, 238, 241, 246, 252, 264, 266, 267, 282, 287, 301, 303, 306, 322, 325, 328, 331, 340, 356, 359, 365, 366, 370–377, 379–381, 384, 389, 395–397, 403, 405, 407–409, 411–414, 416–419, 423, 424, 429, 431, 433, 436–441, 446–451, 474, 489, 497, 513
Total absorption and mass transfer flux, 281, 300, 488
Total absorption mass transfer coefficient, 281
Total concentration, 253, 345, 392, 393, 476
Total friction loss, 47, 56, 69
Total head loss, 512
Total heat transfer coefficient, 221, 232, 238, 482
Total heat transfer rate equation, 209, 210
Total length of branch pipe, 72
Total mass transfer coefficient, 295, 355
Total mass transfer coefficient of gas phase, 346
Total molar concentration, 271
Total plate efficiency, 321
Total pressure, 74, 253, 255, 260, 267, 272, 289, 349, 355, 356
Total resistance, 288–290, 293–295
Total resistance loss, 481, 506
Total volume, 409, 413
Tower diameter, 314, 315, 325, 476
Trace, 17
Transfer rate, 171–173, 178, 190, 192–194, 206, 212, 213, 215, 218, 223, 224, 236, 243, 247, 250, 266, 514
Transfer step, 170, 369
Transition flow, 201
Transition zone, 25, 30, 31, 42, 511
Trapezoidal Weir, 105
Trapezoid method, 317
Trial and error method, 64, 66, 67, 69, 117, 326
Triangular weir, 105–107, 165, 509
Tributary, 83
Tubular heat exchanger, 216, 226, 230, 231, 237–239, 242
Turbulence, 24, 25, 30, 31, 33, 42, 117, 158, 160, 161, 188, 194, 200, 237, 264, 279, 494
Turbulent boundary layer, 29, 30, 194
Turbulent boundary layer region, 114
Turbulent flow, 30, 31, 38, 39, 41–43, 114, 162, 190, 194, 199, 201, 263, 264
Turbulent region, 25, 31, 42, 82, 511
Two-film theory, 278–280, 355
- U**
Ubbelohde viscometer, 109–111
Ultrafiltration, 247
Unidirectional diffusion, 252, 255, 257, 347, 488
Uniform flow, 20, 82–85, 91, 96, 97, 100, 102
Uniform flow in open channel, 83, 84
Unit time, 2, 17, 51, 55, 123, 152, 172, 173, 178, 186, 209, 210, 238, 250, 258, 397, 438, 493, 494, 498, 512, 514
Unit volume, 16, 55, 230, 236, 305, 308, 339, 426, 472
Unsteady flow, 15, 19, 82
- V**
Vacuum degree, 128
Van Hoff approximation law, 384
Vaporization heat, 252, 458, 459, 462, 463
Velocity, 24
Velocity distribution, 15, 27, 31, 84, 191
Velocity of flow, 55, 69, 88, 158
Venturi flowmeter, 108, 109
Viscosity, 20–24, 27, 29, 39, 43, 64, 66, 69, 111, 112, 114, 117, 118, 120, 129, 140, 153, 154, 156, 158–160, 189, 191–193, 195, 198–200, 262, 264, 299, 326, 454, 455, 458, 459, 462, 463, 465, 466, 498, 509, 510
Viscosity coefficient, 21
Viscosity of mixed liquid, 22
Viscous force, 20, 24, 40, 159, 494

Voidage, 508

Volume, 7, 16, 52, 54, 95, 96, 111, 142,
144, 145, 147, 149, 152, 154, 157,
195, 196, 227, 246, 260, 261, 308,
336, 365, 366, 396, 402, 406,
409–418, 422, 423, 426–430, 437,
441, 444, 450, 451, 454, 455, 462,
463, 494, 499, 508, 509, 515

Volume flow, 17, 18, 57, 64, 161, 325, 415,
421, 494

Volume loss, 149

Volumetric efficiency, 145, 428–430

Vortex, 35, 114, 263

Vortex pump, 155

W

Water hammer, 122, 137

Water level, 83, 108, 166

Water tongue, 104

Width, 86–88, 93–95, 97, 99, 100, 102,
104, 106, 107, 119, 164, 165, 408,
409, 472, 504

Work, 52–55, 57, 121, 127, 131, 137, 147,
154, 165, 167, 493, 499, 500

Y

Yield, 388, 419, 426, 430–435, 438, 439,
442–445, 515

Z

Zero order reaction, 364, 370, 393, 411,
416, 419, 428, 429, 444, 445, 447,
451, 473

Zero-order reaction kinetic constant, 370